Noname manuscript No.
(will be inserted by the editor)

Compress sensing algorithm for estimation of signals in sensor networks

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Received: date / Accepted: date

Abstract In this research, we present a data recovery scheme for wireless sensor networks. In some sensor networks, each node must be able to recover the complete information of the network, which leads to the problem of the high cost of energy in communication and storage of information. We proposed a modified gossip algorithm for acquire distributed measurements and communicate the information across all nodes of the network using compressive sampling and Gossip algorithms to compact the data to be stored and transmitted through a network. The experimental results on synthetic data show that the proposed method reconstruct better the signal and in less iterations than with a similar method using a thresholding algorithm.

Keywords sensor networks · compressive sampling · gossip algorithms

1 Introduction

Sensor networks, both wired and wireless, have found a wide variety of applications, which has caused their growth, and with this, a greater amount of information to propagate [24]. Thus, there exists a constantly search for optimization in both speed and in the amount of data to be transmitted. In a network with thousands of nodes, for unify their information it would be necessary at least n transmissions, which n in the number of nodes, in practice this could be very slow or not at all useful, also this can carry out other problems such as: complications to detect possible failures, maintenance, or even to detect possible attacks on the network [13]. To solve this problems, an alternative is the use of compressed sensing (CS) algorithms [4, 11], which seek to reconstruct a signal starting from a much smaller amount of data, this is possible by expressing the signal in sparse domain (Fourier, Wavelet, etc.), where most of its elements are null, this could be beneficial in both speed, amount of data to be transmitted, power savings and facilitate the analysis of information.

The use of optimization and CS has been used in a distributed setting in several works. In [22] it is presented a distributed projected consensus algorithm were nodes combine their local average with projection on their individual constraint sets. In [20] are proposed several strategies based on distributed iterated hard thresholding algorithm over a network that employ diffusion mechanisms, the developed algorithms have low complexity in terms of communication in the network. In the work of [12] is proposed the use of approximated message passing to reduce the amount of data transmitted in the sensor network, in their algorithm they try to reduce the communication cost, while maintaining the same recovery solution as the centralized scheme. However, their algoritm relies in know data of every sensor and the performace suffer when the number of sensor increases. In [26] a distributed greedy algorithm for sparse learning is proposed, the algorithm is not based in a consensus scheme but instead it is designed to achieve performance through cooperation and information exchange. In [6] a framework for distributed minimization of non-convex functions based on difference of gradients using successives convex approximations and consensus to distribute the computation among network nodes, each node solves a local convex approximation problem following by local averaging operations. In [21] is introduced an algorithm based on distributed inexact gradient and gradient tracking techniques, and doubly stochastic mixing matrices, at each step of the algorithm all nodes iterates to a global and consensual minimizer.

All of these described algorithms, make use of a type of consensus optimization paradigm, in this research we explore gossip schemes for efficient implementation of CS algorithms in the distributed setting to achieve consensus within the nodes in the network, in order to reduce the computational complexity and minimize the number of active nodes at each time step.

Here, we proposed a distributed CS scheme for application in sensor networks to solve the problem of signal estimation, and where each node has only available a few measurements through linear incoherent sampling. Our proposed scheme is based on gossip algorithms and approximated message passing, this permits a rapid convergence over a sensor network and reduce the computational complexity as compared with the work of [12]. To the best of our knowledge, the proposed scheme is the first to combine gossip algorithms with the approximated message passing in a distributed setting. Our main contributions are a novel algorithm that combines approximated message passing and gossip methods. approximated message passing, the proposed algorithm is robust to diverse topologies, in the sense that it can work even if each node has only one neighbor, and each step is easy to compute.

The rest of the paper is organised as follows. In the section 2, the CS and gossip algorithms are reviewed. In the section 3 the proposed algorithm is presented. The section 4 details the experiments, and results are presented. Finally, the conclusions are provided in the section 5.

2 Theory

In this work we represent the network of N nodes as a graph $G = (V, E)$, where $V = \{1, 2, ..., N\}$ is a set of nodes, and $E \subset V \times V$, represent communication links between two nodes (edges)

$$E = (u, v) : u \neq w \tag{1}$$

where $v, w \in V$. Also, we denote the data at each node $v \in V$ at a time t as $x_v(t)$.

