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A Deconvolution Approach for Degradation Modeling With Measurement Error

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ABSTRACT Degradation trajectories over time provide information that is important for the life estimation of products and systems. However, most of the time the degradation measurements are disturbed by different conditions that cause uncertainty. This is an important problem in the area of reliability assessment based on degradation data, because the multiple observed measurements characterize the degradation path, which ends defining a failure time. Thus, in the presence of measurement error the observed failure time may be different from the true failure time. As the measurement error is inherent to the degradation testing, it results important to establish models that allow to obtain the true degradation from the observed degradation and some measurement error. In this article, a modeling approach to assess reliability under measurement error is proposed. It is considered that the true degradation is obtained by deconvoluting the observed degradation and the measurement error. We considered the inverse Gaussian and Wiener processes to describe the observed degradation of a particular case study. Then, the obtaining of the true degradation is performed by developing the proposed deconvolution method which considers that the measurement error follows a Gaussian distribution. An illustrative example is presented to implement the proposed modeling, and some important insights are provided about the reliability assessment.

INDEX TERMS Deconvolution, fast Fourier transform, inverse Gaussian process, measurement error, reliability, Wiener process.

I. INTRODUCTION

The degradation modeling has become an important tool in the area of reliability inference of highly reliable products. One of the main reasons is because as modern products and systems are developed with high quality standards the traditional reliability analysis approach based on failure times has become unsuitable, this indeed has posed the need of alternative models. On the other hand, most products and systems are expected to naturally degrade over time which causes the reliability to decrease as the degradation process evolves [1]. In general, different modeling approaches have been presented in the literature for reliability assessment based on degradation. Zio [2] identified four different approaches as: statistical models of time to failure, which are based on degradation data, stochastic degradation modeling, physics-based models and multi-state models. These four approaches consist in finding the model that describes the process of degradation, determining the failure time when the degradation accumulates to a certain critical level, characterize the failure time distribution, and finally perform the reliability assessment. Sun *et al.* [3] studied these four approaches in the analysis of high-power white LEDs. They mentioned that more work should be done in the aims of improve the performance of the models in terms of addressing different sources of uncertainty.

Specifically, degradation models based on stochastic processes have been given special attention in the last years in the literature. The gamma process is one of the first stochastic processes used in degradation modeling and has been widely used in different applications. Sun *et al.* [4] used the gamma process to model the sealing performance of rubber O'rings, they also considered the Arrhenius relationship to model the effect of temperature given that a constant stress degradation

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test was performed. Yung *et al.* [5] used a Lévy subordinator of a mixed Gamma process and compound Poisson process to describe the degradation of process of high-power LEDs. Sun *et al.* [6] used the gamma process to characterize the performance degradation of solar cells considering the general stress-strength interference model. On the other hand, lately, two of the most used degradation models are based on the Inverse Gaussian (IG) and Wiener processes. The IG process has received great attention in the last years given its characteristic that it is a limiting compound Poisson process, which justifies its use in degradation process that are originated by multiple external shocks. In addition, this process produces independent monotone paths and it is flexible to the incorporation of random effects. Such characteristics have been thoroughly studied in the next studies [7]–[12]. The Wiener process, on the other hand, produces independent non-monotone degradation paths and has been also thoroughly explored in the literature [13]–[15]. Specially, for these characteristics the Wiener process has been used in different applications. Kong *et al.* [16] studied a two phase degradation process with change points based on the Wiener process. Jin and Matthews [17] studied reliability demonstration plans with small samples for Rubidium Atomic Frequency Standard by using the Wiener process. Cui *et al.* [18] developed a degradation model based on the Wiener process by considering periodical calibrations.

In general, there are several difficulties for obtaining lifetime estimations based on degradation data. One of the main, is related to the uncertainty that arises in the measuring process of the degradation. For instance, the probabilistic behavior of the degradation increment over time is well modeled by the stochastic process that governs the degradation process. However, an independent process describes the measurement error (ME). Then, the process of the measured degradation is an additive function of the process of the true degradation and the independent ME. Thus, if the reliability assessment is performed by only considering the measured degradation without any consideration about the ME, then the assessment may be inaccurate. In such cases, it is of interest to find the process that describes the true degradation as a function of the observed degradation and the ME. In this way, it would be possible to assess the reliability accurately without error contamination.

The Wiener process is the stochastic process that has received more attention when dealing with ME. Whitmore [19] presented a model considering that the true degradation is governed by a Wiener diffusion process. It is assumed that the expected value of the measured degradation equals the expected value of the true degradation plus the expected value of the measurement error, from there, the modeling is developed. Several approaches have been introduced considering different sources of variation as mixed effects along with the ME modeling in the Wiener process [20]–[25]. Li *et al.* [26] explored the Wiener process model with ME under accelerated conditions. Zhai and Ye [27] investigated the Wiener degradation model with ME considering that the errors follow a t-distribution. While, other studies have considered the same modeling approach, but focusing in estimating the remaining useful life [28], [29].

The Wiener process is a non-monotone process, which means that it may have some negative increments. Thus, this process may not be adequate for many degradation processes. In this sense, the IG process overcomes this drawback given that it is a monotone process. In addition, the IG process with ME has not been thoroughly studied in the literature. Zhang *et al.* [30] considered an IG process to describe the observed depth-growth of corrosion defects on pipelines. They used a linear model to describe the true depth-growth by incorporating constant and non-constant biases and the ME. Although, in their study, they consider that the biases and standard deviation of the ME as deterministic quantities. Thus, more research should be intended in this specific area of degradation modeling. In this study, we propose to use deconvolution to deal with the IG process with ME. Several studies have been presented in the literature considering the deconvolution to deal with ME. The proposed models normally consider a specific form of the distribution of the ME. Then, the distribution of a true measurement is obtained by deconvoluting the characteristic functions of the distribution of the observed measurement and a kernel of the error distribution [31]–[35]. Although, these models do not involve a stochastic modeling as is the case when dealing with degradation processes, which is one of the contributions of this article.

A common assumption when dealing with degradation processes with ME is that the ME follows a Gaussian distribution with $\mu = 0$ and σ. This assumption is adequate as can be noted in several reported works [9], [20], [21], [28], [29]. In addition, there are several studies in the literature in which Gaussian ME is incorporated in a deconvolution process to obtain the true measurements [36]–[38]. In this paper, it is considered that the distribution of the ME is known to be Gaussian and that the Wiener and IG processes govern the observed degradation. As the process of the true degradation is of interest, an approximation is proposed by considering the Fourier transform (FT) of the characteristic function (CF) of the probability density function (PDF) of the true degradation, which is obtained by dividing the CF of the observed degradation distribution and the CF of the ME Gaussian distribution, this procedure is known as deconvolution. Indeed, several works have been reported in the literature where non-Gaussian ME are proposed. Li *et al.* [39] and Li *et al.* [40] considered to model the ME with autoregressive models in the Wiener process. Shen *et al.* [41] considered the logistic distribution to describe the ME by also considering the Wiener process. Whereas, Giorgo *et al.* [42] proposed a perturbed gamma process considering that the ME is a non Gaussian random variable that depends stochastically on the actual degradation level.

As the deconvoluted function results in a quite complex form, the fast Fourier transform (FFT) is implemented in the discrete form of the FT. A code in the R software is developed to obtain the true degradation considering the proposed scheme. The proposed approach is implemented in a simulation study in order to assess the effect of the ME over the true degradation. In addition, the modeling is applied to a case study dataset to obtain the true degradation under an estimated value of the standard deviation of the ME. The reliability assessment is carried out by considering the pseudo-failure times of the true degradation for the IG process, given that the true degradation function have a complex form. In the case of the Wiener process, the reliability assessment is provided with the first-passage time distributions. From both cases, it is observed that the reliability assessment of the product is miss-estimated if the ME is not considered in the modeling. The deconvolution approach is considered given that the true measurements in degradation analysis are not observed directly, which means that they represent a hidden state of the degradation process. Instead, the observed degradation is the only known state of the degradation process. Because of this, in most of the degradation process analysis it is considered that the observed degradation is governed by a specific stochastic process. Then, the hidden true degradation may be obtained as an approximation given that the observed degradation is modeled by a certain model and the ME is known to be described by a specific distribution.

