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Reliability analysis using exponentiated Weibull distribution and inverse power law

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Abstract

Today in reliability analysis, the most used distribution to describe the behavior of devices is the Weibull distribution. Nonetheless, the Weibull distribution does not provide an excellent fit to lifetime datasets that exhibit bathtub shaped or upside-down bathtub shaped (unimodal) failure rates, which are often encountered in the performance of products such as electronic devices (ED). In this paper, a reliability model based on the exponentiated Weibull distribution and the inverse power law model is proposed, this new model provides a better approach to model the performance and fit of the lifetimes of electronic devices. A case study based on the lifetime of a surface-mounted electrolytic capacitor is presented in this paper. Besides, it was found that the estimation of the proposed model differs from the Weibull classical model and that affects the mean time to failure (MTTF) of the capacitor under analysis.

KEYWORDS

exponentiated Weibull distribution, inverse power law, nonmonotonic failure rate, Weibull distribution

1 | INTRODUCTION

In reliability analysis, the Weibull distribution is very popular to describe the behavior and life cycle of devices with a monotone failure rate. This distribution may be an initial choice because of its negatively and positively skewed density shape Carrasco et al.[1] On the other hand, if the device under analysis has a complex system constitution such as electronic devices (ED), the failure rate observed often exhibits nonmonotonic behavior; this is because ED can involve high initial failure rate (infant mortality) and eventual high failure rates due to aging and wear out, indicating a bathtub failure rate.[2] In those situations, the Weibull distribution, which allows a monotone failure rate might not be appropriate for describe the behavior of the data obtained in Accelerated Life Testing (ALT) and provide an excellent fit to lifetime datasets with bathtub shaped or upside-down bathtub shaped (unimodal) failure rates.

The models that present bathtub shape failure rate are very profitable in reliability analysis. Nevertheless, distributions, which describe the bathtub shape failure rate can be complicated, difficult to be modeled and by consequence cannot be implemented in a real reliability analysis. In recent years, new distributions have been proposed for modeling bathtub shape failure rate based on exponentiated family. For example, Gupta and Kundu[3] studied the exponentiated exponential family; the properties of this family can be considerate as an alternative to gamma and Weibull distributions because of the similar scale and shape parameters presented in both families. Nadarajah and Kotz[4] introduced four more exponentiated type distributions that generalize the standard gamma, standard Weibull, standard Gumbel, and the standard Fréchet distributions in the same way the exponentiated exponential distribution generalizes the standard exponential distribution. However, in recent years, research of exponentiated family has focused on Weibull distribution and their applications in survival analysis. Mudholkar et al[2] introduced the Exponentiated Weibull Distribution (EWD) and their applications to goodness of fit in survival analysis. Pal et al[5] studied the failure rate function of the EWD obtained in survival analysis and compared the obtained behavior with gamma and Weibull distribution. Nadarajah et al[6] present a comprehensive review of the EWD and include structural properties, characterizations, generalizations and related distributions, transformations, estimation methods, discrimination, the goodness of t tests, regression models, multivariate generalizations, and computer software. Modified models of EWD and its properties can be found in Aryal and Tsokos, Flaih et al, Gera, and Mahmoudi and Sepahdar.[7-10] Additionally, applications in reliability analysis of the EWD can be found in Barriga et al,[11] which combine the EWD and Arrhenius model to analyze the behavior of simulated temperature process obtained in an ALT. Other reliability applications form EWD and their variations can be seen in Ahmad et al, Ahmad et al, and Bargout.[12-14]

On the basis of the background and literature review, in this paper, we use the EWD and Inverse Power Law (IPL) to obtain a life-stress relationship, which can describe the performance of devices under an ALT. In this case, we use the proposed model, which is used to estimate the lifetime of a surface-mounted electrolytic capacitor (SMEC) subjected to ALT under a voltage profile. A comparative study between Inverse Power Law-Weibull (IPLW) (classical life-stress relationship) and Inverse Power Law-Exponentiated Weibull Distribution (IPLEWD) (proposed life-stress relationship) is presented in order to contrast the differences obtained on both models, the observed results show that the proposed model can model and describe in better form the life cycling of the device under analysis that the classical model.