2.1 Gossip algorithms

Gossip algorithms for distributed systems were introduced for reliable transfer data in communication systems. In a network using these algorithms, information is exchanged asynchronously and no specialized routing is necessary [16, 1]. If all nodes in a network have access to a subset of the data, then, under the Gossip scheme, information is exchanged iteratively between a subset of nodes at a time, until all information is propagated [5]. At each step a node share information with a neighbor to compute a local update. Here we are interested in pairwise randomized gossiping [3], and one classic application example is the so called average consensus which consist of distributed averaging in network given by G were each node $v \in V$ has a initial measurement $x_v(0)$ and it is required that the whole network knows and estimate of the average of all the measurements of the nodes of G [7]. Denoting $x_v(t)$ as the estimation in node v at iteration t, then at each iteration of the algorithm the following steps are done:

1. A pair of nodes $v, w \in V$ are selected randomly.
2. The selected nodes exchange their current estimates, $x_v(t)$, $x_w(t)$.
3. Each node updates their estimates as

$$x_v(t+1) = x_w(t+1) = (x_v(t) + x_w(t))/2. \tag{2}$$

Convergence of the pairwise gossip algorithm to the true average is guarantee if the nodes keep gossiping each other for enough time [7].

2.2 Compressed sensing

The theory of CS has been integrated in many areas of image processing, signal processing, and communications. There are CS applications in MRI [17], signal compression [18], radar [15], cognitive radio [25], among others.

Given a compressible signal, CS techniques are able to reconstruct that signal using only few linear measurements [2,4]. Considering a signal $x \in \mathbb{R}^N$ that is K-sparse, that is where only K of N coefficients are nonzero, then using CS theory it is possible to reconstruct x from a measurement vector $y \in \mathbb{R}^m$ obtained through linear samples using a measurement matrix A, CS theory establish the condition for selecting a suitable A. A common choice that works with high probability is to select A as a matrix of random numbers, and then x can be recovered exactly or approximately from the measurements y by solving

$$\hat{x} = \underset{x}{\operatorname{argmin}} ||x||_0 \quad \text{such that} \quad ||Ax - y|| \leq \epsilon \tag{3}$$

Were $\epsilon > 0$ is a error tolerance. This optimization problem is a convex known as basis pursuit [2,19,14]. There exist several techniques that can be used to solve (3). Here we are interested to solve (3) under a distributed setting.

3 Methods

In this paper we consider the problem of estimate a signal $x \in \mathbb{R}^n$ employing a sensor network G, where each node v of G has available $y_v \in \mathbb{R}^m$ measurements given by

$$y_v = A_v x + e_v, \tag{4}$$

here $m << n$, $A_v \in \mathbb{R}^{m\times n}$, e_v is noise, and x is assumed to be $k-sparse$. Thus, one way to estimate the signal x at each node is solving the following optimization problem

$$\hat{x} = \operatorname*{argmin}_x \sum_{v\in V} ||y_v - A_v x||_2^2 \quad \text{such that} \quad ||x||_0 < k \tag{5}$$

This can be solved using CS theory in a distributed setting, the nodes in the network could exchange information, helping to attain an estimate of the signal in a faster or with better precision as compared to non-distributed setting [20].

To solve the optimization problem using a collaborative scheme, Gossip Pairwise algorithms are combined with threshold reconstruction methods, this leads to obtain an approximation of the signal to be recovered in each iteration of the gossip algorithm.

There exists several algorithms for distributed reconstruction, in which, each node obtains an approximation of the original signal that is updated iteratively according to the information provided by the neighbors, one of them is the iterative hard thresholding (IHT) with gossip [23]. Here we propose an algorithm based on the optimization scheme called Approximate Message Passing Algorithm (AMP) [8–10], which is an improvement of the methods based on a thresholding schemes, it adds a term based on the theory of belief propagation in graphical models. The updates of the AMP algorithm are as follows

$$x(t+1) = \sigma_t(A^T z(t) + x(t)) \tag{6}$$

where $z(t)$ is the residue between the encoded information and the estimates, and σ_t is a thresholding function such as soft thresholding.

$$z(t) = y - Ax(t) + m(t) \tag{7}$$

where $m(t)$ is a term that acts a a momentum and it is one of the key modifications over IHT, this term is given by

$$m(t) = \frac{1}{\delta} z(t-1)\langle \sigma_t'(A^T z(t-1) + x(t-1))\rangle \tag{8}$$

where σ_t' is the derivative of the threshold function and

$$\langle f(n)\rangle = \sum_1^n \frac{f(n)}{n} \tag{9}$$

indicates the sampling mean.