The rest of the paper is organized as follows, in Section 2 the deconvolution approach to obtain the true degradation from the observed degradation and the ME is described. In Section 3, we present the IG and Wiener processes to describe the observed degradation. In addition, the deconvolution for these two processes is introduced. In Section 4, the estimation of parameters of the observed degradation and the ME is discussed. In Section 5, a simulation study is performed to visualize the effect of the ME over the true degradation. In Section 6, an illustrative example is presented which consists in the application of the proposed modeling to a fatigue crack growth dataset, the reliability assessment of the product is provided and some important insights are denoted. Finally, in Section 7, the concluding remarks are provided.

II. A DECONVOLUTION APPROACH TO OBTAIN TRUE MEASUREMENTS

Considering a degradation process as a stochastic process, the degradation path over time can be modeled as $\{Z(t); t > 0\}$. Thus, $\Delta Z_i\left(t_j\right) = Z_i\left(t_j\right) - Z_i\left(t_{j-1}\right)$ for $i = 1, \ldots, n$ units at corresponding times t_j for $j = 1, \ldots, m$. In this paper, it is considered that the observed degradation $Z_i\left(t_j\right)$ is contaminated with ME, such that for any i and t_j the observed degradation is denoted as $Z_i\left(t_j\right) = S_i\left(t_j\right) + \varepsilon$, where $S_i\left(t_j\right)$ represents the true degradation, and ε the ME, observed at each $Z_i\left(t_j\right)$ for the $i = 1, \ldots, n$ units at corresponding times t_j for $j = 1, \ldots, m$. When the ME's PDF is known and the observed degradation is assumed to be governed by a specific stochastic process. Then, the true degradation may be obtained via deconvolution at every $j = 1, \ldots, m$. In this Section, we present general information about the deconvolution, while in Section IIIA the proposed deconvolution approach is developed.

Considering the convolution as $g * h = u$, where $(*)$ represents the convolution operator and u represents the convoluted function. Let u be the PDF of some observed measurement $\left(Z_i\left(t_j\right)\right)$, g is the PDF of the true measurement $\left(S_i\left(t_j\right)\right)$, and h is the PDF that describes a known ME (ε). Then, if the functions and parameters of u and h are known, the deconvolution can be performed by obtaining the CF of u and h and applying the deconvolution as $\varphi_G(\zeta) = \varphi_U(\zeta)/\varphi_H(\zeta)$, where $\varphi_G(\zeta), \varphi_H(\zeta)$, and $\varphi_U(\zeta)$ are the respective CF. In general, for any random variable X with distribution $f(X)$ the CF function is defined by $\varphi_X(\zeta) = E[exp\{i\zeta X\}]$, where ζ is real and $i = \sqrt{-1}$. Then, $\varphi_X(\zeta)$ is defined on a real line as a function of ζ, which provides and alternative approach to obtain characteristics of f and perform analysis of the function f when this is complex.

Finally, the deconvoluted PDF of the true measurement g can be obtained by computing the inverse Fourier transform (IFT) of φ_G. Thus, in first instance the CF of the true measurement PDF φ_G is obtained as in (1).

$$\varphi_G(\zeta) = \frac{\varphi_U(\zeta)}{\varphi_H(\zeta)}. \tag{1}$$

Then, as mentioned earlier the PDF of g is obtained by computing the IFT of (1), as denoted in (2).

$$f(g) = \int_{-\infty}^{\infty} \varphi_G(\zeta) \times exp\{-i\zeta g\} d\zeta. \tag{2}$$

It can be noted that the expressed integral may results in a quite complex form. However, some methods can be used to obtain an approximation of the PDF of g. In this article, the fast Fourier transform (FFT) is considered as an approximation method to obtain the PDF of the true degradation. The FFT is an algorithm to compute the discrete Fourier transform (DFT), and allows an efficient calculation of the DFT coefficients of a periodic function sampled on a regular grid of 2^p points. The DFT considers that the FT can be obtained as a Riemann sum approximation. We consider P equally spaced sub-intervals ranging from a regular grid of $[D_0, D_1]$, where, the lower interval is $D_0 = 0$, representing the minimum true value. The upper interval is defined as $D_1 = \mu_\varepsilon + 5\sigma + q$, which represents the maximum true value. In this case, we consider the maximum true value as the sum of the maximum observed measurement and the maximum value of the ME. For this, we consider the 99.999th quantile of the assumed distribution for the observed degradation defined as q and $\mu_\varepsilon + 5\sigma$ for the ME, where $(\mu_\varepsilon, \sigma)$ are the parameters of the ME distribution.

In this way, the approximation of (2) is obtained in (3).

$$\begin{aligned} f(g) &\simeq \int_{D_0}^{D_1} \varphi_G(\zeta) \, exp\{-i\zeta g\} d\zeta, \\ &\simeq \frac{2D_1}{P} \sum_{r=0}^{P-1} \varphi_G\left(\frac{2D_1}{P}(r-1) - D_0\right) \\ &\quad \times exp\left\{-i\zeta g\left(\frac{2D_1}{P}(r-1)\right) - D_0\right\}, \end{aligned} \tag{3}$$

where, the P equally spaced sub-intervals are considered to have a width of $2D_1/P$. In the literature, it has been suggested that the number of sub-intervals should greater or equal to the sample size of the function to be deconvoluted. Indeed, the greater P is, the closer the deconvoluted function will be to the true function [43]. For the implementation of the FFT in (3) we considered the "NormalGamma" package [44] from R. The source code of the "dnormgam" function, which is oriented to convolution problems was modified in order to implement the proposed deconvolution approach in (3). The implementation of the FFT in the R software for a sampled vector $k = 0, 1, \ldots, P-1$ is defined as [38],

$$W[k] = \frac{D_1}{P\pi} exp\{i\pi(k-1)\} \sum_{r=1}^{P} \varphi_G\left(\frac{2D_1}{P}(r-1)\right) \times exp\left\{\frac{2i\pi}{P}(r-1)(k-1)\right\}. \quad (4)$$

Thus, $W[k]$ is a vector of observations free of error. With $W[k]$, degradation paths can be constructed from the observed degradation in the aims to assess the reliability based on the true degradation.

III. INVERSE GAUSSIAN AND WIENER STOCHASTIC DEGRADATION MODELS

For the implementation of the previously discussed deconvolution approach, it is considered that observed degradation is governed by two widely used stochastic processes in degradation modeling. We consider the IG and Wiener stochastic processes. We first discuss the details of the IG process. It is considered that $\{Z(t), t \geq 0\}$ is governed by an IG process with the following properties: the increment $Z(t+\Delta t) - Z(t)$ follows an IG distribution with parameters μ and λ, $Z(t)$ has independent increments, $Z(t_4) - Z(t_3)$, and $Z(t_2) - Z(t_1)$ are independent $\forall\, t_1 < t_2 < t_3 < t_4$. Thus, the PDF of $Z(t)$ is given as,

$$f_{IG}(Z(t)) = \sqrt{\frac{\lambda t^2}{2\pi Z^3(t)}} \times exp\left\{-\frac{\lambda(Z(t)-\mu t)^2}{2\mu^2 Z(t)}\right\}. \quad (5)$$

An important aspect to consider when dealing with the reliability assessment of products is the first-passage time distributions. Such are obtained when the degradation paths reach a critical degradation level ω. Then, the lifetime T_ω of the product is defined as $T_\omega = inf\{Z(t) \geq \omega\}$. Given the monotonicity property of the IG process, it is well known that the cumulative distribution function (CDF) of T_ω can be obtained as $P(Z(t) \geq \omega) = 1 - F\left(\omega, \mu t, \lambda t^2\right)$. Thus, the reliability function is obtained as follows

$$P(Z(t) < \omega) = \Phi\left[\sqrt{\frac{\lambda}{\omega}}\left(\frac{\omega}{\mu} - t\right)\right] + exp\left\{\frac{2\lambda t}{\mu}\right\} \times \Phi\left[-\sqrt{\frac{\lambda}{\omega}}\left(\frac{\omega}{\mu} + t\right)\right], \quad (6)$$

where Φ denotes the standard Gaussian CDF.