Finally, this paper is organized as follows. Section 2 presents a preliminary notation related to four parameters EWD. Section 3 presents the construction of the reliability model. Section 4 presents the moment function of the proposed model. Section 5 presents the likelihood function to calculate the parameters proposed in section 3. Section 6 presents the case study based on the SMEC. Section 7 presents the discussion of the paper that compares the classical model and the proposed model. The last section provides concluding remarks and future work.

2 | PRELIMINARY NOTATION

Before the defined the proposed reliability model, it is necessary to introduce the concept of exponentiated distribution (EXD). On probability space (Ω, F, P), we define a random variable X. Let $g(x)$ be a Probability Density Function (PDF) and $G(x) = \int_{-\infty}^{0} g(s)ds$ be a Cumulative Distribution Function (CDF) of random variable X. Next, we define D as the class of EXD, which is a set of PDF indexed by parameter θ:

$$D = \left\{g_\theta \,:\, g_\theta(x) = \theta\left(G(x)^{(\theta-1)}g(x)\right)\right\}. \tag{1}$$

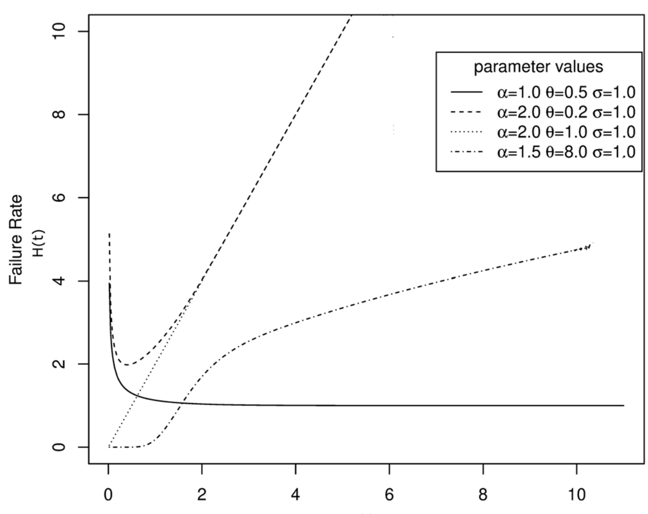

FIGURE 1 Failure rate for some parameter values of Exponentiated Weibull Distribution (EWD)

By setting $g(x)$ the PDF of Weibull distribution with scale parameter λ, a shape parameter k and $G(X)$ as the CDF of Weibull distribution and following the equation in (1) is obtained

$$f(x) = \frac{\theta\alpha}{\lambda}\left(\frac{x}{\lambda}\right)^{\alpha-1}\left[1-e^{-(x/\lambda)^{\alpha}}\right]^{\theta-1}e^{-(x/\lambda)^{\alpha}}. \tag{2}$$

Equation 2 represents the EWD for $x > 0$, with two shape parameters $\theta > 0$ and $\alpha > 0$ and one scale parameter λ. Other important distributions derived from EWD as special cases are discussed in AL-Hussaini.[15] In Figure 1, it is presented a comparison of different failure rate for the EWD expressed by Equation 2.

In the following section, we use the EWD described in (2) and an ALT to obtain the reliability model, which describes the behavior of devices under voltage stress scenario.

3 | RELIABILITY MODEL

In reliability, the model that describes the effect of the voltage on the device‘s life is the IPL, this model is written as

$$\lambda = \frac{1}{kV^n}, \tag{3}$$

where $k > 0$ is a characteristic parameter and depends on material properties, product design, and other factors in the ED. Parameter $n > 0$ measures the effect of the stress on the device's life. Parameter $V > 0$, represents the voltage stress level applied in the piece.

Thus, by substituting (3) in (2) and replacing x to t as the failure time of piece under ALT, the PDF of the life-stress relationship that describes the performance of pieces under voltage profiles and following EWD behavior is obtained

$$f(t) = \gamma\theta\alpha(\gamma t)^{\alpha-1}\left[1-e^{-(\gamma t)^{\alpha}}\right]^{\theta-1}e^{-(\gamma t)^{\alpha}}, \tag{4}$$

where $\gamma = kV^n$.