Here we use AMP in conjunction with Gossip to solve the aforementioned problem in a sensor network. Thus, given a node v selected to gossip a node w we calculate the updates for each of the gossiping nodes as

$$x_v(t+1) = \sigma_k[\frac{x_v(t) + x_w(t)}{2} + \tau A_v^T z_v(t)], \tag{10}$$

and

$$x_w(t+1) = \sigma_k[\frac{x_v(t) + x_w(t)}{2} + \tau A_w^T z_w(t)] \tag{11}$$

where we expect that the inclusion of term $z_w(t)$ will help achieve a rapid convergence, this term is calculated as

$$z_v(t) = y_v - A_v \frac{x_v(t) + x_w(t)}{2} + \frac{1}{\delta} z_v(t-1) \langle \sigma_t'(A^T z(t-1) + x_v(t-1)) \rangle, \tag{12}$$

$$z_w(t) = y_w - A_w \frac{x_v(t) + x_w(t)}{2} + \frac{1}{\delta} z_w(t-1) \langle \sigma_t'(A^T z(t-1) + x_w^(t-1)) \rangle. \tag{13}$$

In the next section we demonstrate the effectiveness of the proposed method and offer comparisons with Gossip-IHT of [23]. Table 3 shows the proposed algorithm.

proposed algorithm
1:Initialize: $x_v = 0$ for all $v \in V$, set $\tau > 0$, m(0)=0
2:for t=0,1,..., stop iter do
3:Select randomly a communication link $E = u, w$
4:$z_v(t) = y_v - A_v \frac{x_v(t)+x_w(t)}{2} + \frac{1}{\delta} z_v(t-1) \langle \sigma_t'(A^T z(t-1) + x_v(t-1)) \rangle$
5:$x_v(t+1) = \sigma_k[\frac{x_v(t)+x_w(t)}{2} + \tau A_v^T z_v(t)]$
6:$z_w(t) = y_w - A_w \frac{x_v(t)+x_w(t)}{2} + \frac{1}{\delta} z_w(t-1) \langle \sigma_t'(A^T z(t-1) + x_w(t-1)) \rangle$
7:$x_w(t+1) = \sigma_k[\frac{x_v(t)+x_w(t)}{2} + \tau A_w^T z_w(t)]$
8:$x_h(t+1) = x_h(t)$ for any $h \neq v, w$
9:end for

4 Results

In this section we present the different experiments carried out during the investigation, as well as the analysis and evaluation of the results obtained. We compare gossip IHT [23] with our proposed method, gossip AMP. We simulated a sensor network with 100 nodes, and we select the communication range of each sensor such that every node has at least one neighbor and and there are no nodes without communicating. In Figure 1 it is shown a realization of a simulated network.

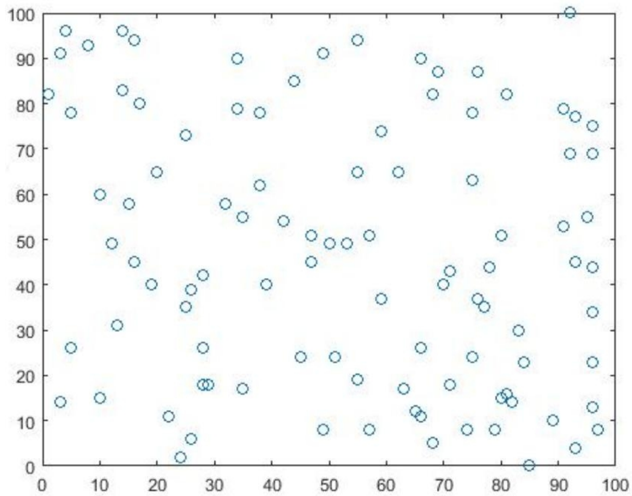

Fig. 1 A realization of a sensor network with 100 nodes.

For the experiments, we also simulated a signal with values in the range of $[-1, 1]$ and then we use a k-sparse version of it. Additionally, a measurement matrix A was created inline for each node, by using a predefined seed for generating random numbers, this has the advantage of not being expensive in storage. As a measure of error we used the mean squared error.

In the first experiment we generated a 100 samples signal with a sparsity of $k = 15$. We use this data to obtain values for the parameters of each algorithm. In the Gossip IHT algorithm the parameter τ was modified between $[0, 1]$, until finding a small recovery error in all the nodes, in this case the mean square error. In Figure 2, the decrease in the error is shown for several iterations of the gossip IHT algorithm with different values of τ. We show the two curves for which the reconstruction gives the less error, which correspond to $\tau = 0.006$ and $\tau = 0.007$, the minimum error is attained at iteration 1200 were can be observed that the most suitable value for τ is 0.007 with an error of 0.000233.