Considering a degradation test (DT) where n units are tested and m measurements for all the units are observed up to the termination time T, which results in degradation measurements $Z_i\left(t_j\right)$ of the ith unit at the corresponding time t_j, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, and according to the independent increment property of the IG process, and $\Delta Z_i\left(t_j\right) = Z_i\left(t_j\right) - Z_i\left(t_{(j-1)}\right)$, $t_0 = 0$, $\Delta t_j = t_j - t_{j-1}$, for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. Thus, it is possible to obtain independent random variables $\Delta Z_i(t_j) \sim IG\left(\mu \Delta t_j, \lambda \Delta t_j^2\right)$.

The Wiener process is also considered for the implementation of the deconvolution approach, given that this process has been widely used in degradation modeling. In addition, this process has been widely studied when dealing with ME, as discussed in Section I. Thus we consider this process in the aims of establishing a point of comparison with the proposed approach. The theoretical foundation for the use of this process is based on the fact the often a gradual drift of the mean value of degradation characterizes most of the degradation processes. Thus, the selection of a stochastic process can be based on the assumption of an additive accumulation of degradation with constant wear intensity. Considering every degradation increment as an additive superposition of a large number of small effects, then it is possible to assume normality for the degradation process [45]. The general function of the Wiener processes with drift is described as,

$$Z(t) = Z(0) + \alpha t + \beta B(t), \quad (7)$$

where α is a drift parameter, β is a diffusion parameter, $Z(0)$ is the initial level of degradation, which in this paper is considered as $Z(0) = 0$, and $B(t)$ is the standard Wiener process, with the next characteristics: $B(0) = 0$, $B(t)$ has independent and stationary increments $B(t) - B(s) = \Delta B(t)$ with a Gaussian distribution $\Delta B(t) \sim G(0, \sqrt{\Delta t})$.

The moment of a failure caused by degradation is the moment when the degradation path reaches a critical level ω. Thus the lifetime is defined as $T_\omega = inf\{Z(t) \geq \omega\}$. It is well known that T_ω follows an IG distribution, with reliability function defined as,

$$P(Z(t) < \omega) = 1 - \Phi\left[\sqrt{\frac{1}{\beta^2 t}}(\alpha t - \omega)\right] + exp\left\{\frac{2\alpha\omega}{\beta^2}\right\} \times \Phi\left[-\sqrt{\frac{1}{\beta^2 t}}(\alpha t + \omega)\right]. \quad (8)$$

By also considering a DT where degradation measurements $Z_i\left(t_j\right)$ of the ith unit at the corresponding time t_j are obtained. And, according to the independent property of the Wiener process, and $\Delta Z_i\left(t_j\right) = Z_i\left(t_j\right) - Z_i\left(t_{(j-1)}\right)$, $t_0 = 0$, $\Delta t_j = t_j - t_{j-1}$, for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. Thus, it is possible to obtain independent random variables $\Delta Z_i(t_j) \sim G\left(\alpha \Delta t_j, \beta\sqrt{\Delta t_j}\right)$, with the next PDF and CDF.

$$f_W\left(\Delta Z_i\left(t_j\right)\right) = \frac{1}{\sqrt{2\pi\Delta t_j}\beta} exp\left\{-\frac{\left(\Delta Z_i\left(t_j\right)-\alpha\Delta t_j\right)}{2\beta^2\Delta t_j}\right\}. \tag{9}$$

$$F\left(\Delta Z_i\left(t_j\right)\right) = \Phi\left\{\frac{\Delta Z_i\left(t_j\right)-\alpha\Delta t_j}{\beta\sqrt{\Delta t_j}}\right\}. \tag{10}$$

A. DECONVOLUTION FOR INVERSE GAUSSIAN AND WIENER PROCESSES

In this paper, it is considered that the ME follows a Gaussian distribution with $(\mu_\varepsilon = 0, \sigma)$, and is assumed to be independent of the cross time. Considering that degradation measurements have been observed for $Z_i(t_j)$ of the *ith* unit at the corresponding time t_j, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. Then, ε is also observed at t_j for each $Z_i\left(t_j\right)$. We denote $\boldsymbol{Z_j} = Z_i(t_j)$, where $\boldsymbol{Z_j} = \left\{Z_1(t_j), Z_2(t_j), \ldots, Z_n(t_j)\right\}$, for $j = 1, 2, \ldots, m$, contains the degradation measurements for all $i = 1, 2, \ldots, n$ at j. Then, $\boldsymbol{\Delta Z_j} = \boldsymbol{Z_j} - \boldsymbol{Z_{j-1}}$, represents a vector of degradation increments for all $i = 1, 2, \ldots, n$ at j, as

$$\boldsymbol{\Delta Z_j} = \left\{\left(Z_1\left(t_j\right) - Z_1\left(t_{j-1}\right)\right), \left(Z_2\left(t_j\right) - Z_2\left(t_{j-1}\right)\right), \ldots, \left(Z_n\left(t_j\right) - Z_n\left(t_{j-1}\right)\right)\right\},$$

denoted as $\boldsymbol{\Delta Z_j} = \left\{\Delta Z_1\left(t_j\right), \Delta Z_2\left(t_j\right), \ldots, \Delta Z_n\left(t_j\right)\right\}$ for $j = 1, 2, \ldots, m$. Thus, for the degradation measurements it follows that $\boldsymbol{Z_j} \sim IG\left(\mu_j, \lambda_j\right)$, where $\mu_j = \mu t_j$, and $\lambda_j = \lambda t_j^2$, for the IG process. And, $\boldsymbol{Z_j} \sim G\left(\alpha_j, \beta_j\right)$, where, $\alpha_j = \alpha t_j$, and $\beta_j = \beta\sqrt{t_j}$, for the Wiener process. The deconvolution approach described in Section 2 is implemented in $IG\left(\mu_j, \lambda_j\right)$ and $G\left(\alpha_j, \beta_j\right)$ for $j = 1, 2, \ldots, m$, considering $G\left(\mu_\varepsilon = 0, \sigma\right)$ to obtain the true degradation.