On the basis of (4), the CDF ($F(t)$), the survival ($S(t)$), hazard ($H(t)$), and quantile ($Q(u)$) functions are given by

$$F(t) = \left[1-e^{-(\gamma t)^{\alpha}}\right]^{\theta-1}, \tag{5}$$

$$S(t) = 1-\left[1-e^{-(\gamma t)^{\alpha}}\right]^{\theta-1}, \tag{6}$$

$$H(t) = \frac{\gamma\theta\alpha(\gamma t)^{\alpha-1}\left[1-e^{-(\gamma t)^{\alpha}}\right]^{\theta-1}e^{-(\gamma t)^{\alpha}}}{1-\left[1-e^{-(\gamma t)^{\alpha}}\right]^{\theta-1}}, \tag{7}$$

$$Q(u) = F^{-1}(u) = \frac{1}{\gamma}\left[-ln\left(1-u^{1/\theta}\right)\right]^{1/\alpha} \quad 0 \le u \le 1. \tag{8}$$

4 | MOMENT FUNCTION OF PROPOSED MODEL

On the basis of the mathematical approach proposed in AL-Hussaini and Ahsanullah,[15] the ℓth moment of random variable t following the CDF expressed by Equation 5 is given by

$$M\left(t^{\ell}\right) = \ell\sum_{J=1}^{r} c_J I_J(\ell),$$

where

$$r = \begin{cases} & \theta = 1, 2, \ldots \\ \infty, & \theta \text{ is positive nonintegrable value,} \end{cases}$$

$$c_J = (-1)^{J-1}\,\theta(\theta-1)\ldots\frac{(\theta-J+1)}{J!},$$

$$I_J = \int_0^{\infty} t^{\ell-1}e^{-Ju(t)}dt.$$

Also $u(t) = -lnR_{G(t)}$ is such that $S_{G(t)}$ a survival function so that $u(x)$ is a continuous, monotone increasing differentiable function of t.

For the case of EWD, $u(t) = \gamma t^{\alpha}$, I_j is given by

$$I_j = (\ell) = \int_0^{\infty} t^{\ell-1} e^{-j\gamma t^{\alpha}} dt.$$

By transforming $z = t^{\alpha}$, the integral becomes

$$I_j(\ell) = \frac{1}{\alpha\gamma}\int_0^{\infty} z^{\frac{\ell}{\alpha}-1} e^{-jz} dz.$$

By solving the last integral, the moment generating function follows:

$$E\left(t^{\ell}\right) = \ell\sum_{j=1}^{r} c_j M_j(\ell) = \frac{\Gamma\left[1+(\ell/\alpha)\right]}{\gamma^{\ell/\alpha}}\sum_{j=1}^{r}\frac{c_j}{j^{\ell/\alpha}}. \tag{9}$$

5 | PARAMETER ESTIMATION OF THE MODEL

On the basis of the Equation 4, let $V_{\iota}, \iota = 1, 2, \ldots,$ the voltage applied in the piece under ALT and $t_{\iota}, \iota = 1, 2, \ldots, \omega$, the failure time of the piece induced by the stress level V_{ι}, thus, the ln-likelihood function is defined as

$$\begin{aligned}\Lambda = w\left\{ln(k) + ln(\alpha) + n\sum_{\zeta=1}^{w}\left(ln\left(v_j\right)\right)\right\} + \left\{(\theta-1)\cdot\sum_{\zeta=1}^{w}\left[ln\left(1-e^{-\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}}\right)\right]\right\} \\ -\sum_{\zeta=1}^{w}\left[\left(kv_j^{n}t_{\zeta}\right)^{\alpha}\right] + \left\{(\alpha-1)\cdot\sum_{\zeta=1}^{w}\left[ln\left(kv_{\zeta}^{n}t_{\zeta}\right)\right]\right\}.\end{aligned} \tag{10}$$