In the evaluation of the proposed Gossip AMP algorithm we worked with the parameters τ and δ, which were modified in the intervals [0, 1] and [1, 5] respectively. In Figure 2, it is shown the two curves for which the reconstruction gives the less error, that correspond to the combination of parameters: $\tau = 0.01$ and $\delta = 1$; and $\tau = 0.007$ and $\delta = 0.9$. The minimum error at 1200 iterations was achieved with $\tau = 0.007$ and $\delta = 0.9$ with an error of 0.0000181.

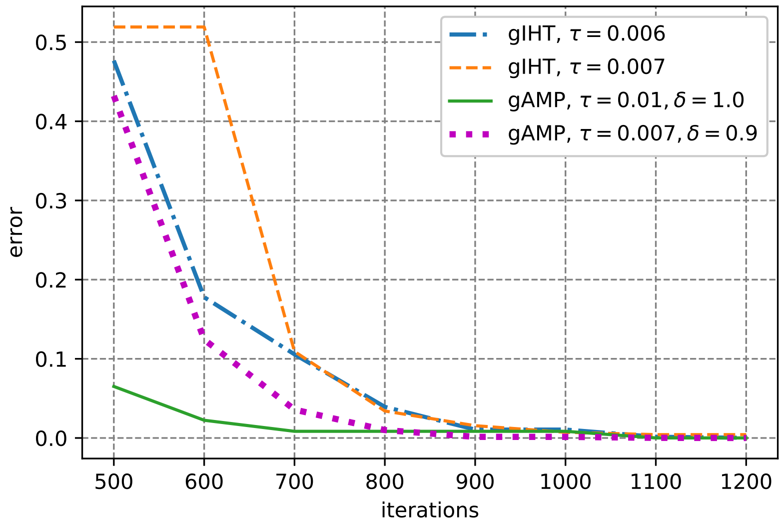

Fig. 2 The graph shows the error of reconstruction of the algorithms for each iteration of the signal. We show the best curves for the gossip IHT (gIHT) and the proposed algorithm, gossip AMP (gAMP).

Individual progress of the estimates during the iterations of the Gossip IHT and AMP algorithms can be visualized in Figure 3, it can be noted that in the first iterations the gossip IHT algorithm has large oscillations in comparison with the proposed algorithm. In addition, some values never stop oscillating in the IHT algorithm, for example the point above, while in the proposed algorithm all the test points converge.

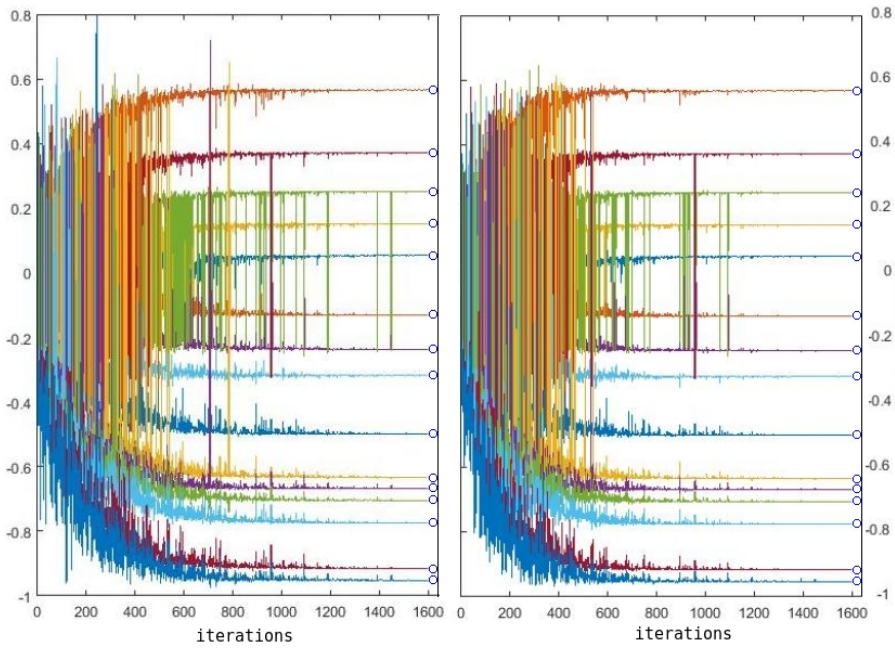

Fig. 3 The graphs show the convergence of the algorithms for 15 points of the signal, whose true value are denoted by the circles. The left graph shows the gossip IHT algorithm, while the left graph shows the proposed algorithm.