We first consider the CF of the Gaussian ME $\varphi_\varepsilon(\zeta) = E\left[exp\left\{i\zeta\varepsilon\right\}\right]$, which is constant for every t_j of the specified stochastic process. The CF for the Gaussian distribution is well known and it is presented in (11) to implement the deconvolution approach.

$$\varphi_\varepsilon(\zeta) = exp\left\{i\mu_\varepsilon\zeta - \sigma^2\zeta^2/2\right\}. \tag{11}$$

In addition, it is considered that the observed degradation at t_j is governed by an IG process and a Wiener process. Thus, the CF for the observed degradation based on these two processes at t_j can be obtained as $\varphi_{Z_j}(\zeta) = E\left[exp\left\{i\zeta Z_j\right\}\right]$, by considering that $\boldsymbol{Z_j} \sim IG\left(\mu_j, \lambda_j\right)$ and $\boldsymbol{Z_j} \sim G\left(\alpha_j, \beta_j\right)$, for the IG and Wiener processes respectively. Then, the CFs can be obtained for each $t_j, j = 1, 2, \ldots, m$ as,

$$\varphi_{\boldsymbol{Z_j}}(\zeta) = \int_{-\infty}^{\infty} exp\left\{i\zeta\boldsymbol{Z_j}\right\} f\left(\boldsymbol{Z_j}\right) dZ_j, \quad j = 1, 2, \ldots, m.$$

The PDF $f\left(\boldsymbol{Z_j}\right)$ can be considered as $IG\left(\mu_j, \lambda_j\right)$ or $G\left(\alpha_j, \beta_j\right)$. Fortunately, these CFs have closed form, these are presented in (12) and (13), respectively.

$$\varphi_{\boldsymbol{Z_j}}^{(1)}(\zeta) = exp\left\{\frac{\lambda_j}{\mu_j}\left(1 - \sqrt{1 - \frac{2\mu_j^2 i\zeta}{\lambda_j}}\right)\right\}, \quad j = 1, 2, \ldots, m. \tag{12}$$

$$\varphi_{\boldsymbol{Z_j}}^{(2)}(\zeta) = exp\left\{i\alpha_j\zeta - \beta_j^2\zeta^2/2\right\}, \quad j = 1, 2, \ldots, m. \tag{13}$$

Thus, if an IG process governs the observed degradation, and by considering the Gaussian ME, the CF of the true degradation would be defined as

$$\varphi_{\boldsymbol{S_j}}^{(1)}(\zeta) = exp\left\{\frac{\lambda_j}{\mu_j}\left(1 - \sqrt{1 - \frac{2\mu_j^2 i\zeta}{\lambda_j}}\right) - i\mu_\varepsilon\zeta - \sigma^2\zeta^2/2\right\}, \quad j = 1, 2, \ldots, m. \tag{14}$$

If the Wiener process is considered to govern the observed degradation, then the CF of the true degradation would be obtained as

$$\varphi_{\boldsymbol{S_j}}^{(2)}(\zeta) = \frac{exp\left\{i\alpha_j\zeta - \beta_j^2\zeta^2/2\right\}}{exp\left\{i\mu_\varepsilon\zeta - \sigma^2\zeta^2/2\right\}},$$
$$\varphi_{\boldsymbol{S_j}}^{(2)}(\zeta) = exp\left\{i\zeta\left(\alpha_j - \mu_\varepsilon\right) - \zeta^2/2\left(\beta_j^2 - \sigma^2\right)\right\}, \quad j = 1, 2, \ldots, m. \tag{15}$$

It is well known that the IFT from (15) returns the Gaussian distribution. Thus, when the observed degradation is known to be governed by a Wiener process and the ME is Gaussian, the true degradation is described as $\boldsymbol{S_j} \sim G\left(\left(\alpha_j - \mu_\varepsilon\right), \left(\beta_j - \sigma\right)\right)$ for $j = 1, 2, \ldots, m$. However, the exact solution of (14) for the true IG degradation can not be directly found as in the case of (15). Thus, an approximation of the PDF for the true degradation may be obtained by computing the IFT via FFT of $\varphi_{\boldsymbol{S_j}}^{(1)}$, this by considering (3) as denoted in Section 2.

$$f\left(s_j\right) \simeq \int_{-D_0}^{D_0} \varphi_{\boldsymbol{S_j}}^{(1)}(\zeta)\, exp\left\{-i\zeta s_j\right\} d\zeta,$$
$$\simeq \frac{2D_0}{P}\sum_{r=0}^{P-1}\varphi_{\boldsymbol{S_j}}^{(1)}\left(\frac{2D_0}{P}(r-1) - D_0\right) \times exp\left\{-i\zeta s_j\left(\frac{2D_0}{P}(r-1)\right) - D_0\right\}.$$

The previously proposed approach for the IG and Wiener processes is illustrated in Fig 1. In general, the proposed deconvolution approach can be implemented considering the next steps:

1) Estimate the parameters of the observed degradation and the parameters of the assumed distribution for the ME (this step is discussed in the next section).
2) Define the CF at t_j for the observed degradation based on the assumed stochastic process, (12) and (13) for the IG and Wiener processes, respectively.

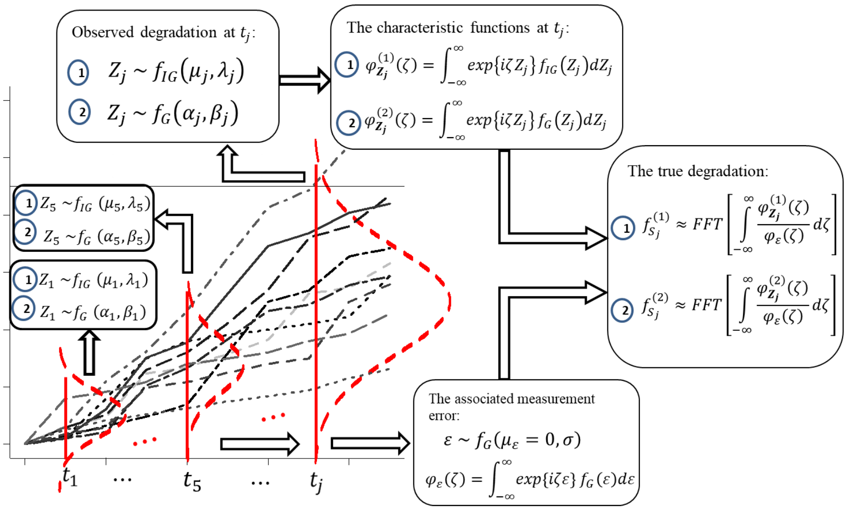

FIGURE 1. Illustration of the proposed deconvolution approach.

3) Obtain the CF of the true degradation based on the ratio of the CF of the observed degradation and the CF of the ME, (14) and (15) for the IG and Wiener processes, respectively.
4) Apply the FFT to the CF of the true degradation to obtain the true PDFs at t_j, consider the general form presented in (3).
5) Characterize the true degradation paths based on the true PDFs at t_j.
6) Perform the reliability assessment based on the true first-passage time distribution.

IV. ESTIMATION OF PARAMETERS FOR THE OBSERVED DEGRADATION AND THE MEASUREMENT ERROR

In this Section, we discuss the estimation of parameters that are needed to perform the previously presented deconvolution approach. As described in Section II, the first step to implement the deconvolution approach consists in knowing the functions and parameters of the observed degradation and the ME. The proposed approach considers that $\boldsymbol{Z_j}$ follows either an IG process or a Wiener process, and that the ME follows a Gaussian distribution as $G(\mu_\epsilon = 0, \sigma)$. Thus, for any of the two process for the observed degradation, there are two sets of parameters as $\theta_1 = (\mu, \lambda)$, and $\theta_2 = (\alpha, \beta)$ that need to be estimated, respectively. It should be noted that for the ME only one parameter is to be estimated (σ), and this can be estimated independently of θ_1 and θ_2.

We first discuss an alternative to obtain $\hat{\sigma}$. Given that σ represents the standard deviation of the ME, and that this variation is due to the measurement system, an alternative to estimate σ may consists in performing a repeatability and reproducibility (R&R) study to the measurement system. This method allows to identify how much of the measurement system variability is due to three sources of variation known as variance components, which are the variation of the part, variation due to the measurement device (also known as repeatability), and the variation due to the operator (also known as reproducibility). Although, this method consider an analytic scheme to improve the performance of the measurement system, the main objective for this paper is only focused in obtaining $\hat{\sigma}$. Thus, a result of performing a R&R study consists in estimating the previously discussed variance components, from these components it is known that $\hat{\sigma}_{R\&R} = \hat{\sigma}_{Repet} + \hat{\sigma}_{Reprod}$. Where, $\hat{\sigma}_{R\&R}$ captures the variation due to both the measurement device $\left(\hat{\sigma}_{Repet}\right)$ and the operator $\left(\hat{\sigma}_{Reprod}\right)$. Both of these variance components are independent of the part variation, which is governed by a specific stochastic process. Thus, $\hat{\sigma}_{R\&R}$ may be considered as an estimate of σ, such as $\hat{\sigma} = \hat{\sigma}_{R\&R}$. More details about R&R and measurement system analysis can be found in [46]–[48]. Once $\hat{\sigma}$ is obtained, the parameters of the specific stochastic processes need to be estimated.