The first partial derivatives of Equation 10 with respect each parameter is given by

$$\frac{\partial\Lambda}{\partial k} = \frac{w}{k} + \left\{(\theta-1)\cdot\sum_{\zeta=1}^{w}\left[\frac{\alpha\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}}{k\cdot\left[\left(e^{kv_{\zeta}^{n}t_{\zeta}}\right)^{\alpha}-1\right]}\right]\right\} - \sum_{\zeta=1}^{w}\left[\frac{\alpha\cdot\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}}{k}\right] + \frac{\omega(\alpha-1)}{k}, \tag{11}$$

$$\begin{aligned}\frac{\partial\Lambda}{\partial\alpha} = \frac{w}{\alpha} + \left\{(\theta-1)\cdot\sum_{\zeta=1}^{w}\left[\frac{\left(kv_{\zeta}^{n}t_j\right)^{\alpha}\cdot ln\left(kv_{\zeta}^{n}t_j\right)}{e^{\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}}-1}\right]\right\} \\ -\sum_{\zeta=1}^{w}\left[\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}\cdot ln\left(kv_{\zeta}^{n}t_{\zeta}\right)\right] + \sum_{\zeta=1}^{w}\left[ln\left(kv_{\zeta}^{n}t_{\zeta}\right)\right]\end{aligned}, \tag{12}$$

$$\frac{\partial\Lambda}{\partial\theta} = \frac{w}{\theta} + \sum_{\zeta=1}^{w}\left[ln\left(1-e^{\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}}\right)\right], \tag{13}$$

$$\begin{aligned}\frac{\partial\Lambda}{\partial n} = w\cdot\sum_{\zeta=1}^{w}\left[ln\left(v_{\zeta}\right)\right] + \left\{(\theta-1)\cdot\sum_{j=1}^{w}\left[\frac{\alpha\cdot ln\left(v_{\zeta}\right)\cdot\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}}{e^{\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}}-1}\right]\right\} \\ -\sum_{\zeta=1}^{w}\left[\alpha\cdot ln\left(v_{\zeta}\right)\cdot\left(kv_{\zeta}^{n}t_{\zeta}\right)^{\alpha}\right] + \left\{(\theta-1)\cdot\sum_{\zeta=1}^{w}\left[ln\left(v_{\zeta}\right)\right]\right\}\end{aligned}. \tag{14}$$

The Fisher information matrix is given by

$$J(\delta) = -\begin{bmatrix} O_{kk} & O_{k\alpha} & O_{k\theta} & O_{kn} \\ O_{\alpha k} & O_{\alpha\alpha} & O_{\alpha\theta} & O_{\alpha n} \\ O_{\theta k} & O_{\theta\alpha} & O_{\theta\theta} & O_{\theta n} \\ O_{nk} & O_{n\alpha} & O_{n\theta} & O_{nn} \end{bmatrix}, \tag{15}$$

where the elements of the matrix are given in Appendix A. The computation of the Equation 15 was made in R via MaxLik Package.

To obtain the confidence interval, we based on the asymptotic normality of MLEs. The $100\,(1-\eta)\,\%$ confidence intervals for $k, \alpha, \theta, and\ n$ are given by

$$\left(\hat{k} \pm z_{1-\rho/2}\sqrt{var(\hat{k})}\right), \tag{16}$$

$$\left(\hat{\alpha} \pm z_{1-\rho/2}\sqrt{var\,(\hat{\alpha})}\right), \tag{17}$$

$$\left(\hat{\theta} \pm z_{1-\rho/2}\sqrt{var\,(\hat{\theta})}\right), \tag{18}$$

$$\left(\hat{n} \pm z_{1-\rho/2}\sqrt{var\,(\hat{n})}\right), \tag{19}$$

where $z_{1-\rho/2}$ is the upper $(\rho/2)$ percentile of the standard normal distribution.

6 | CASE STUDY

In this section, the model established in (4) is used to estimate the lifetime of SMEC. The presented data were taken from Yang,[16] which evaluated the reliability of SMEC via ALT. The ALT was performed with the following parameters considerations:

- Three test levels were established at 80 100 and 120 V.
- In each voltage level, eight units were under analysis.
- All units were run to failure, where failure is said to have occurred when the capacitance drifts more than 25%.
- No censored data were considered for this ALT.