Once the values of the parameters in each algorithm are established, we test again on a new data realization on a new network (100 random nodes). A maximum error was obtained every 100 iterations starting from the 500 until arriving to 1200, with the purpose of visualizing the progress of the estimations of each algorithm, this is shown in Figure 4 where it can be observed that Gossip AMP decreases the error faster than Gossip IHT.

To visualize the reconstruction of all the nodes, the signal of each node was overlapped in the same graph for each algorithm, this is shown in Figure 5.

We also explore the results without knowing the sparsity of the signal before hand. For this end, we generated data to obtain a 100 samples signal in the range of $[0, 1]$, next we obtained a sparse version of it by using the equation (14)

$$\sigma(x;\theta) = \begin{cases} (x-\theta) & if x > \theta \\ 0 & -\theta \leq x \leq \theta \\ (x+\theta) & if x < -\theta \end{cases} \tag{14}$$

were we used a value of $\theta = 0.001$ to perform the experiments. The parameters of each algorithm were varied within intervals that were determined empirically and that had greater possibility of containing the optimal values.

We evaluate the Gossip IHT algorithm for different values of the parameter τ in a range of $[0, 1]$ and for several iterations. In Figure 6 we can see the

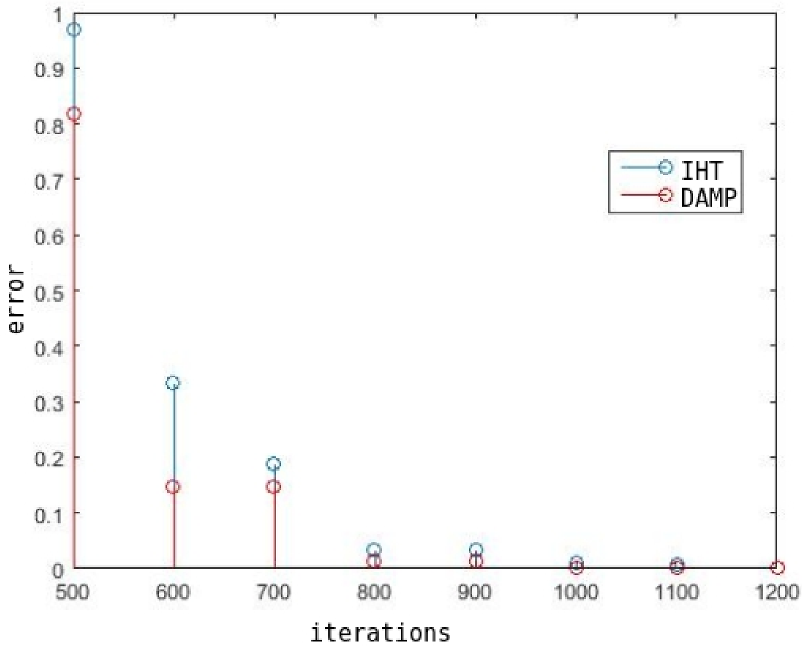

Fig. 4 Progress through iterations of the Gossip AMP algorithm, $\tau = 0.006$ and $\delta = 0.007$

decrease of the error in IHT as τ as varying. We only show the two curves with less error for a given iteration. The best result is found with the value of $\tau = 0.005$, with an error of 0.149. On the other hand, we evaluate the proposed algorithm varying the parameters τ and δ, which were modified in the intervals [0, 1] and [1, 5] respectively. Figure 6 shows the two curves with less error. Here the best value is given by an error of 0.02, using $\tau = 0.007$ and $\delta = 0.9$.

Also, from Figure 6 we can see that for a given iteration, the reconstruction error is less by using the proposed algorithm. At iteration 500 the error is twice between the best combination of parameters for the proposed algorithm and the second best combination of parameter for the gossip IHT.