On the other hand, the estimations of $\hat{\theta}_1$ and $\hat{\theta}_2$ can be obtained considering that $\boldsymbol{Z_j}$ degradation measurements have been observed for the ith unit at the corresponding time t_j with $i = 1, \ldots, n$ and $j = 1, \ldots, m$. It is considered that $\boldsymbol{Z_j}$ are independent measurements that are governed, in first instance by an IG process $IG\left(\mu_j, \lambda_j\right)$ and a Wiener process as $G\left(\alpha_j, \beta_j\right)$. The parameters of interest θ_1, and θ_2 can be estimated through direct optimization of the respective log-likelihood function, which are obtained from (5) and (9), for the IG and Wiener process, as,

$$l(\theta_1) = \sum_{i=1}^{n} \left(\prod_{j=1}^{m} ln\left(f_{IG}\left(\boldsymbol{Z_j}\right)\right) \right) \tag{16}$$

$$l(\theta_2) = \sum_{i=1}^{n} \left(\prod_{j=1}^{m} ln\left(f_{W}\left(\boldsymbol{Z_j}\right)\right) \right) \tag{17}$$

As discussed by [9] and [49] direct maximization of the functions presented in (16) and (17) for both the IG and the Wiener process is consistent and satisfactory, and can be used as maximum likelihood estimators as $\hat{\theta}_1$ and $\hat{\theta}_2$. Thus, for the observed degradation, both functions are solved for θ_1 and θ_2 using the *optim* routine in the *R* software. By obtaining $\hat{\theta}_1, \hat{\theta}_2$, and $\hat{\sigma}$, as previously discussed, the deconvolution approach can be directly implemented. We discuss such implementation in detail in the next sections.

V. SIMULATION STUDY

A simulation study is considered to obtain the true degradation $\boldsymbol{S_j}$ of simulated data considering the approximation described in (14) and (15) for both the IG and Wiener processes. In first instance, an IG process is used to describe different realizations of the observed degradation $\boldsymbol{Z_j}$ by considering (5) with parameters $\mu = 1.3, \lambda = 1.9$. Thus, we considered $n = 10$ degradation paths, with $m = 10$ observation times which are the same for all the $i = 1, 2, \ldots, 10$ as $t_j = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$. In addition, the parameters of the Wiener process that describe the observed degradation are considered as $\alpha = 1.3, \beta = 1.15$. We considered the same characteristics for the peudo-paths as, $n = 10$ degradation paths, with $m = 10$ observation times which are the same for all the $i = 1, 2, \ldots, 10$ as $t_j = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$. The simulated paths for the IG and Wiener processes are presented in Fig. 2. A common assumption when dealing with ME is that ε follows a Gaussian distribution with $\mu_\varepsilon = 0$ and σ. Such an approach is considered in this section, such that a Gaussian distribution with $\mu = 0$, and different values of σ as $(0.1, 0.3, 0.6)$ are considered in order to study the effect of the ME on the true degradation.

Considering that $\boldsymbol{Z_j} = Z_i(t_j)$, where $\boldsymbol{Z_j}$ contains the degradation measurements for all $i = 1, 2, \ldots, n$ at j, then $\boldsymbol{Z_j} \sim IG\left(\mu_j, \lambda_j\right)$, and $\boldsymbol{Z_j} \sim G\left(\alpha_j, \beta_j\right)$, for the IG and Wiener processes, respectively. For the implementation of the deconvolution approach we estimated the percentile q_j from $IG\left(\mu_j, \lambda_j\right)$, and $G\left(\alpha_j, \beta_j\right)$ for $j = 1, 2, \ldots, 10$ and considered $\mu = 0$, and $\sigma = (0.1, 0.3, 0.6)$ for every j. Then, the approximation described in (4) was implemented in R software [50] with $P = 600$ to obtain $f\left(S_j\right)$ for $j = 1, 2, \ldots, 10$. Based on $f\left(S_j\right), j = 1, 2, \ldots, 10$ we constructed pseudo-degradation paths, which are free of ME and considered as the true degradation paths. The respective simulated observed and true degradation paths are compared via box-plots for the different scenarios of σ_i at $j = 1, 2, \ldots, 10$ in Fig. 3.

In Fig 4, we also compare at every t_j the mean degradation and standard deviation of the observed degradation and the three proposed scenarios with ME. Specifically, in Fig. 4a it can be noted that the mean degradation of the IG process with $\sigma = 0.1$ approaches the most to the observed degradation. Whereas, from t_4 to t_9 the mean degradation with $\sigma = 0.3$ and $\sigma = 0.6$ separates the most from the observed degradation. In Fig. 4b, It can also be noted that from t_5 the standard deviation for the observed degradation tends to separate from all the scenarios with ME. For the mean degradation of the Wiener processes in Fig. 4c, it can be noted that the mean degradation with the highest ME, i.e. $\sigma = 0.6$, separates the most from the other mean trajectories from t_7. It can also be noted from Fig. 4d, that the standard deviation from the observed degradation is higher than the ME scenarios in almost all t_j. Such results imply that, if the reliability assessment of the product under study is performed by considering the observed degradation without the ME, it may be inaccurate.

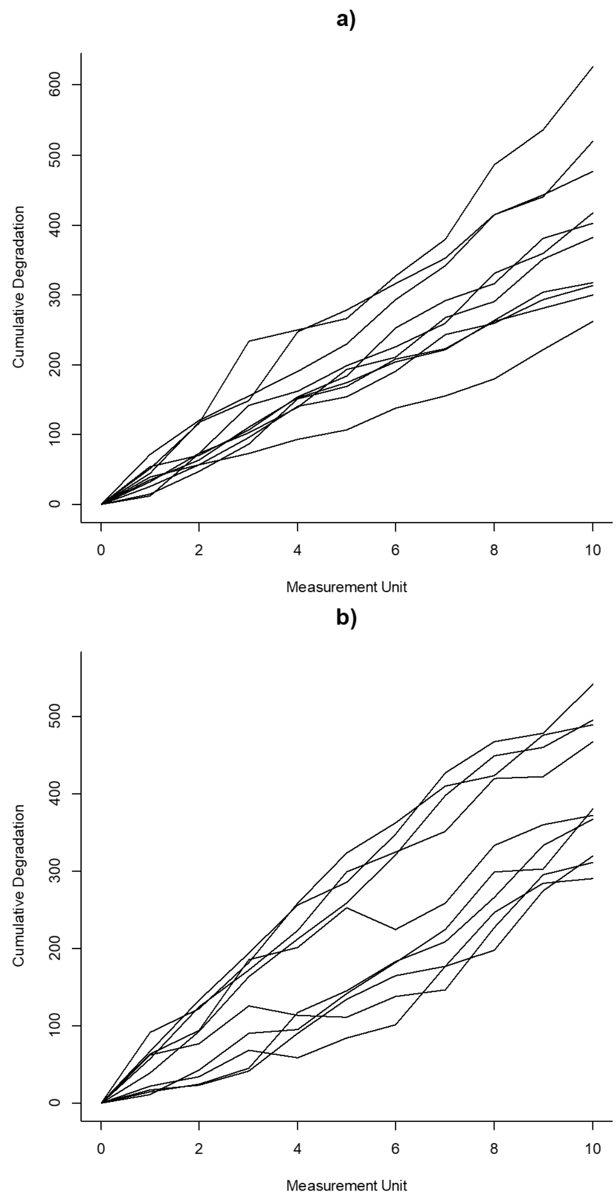

FIGURE 2. Simulated observed pseudo-degradation paths. a) IG process, b) Wiener process.