The results of the ALT experiment performed to SMEC are shown in Table 1.

6.1 | Estimation of IPLEWD model

To estimate the parameters α, θ, k, and n established in the Equation 4, the MLE was programed in R by using the MaxLik package (Gradient Equation 11 to Equation 14 and Hessian Equation 15 were programed in the algorithm). The results of the estimation obtained from the data obtained in Table 1 are presented in Table 2.

TABLE 1 Failure time of SMEC, Yang[16]

Voltage (V)	80	100	120
	1770	1090	630
	2448	1907	848
Failure time	3230	2147	1121
	3445	2645	1307
(Hours)	3538	2903	1321
	5809	3357	1357
	6590	4135	1984
	6744	4381	2331

TABLE 2 Results of estimations obtained via MLE of data provided in Table 1

Parameter	Estimation	Gradient
α	0.53	9.89E + 05
θ	42.8	4.59
k	1.09E - 5	9.42E - 03
n	1.38	253.77

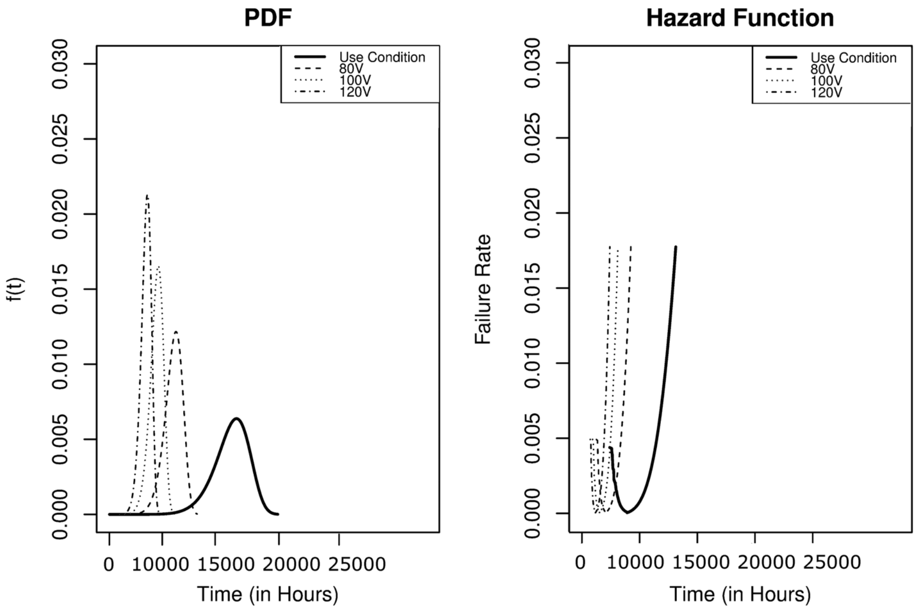

FIGURE 2 Realiability graphs obtanied from the data prsented in Table 1

TABLE 3 Confidence bounds for model parameters presented in Equation 4

Parameter	LCB	Estimation	UCB
α	0.290	0.53	0.769
θ	36.414	42.8	49.185
k	1.088E-05	1.090E-5	1.091E-05
n	1.211	1.38	1.548

Derived from the results obtained in Table 2, reliability graphs, which describe the behavior and performance of SMEC can be drawn by substituting the estimations in Equation 4 and Equation 7; results are shown in Figure 2, for this case consider the use condition of SMEC at 50 volts.

The hazard plot presented in Figure 2 showed that the failure rate of SMEC has a bathtub shape. That behavior is exhibited after 10 000 hours approximately, the probability of the failure rate increases suddenly, which represents the end of the life (wear out) of the SMEC.

The results of fisher matrix presented in Equation 15 are

$$J(\delta) = \begin{bmatrix} 3.94E-17 & 1.39E-14 & -1.18E-12 & -2.61E-13 \\ 1.39E-14 & 1.48E-02 & -4.07E-11 & -9.07E-11 \\ -1.18E-12 & -4.07E-11 & 10.61 & 7.98E-02 \\ -2.61E-13 & -9.06E-11 & 7.99E-02 & 7.35E-03 \end{bmatrix}. \tag{20}$$

The confidence bounds of the parameters obtained in Table 2 can be calculated from the diagonal matrix presented in the Equation 20. The results are presented in the Table 3, consider a 95% confidence interval.