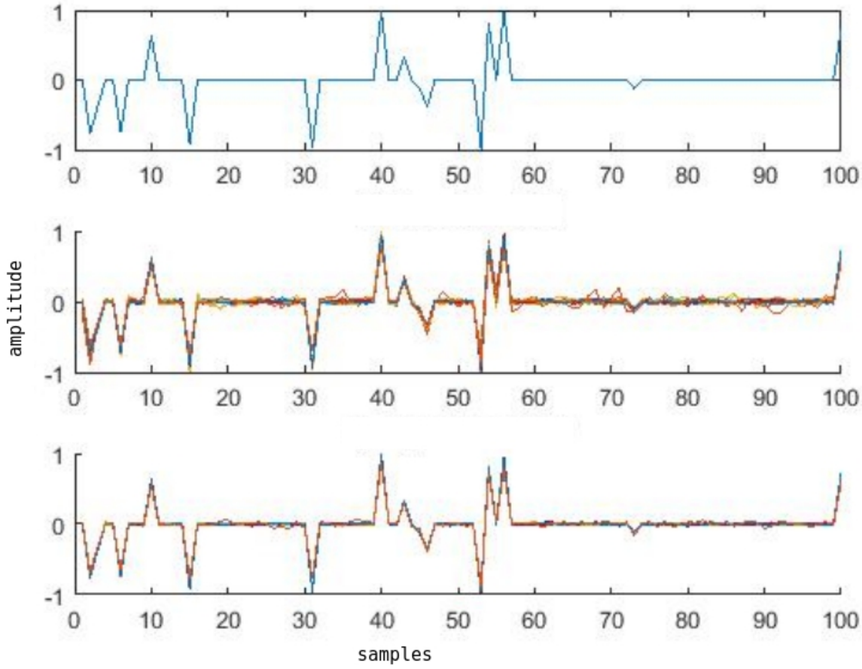

Fig. 5 a) Original signal, b) signal overlap for Gossip IHT and, c) signal overlap for Gossip AMP.

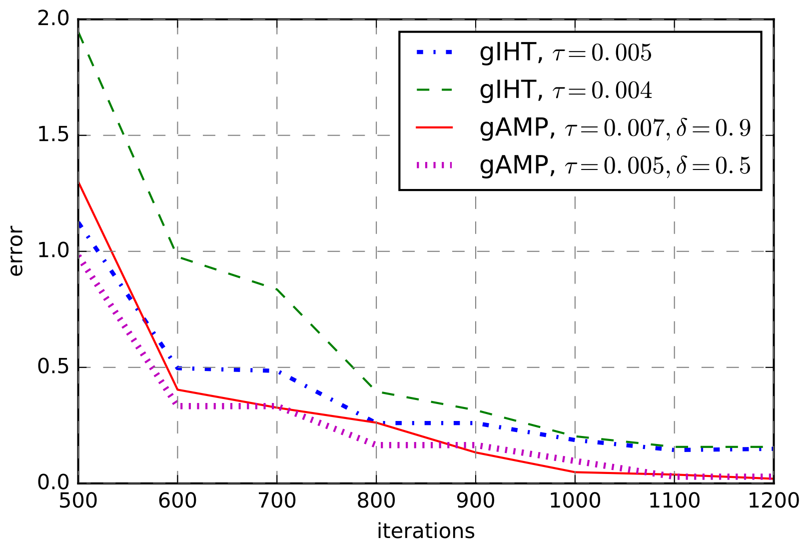

Fig. 6 The graph shows the error of reconstruction of the algorithms for each iteration of the signal. We show the best curves for the gossip IHT (gIHT) and the proposed algorithm, gossip AMP (gAMP).

In Table 1, are shown some advantages and disadvantages of the proposed method.

Table 1 Advantages and disadvantages.

Advantages	Disadvantages
*It can work even if each node has only one neighbor. *Each step is easy to compute	*The parameters of the algorithm have to be determined. *Only consider the information of two nodes at each step

5 Conclusion

In this paper we present a distributed CS scheme for application in sensor networks to solve the problem of signal estimation. We explore the use of the approximated message passing algorithm in conjunction with gossip algorithms in order to adapt CS to a distributed setting, this allows to the proposed algorithm to a rapid convergence in the reconstruction of signal over a sensor network The implementation of the proposed algorithm in a wireless sensor network was carried out satisfactorily, we generated a measurement matrix online, which does not have to be stored or transmitted, because it can be generated in each node of the network, using this matrix allows to create a vector of measurements and transmit less number of in comparison to the size of the original signal.

We carry out experiments using a simulated signal to evaluate a distributed reconstruction using our method, and as comparison the Gossip IHT was used. We found that our method converges signifivantly faster than Gossip IHT, and on most cases the the error in the reconstrucction at each iteration is less in our proposed method than using Gossip IHT. Future research directions include include a regularizing term to reduce possible noise in the signal and explore diffusion schemes intead of gossip algorithms, also a work convergence analysis.

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