VI. ILLUSTRATIVE EXAMPLE

In this section, a fatigue crack growth dataset [51], that has been identified as a case with imperfect degradation measurements, is used to illustrate the degradation modeling with ME based on deconvolution presented in previous sections. The degradation dataset describe the crack propagation over time for 10 devices. In this paper, it is considered that when

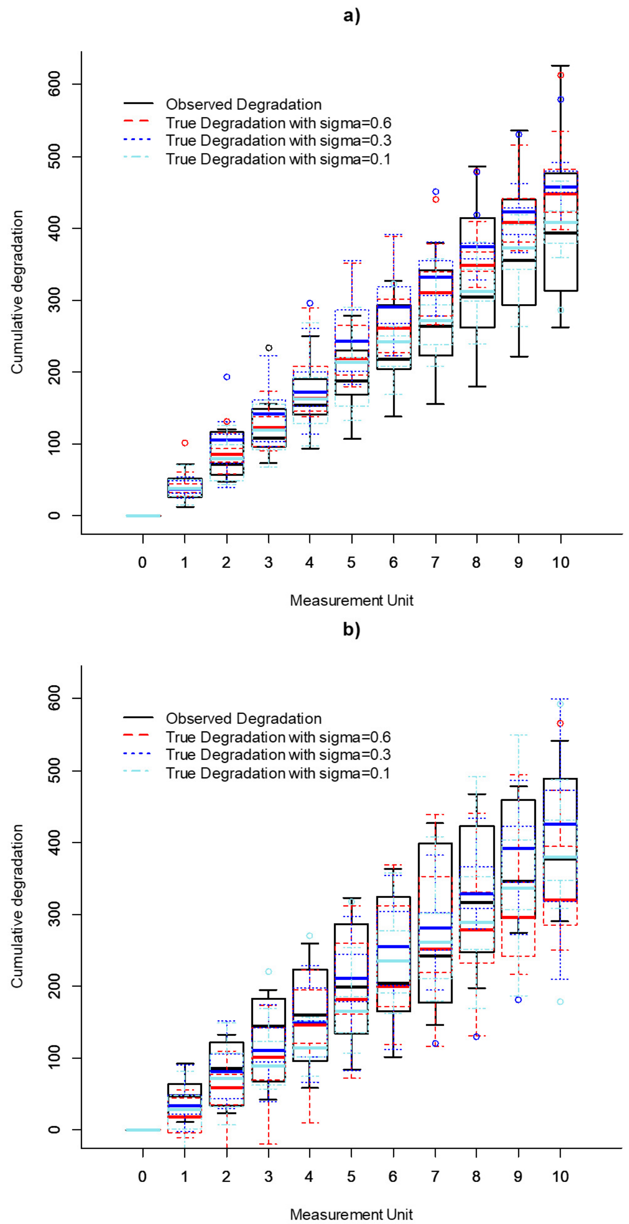

FIGURE 3. Box-plots comparison of the observed pseudo-degradation paths with the true deconvoluted degradation paths under σ_j. a) For the IG process, b) For the Wiener process.

the crack length reaches a critical level of degradation $\omega = 0.4$ mm the device is considered to have failed. The sample size is $n = 10$ such that $i = 1, \ldots, 10$, the observation times t_j, $j = 1, 2, \ldots, 9$, are the same for all the samples with $t_j = (0.1, 0.2, 0.3, \ldots, 0.9)$ hundred thousand cycles. The observed degradation $\boldsymbol{Z_j}$, $i = 1, \ldots, 10$, $j = 1, \ldots, 9$ which describes the crack propagation of the devices is presented in the cumulative degradation paths in Fig. 5. The measurement of the cracks lengths at every t_j was performed using a vision system.

In the aims of illustrate the deconvolution approach to obtain the true degradation from contaminated data for the IG and Wiener processes, it is considered that $\mu_\varepsilon = 0$.

TABLE 1. Estimation of parameters for observed degradation.

Stochastic Process	Parameter	Estimate	Lower CI	Upper CI
IG	μ	0.386	0.328	0.461
	λ	6.018	4.398	7.888
Wiener	α	0.377	0.323	0.431
	β	0.0822	0.0711	0.0953

As mentioned in Section III-A, the first step of the proposed approach consists in estimating the parameters of the observed degradation under the IG and Wiener processes and the parameter of the ME. In the next section, the log-likelihood functions of the observed degradation are maximized to estimate the parameters for the observed degradation and the alternative presented in Section IV to obtain the standard deviation of the ME is implemented.

A. ESTIMATION FOR THE OBSERVED DEGRADATION AND MEASUREMENT ERROR

By considering the crack growth dataset, such that $\boldsymbol{Z_j}$ degradation measurements have been observed for the ith unit at the corresponding time t_j with $i = 1, \ldots, 10$ and $j = 1, \ldots, 9$. It is considered that $\boldsymbol{Z_j}$ follow, in first instance an IG distribution $IG\left(\mu_j, \lambda_j\right)$ and a Gaussian distribution $G\left(\alpha_j, \beta_j\right)$. The parameters of interest $\theta_1 = (\mu, \lambda)$, and $\theta_2 = (\alpha, \beta)$ are estimated via MLE as described in Section V.

The *optim* routine in *R* was used to maximize the log-likelihood functions (16) and (17). The obtained estimations for the parameters of the IG and Wiener processes are presented in Table 1 along with their respective 95% confidence intervals. In addition, in the aims of comparing both stochastic processes we also calculated the Akaike information criterion (AIC) for both models, it was found that for the crack growth dataset the IG process has an AIC of -438.1 and the Wiener has an AIC of -401.1. It can be noted that IG process has the lowest value of AIC and thus can be considered as the best fitting model. This makes sense as the degradation paths in Fig. (5) are monotone (only increasing). It is well known that the Wiener process is adequate to model non-monotone degradation processes, whereas the IG process is adequate to model monotone degradation processes. In the aims of applicability of the proposed approach we consider both processes in the next sections to obtain the true degradation.

For this case study, a Gage R&R study was performed to assess the performance of the measurement system. The characteristics of the study were: three people were needed to perform the study, ten devices with cracks were selected, and three replicates were performed, making a total of 60 readings. The results of the study showed that the standard deviation of the Gage R&R study is $\hat{\sigma}_{R\&R} = 0.0006058$, which as mentioned before is the total variation due to the measurement system. Thus, in this paper it is considered that $\hat{\sigma} = \hat{\sigma}_{R\&R} = 0.0006058$. With the estimated parameters of the stochastic processes and the estimation of the standard deviation of the measurement error it is possible to implement the deconvolution approach proposed in section III-A, we illustrate the implementation in the next section.