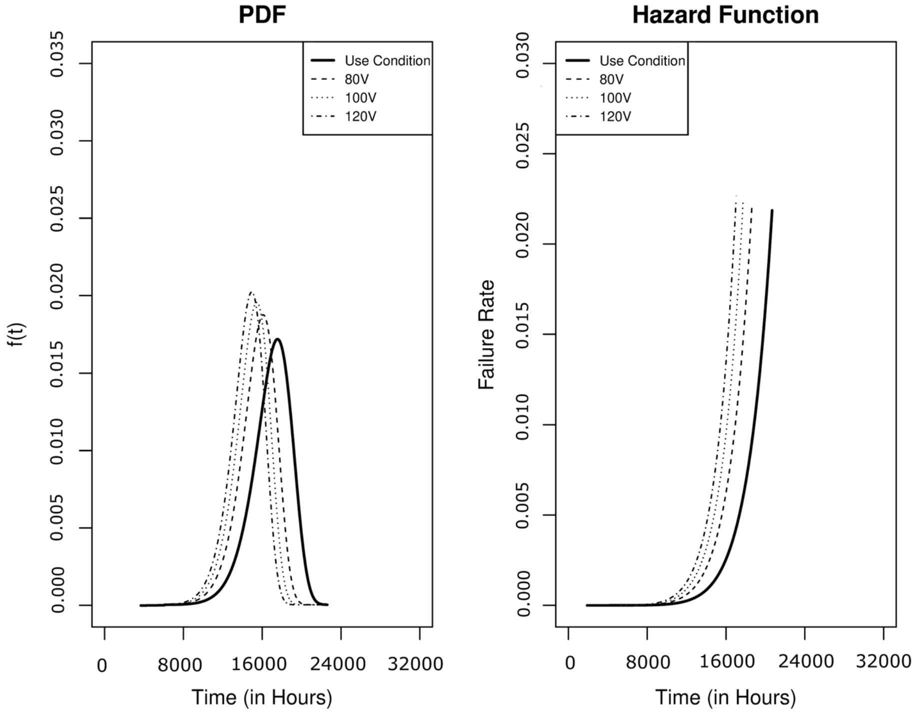

FIGURE 3 Reliability graphs of surface-mounted electrolytic capacitor (SMEC) under Inverse Power Law-Weibull (IPLW)

6.2 | Estimation of IPLW model

To contrast, the results obtained in section 6.1, a reliability analysis with IPLW model is presented. For this case, the IPLW model is defined as

$$f(t, V) = \alpha k v^{n} \cdot \left(k v^{n} t\right)^{\alpha-1} \cdot e^{-(k v^{n} t)^{\alpha}}, \tag{21}$$

and the MLE is given by

$$\begin{aligned}\Lambda_{IPLW} = w\left[ln(\alpha) + ln(k) + n \cdot \sum_{\zeta=1}^{w} ln\left(v_{\zeta}\right)\right] \\ + \left[(\alpha-1)\cdot\sum_{\zeta=1}^{w}\left\{ln\left(k v_{\zeta}^{n} t_{\zeta}\right)\right\}\right] - \sum_{\zeta=1}^{w}\left\{\left(k v_{\zeta}^{n} t_{\zeta}\right)^{\alpha}\right\}.\end{aligned} \tag{22}$$

By taking the values from Table 1, the parameters estimated by MLE in the Equation 21 are

in Figure 3 is shown the reliability graphs of SMEC under IPLW model.

In this case, the the hazard plot showed that the failure rate increase suddenly after 1300 hours and exhibit a monotone behavior.

7 | DISCUSSION

On the basis of the results obtained in sections 6.1 and 6.2, difference between presented models come out. The parameter K, measures the damage induced into the internal components of SMEC such as Aluminum foil, electrolyte paper, metal case, and sealant. The information provided by Tables 2 and 3 showed that the SMEC components suffer more damage under IPLEWD, which the failure rate is increased after 10 000 hours in comparison with IPLW, which the failure is increased after 15 000 hours. In practice, the parameter K depends directly on the quality and the fabrication process of the materials that the SMEC was built. Moreover, the parameter K measures the capacity of the SMEC to hold back the energy in the cells.