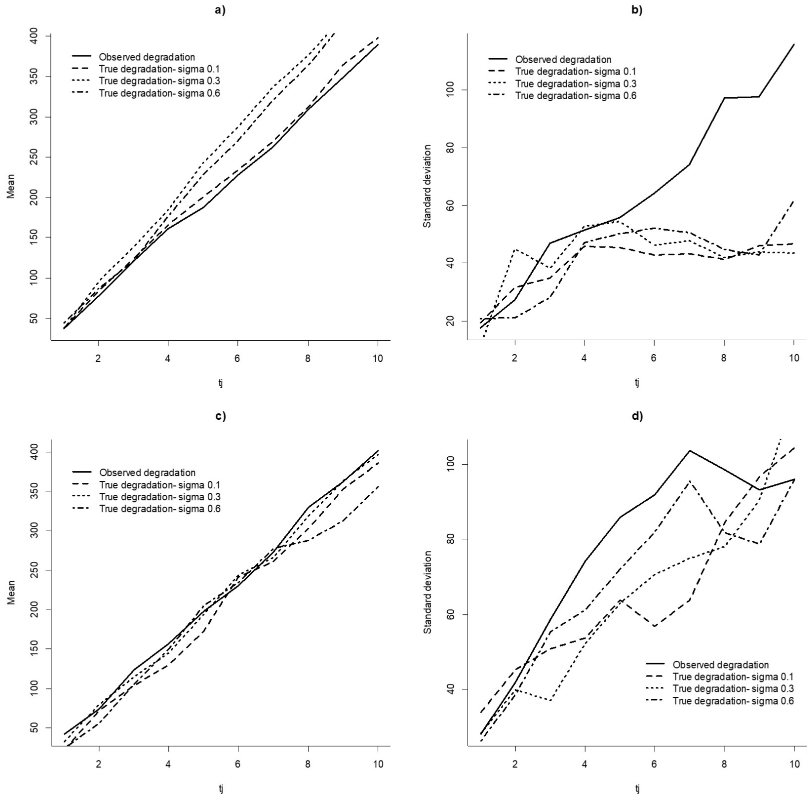

FIGURE 4. **Comparisons at t_j of the means and standard deviations of the observed degradation and the true degradation under the three ME scenarios. a) Means of the IG processes, b) Standard deviations of the IG processes, c) Means of the Wiener processes, d) Standard deviations of the Wiener process.**

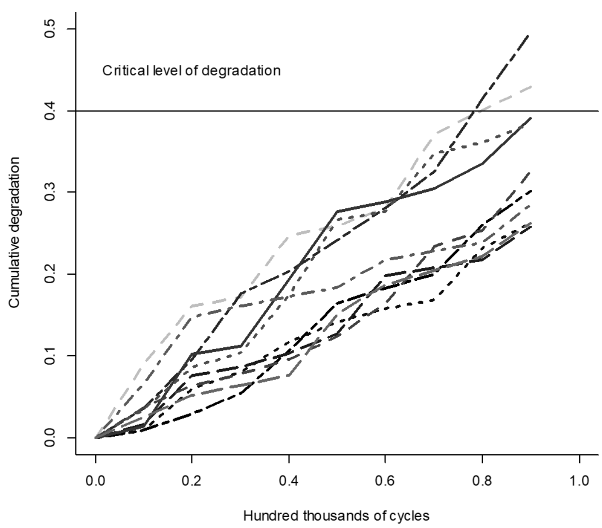

FIGURE 5. **Cumulative degradation paths for the degradation dataset.**

B. OBTAINING THE TRUE DEGRADATION

The proposed method described in Section III is implemented to obtain the true degradation by considering the observed degradation parameters estimated in Section VI-A. As mentioned before, we considered the IG and Wiener process to describe the observed degradation, and the parameters estimated in Table 1 are considered to characterize such processes. As previously discussed, it is considered that the ME follows a Gaussian distribution with $\mu_\varepsilon = 0$ and σ, as the ME for this particular case study is available as $\hat{\sigma} = \hat{\sigma}_{R\&R} = 0.0006058$. Based on this value of $\hat{\sigma}$ and the mean estimates from Table 1, the proposed deconvolution approach is implemented in the R software. Following the steps presented in Section III-A, for the step 2 the CFs defined in (12) and (13) are considered for the IG and Wiener processes, respectively. For the step 3, the CFs of the true degradation were considered as (14) and (15) for the IG and Wiener observed degradation cases. For the implementation of the deconvolution, step 4, we firstly estimated the percentile q_j considering the mean estimates in Table 1 for the IG and Wiener processes at $j = 1, 2, \ldots, 9$. Then, with $\mu_\varepsilon = 0$, and $\hat{\sigma}$ we defined the regular grid $[D_0, D_1]$, with $D_0 = 0$, $D_1 = \mu_\varepsilon + 5\hat{\sigma} + q_j$ for every j of the observed degradation $\boldsymbol{Z_j}$. The approximation described in (4) was performed with $P = 1000$. The implementation of the step 5 is illustrated in Fig. 6, where a comparison of the observed degradation and true degradation under $\hat{\sigma}$ via box-plots for every j is presented.

In Fig. 6, we present a box-plot comparison for every t_j of the crack growth dataset. As previously described, the true degradation for the IG and Wiener processes was obtained by

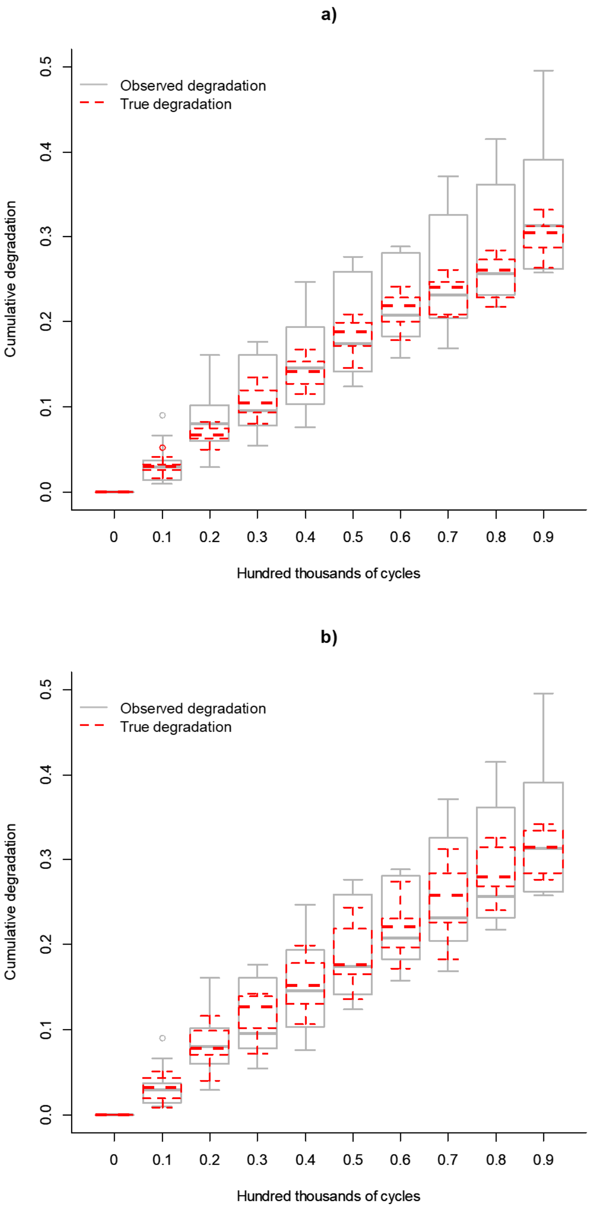

FIGURE 6. **Comparison of the observed degradation paths with the true deconvoluted degradation paths. a) For the IG process, b) For the Wiener process.**

considering the ME as $\left(\mu_{\varepsilon}=0, \hat{\sigma}=0.0006058\right)$ as depicted in Fig. 6. From both figures, it can be noted the effect of the ME. Specifically in the true degradation for both the IG and Wiener processes, it can be noted that the variation at $j = 1, 2, \ldots, 9$ tends to be smaller. Such behavior of the true degradation has also be observed in the works of Si *et al.* [52], and Pulcini [53]. Furthermore, as seen in Fig. 6, the degradation paths appear to have a lower variation of the degradation rate. This denotes that the ME has been excluded from the true degradation. Such characteristics indeed have an effect over the reliability assessment of the product, given that the time when the cumulative degradation reaches the critical path may occur later or before in time.

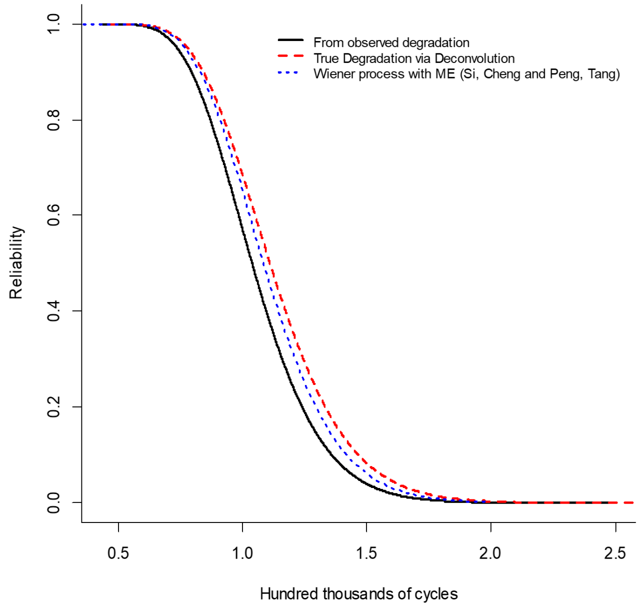

FIGURE 7. **Comparison of the reliability functions for the observed degradation, the true deconvoluted degradation and the Wiener model with ME.**

C. RELIABILITY ASSESSMENT

As can be noted in in Fig. 6, it is expected that the reliability of the product may be miss-estimated if the assessment is provided by considering the observed degradation. In this section, according to the step 6 presented in Section III-A, we present the reliability assessment for the Wiener and IG processes under ME. For the Wiener processes, it is considered that $S_j \sim G\left(\alpha_j, \left(\beta_j - \hat{\sigma}\right)\right)$, thus the reliability function can be directly obtained from (8) by considering the true degradation paths in Fig. 6b under $\hat{\sigma}$. In the aims of compare the proposed approach based on the Wiener process with existing models, we also estimated the reliability considering the Wiener process model with ME presented by Si *et al.* [52], Cheng and Peng [54] and Tang *et al.* [28]. In Fig. 7, we present the comparison of the reliability functions for the observed degradation, the proposed approach based on true degradation via deconvolution and the Wiener model with ME.

It can be noted from Fig. 7, that indeed there are obvious differences between the reliability functions from the observed degradation and the two reliability functions from the models based on ME. Furthermore, it should be noted that both the proposed approach and the Wiener model with ME agree in that the reliability is greater compared with the observed degradation. Which means that the reliability may be underestimated if the observed degradation is considered to assess the product. On the other hand, the reliability function from the proposed approach is also different from the reliability function obtained from the literature Wiener model with ME. This difference is due because in our approach we filtered out the ME via deconvolution to obtain the true degradation, whereas in the Wiener model with ME the error

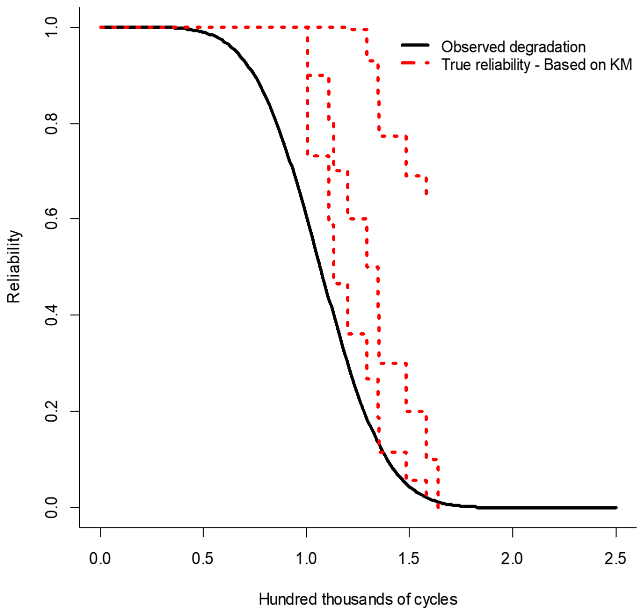

FIGURE 8. **Comparison of the reliability functions for the observed degradation and the true deconvoluted degradation paths under the empirical KM method.**

is estimated along with the observed degradation. An important advantage of the proposed approach is the possibility of integrating in the modeling, the performance of the measuring system, which was measured via an independent statistical tool, i.e. the R&R study. This is a practical advantage, as most of products require the use of specialized measuring systems to perform the measurement of degradation in reliability tests and the measuring process is independent of the evolving of the degradation process.

The failure-time distribution for the observed degradation under the IG process has closed form in terms of the IG distribution. In this manner, the observed reliability function can be easily obtained from (6). However, the PDF of the true degradation $f\left(S_j\right)$ has a complex form. For this we considered the deconvolution approach to obtain an approximation of $f\left(S_j\right)$. As described in Section VI-B, the true cumulative degradation paths were constructed by considering S_j for $j = 1, 2, \cdots, 9$. Thus, for the true degradation under the IG process we present the failure time distribution considering $\hat{\sigma} = 0.0006058$ based on the Kaplan-Meier (KM) method. For this, we estimated the pseudo-failure times of the true degradation paths presented in Fig. 6a using the approach presented by [55] and estimated the true reliability function. In Fig. 8, we present the comparison of the reliability function of the observed degradation with the reliability function of the true degradation estimated via KM along with the corresponding 95% confidence intervals.

It can be noted from Fig. 8 that the empirical reliability based on KM is different from the reliability function estimated from the observed degradation. Specifically, it can be noted that, as it was found in the reliability functions of the Wiener processes, the reliability function based on the true degradation is greater than the reliability of the observed degradation. It should also be noted that the lower 95% pointwise confidence band of the true reliability based on KM does not include the reliability of the observed degradation. Thus, it can also be said that the reliability may be underestimated if the assessment is performed by considering the observed degradation instead of the true degradation.

VII. CONCLUDING REMARKS

In this paper, we propose the use of deconvolution to obtain the true degradation from the observed degradation and ME. We considered the Wiener and IG stochastic processes to describe the observed degradation and a Gaussian distribution to describe the ME. For the implementation of the deconvolution approach we considered the discrete form of the FT and implemented the FFT algorithm to obtain the true degradation from the observed degradation. From the simulation study we found that the parameter σ from the ME has an effect on the variation of the degradation paths in every j and also it has an effect over the mean degradation rate. Such effects have an impact over the reliability assessment. To illustrate such impact, we considered a crack-growth degradation dataset and implemented the proposed approach by considering an estimation of the ME standard deviation as $\hat{\sigma} = \hat{\sigma}_{R\&R} = 0.0006058$, once obtained the true degradation we characterized the reliability functions for the Wiener and IG processes with and without ME. From Fig. 7 and 8, it can be noted that indeed the ME has an impact over the first-passage time distributions. From the Wiener process, the reliability tends to be greater compared with the reliability of the observed degradation. From the IG process, the 95% confidence interval for the KM reliability under $\hat{\sigma} = 0.0006058$ does not include the reliability of the observed degradation. Such impacts are originated because the true degradation obtained via deconvolution causes that the mean degradation rate among degradation paths and the variation within paths be smaller compared to the observed degradation. Although, in degradation analysis the observed measurements are normally contaminated with ME, there are other sources of variation that affect the observed degradation. Such sources of variation have been modeled considering random effects in the stochastic model that governs the observed degradation. Thus, more research efforts may be directed in dealing with the obtaining of true degradation from observed degradation with random effects via deconvolution. In addition, other distribution functions can be considered to describe the ME, such as the t-distribution.

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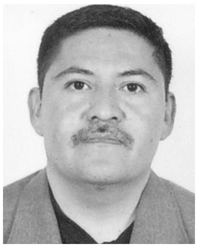

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