The parameter n measures the effects of the voltage into the SMEC has a significant difference between the IPLEWD and the IPLW. A high value of parameter n, means that the saturation voltage in the SMEC can reduce the performance

TABLE 4 Results of parameter estimation for Inverse Power Law-Weibull (IPLW) model

Parameter	Estimation
α	2.68
k	9.95E-10
n	2.78

TABLE 5 Decision criteria obtained from Inverse Power Law-Exponentiated Weibull Distribution (IPLEWD) and Inverse Power Law-Weibull (IPLW) models

Model	AIC
IPLEWD	−65147.11
IPLW	−31476.47

Abbreviation: AIC: Akaike information criterion

of the component especially between the metal plate and the dielectric; this failure can be due to a degradation process of the metal and the current flowing through the capacitor. For this particular case, the IPLW model showed that the effect of the voltage in the SMEC is higher than the IPLEWD. This difference is because the IPLW model shows that the failure rate increases as the device consumes its useful life. However, this behavior does not reflect what the curve of the bathtub establishes as the behavior of a product so that although the failure increases with time, in practice this rate may not reflect the behavior of the device under voltage stress.

Parameters α and θ in the IPLEWD and parameter α in IPLW, represent a shape parameter and marked effects on the behavior of the lifetime and failure time distribution of SMEC. Moreover, the differences of those parameters affect the estimation of the lifetime and the performance of the SMEC directly. A way to show this difference is calculating the mean time to failure (MTTF). The MTTF of the SMEC under IPLBWD is calculated via the first moment of Equation 9. Thus, by computing the results obtained in Table 2 and setting the voltage applied to the component as $V = 50$ volts the $MTTF_{IPLEWD} = 14\,881$ hours.The MTTF for IPLW is calculated as $MTTF = \frac{1}{KV^n} \cdot \Gamma\left(\frac{1}{\alpha} + 1\right)$. In consequence, by taking the values obtained in Table 4 and setting the operational voltage of the device as $V = 50$ volts, the $MTTF_{IPLW} = 16\,485$ hours. The differences of estimations obtained in the MTTF of both models showed that the SMEC analyzed under IPLEWD model estimates 1604 hours less than IPLW model. That difference can be crucial in determining the warranty time of the capacitor when it is under real environmental conditions.

Nevertheless, the MTTF and the failure rate are not suitable information of the IPLEWD describes in better form the performance and reliability of ED than IPLW. Thus, the Akaike information criterion (AIC) is presented for both models in order to show more evidence of the strength of the proposed model. The results can be seen in Table 5.

Additionally, a Kolmogorov-Smirnov (K-S) test was performed by using the *fitdistrplus* package in R. For IPLEWD, the K-S statistics is 0.11 with AIC 404.15, and for IPLW, the K-S statistic is 0.12 with AIC 423.04.

In general, the evidence showed that IPLEWD estimates in better way the reliability performance of SMEC than IPLW.

8 | CONCLUSION AND FUTURE SCOPE

The presented article shows a reliability model, which analyzes the behavior of ED by fitting the failure time obtained in an ALT to EWD, and this distribution has the ability to modeling the nonmonotonic failure rate. By associate the IPL model with EWD, the performance and lifetime of ED are described in a better way when the ED is subjected to voltage profiles; additionally, IPLEWD model is more flexible than the IPLW, which is the first selection in reliability analysis when an ED is under ALT.

The comparison study made between the IPLEWD and IPLW model was performed to measure the difference of the estimation, the failure rate and the MTTF of the device under analysis. The results obtained in the case study showed that the SMEC under IPLEWD reduces its lifetime in around 1604 hours. That time can be crucial when a warranty or maintenance time is calculated.

Further, the difference obtained in the failure rate graphs under IPLEWD and IPLW model showed that the failure rate of SMEC is nonmonotone under the proposed model. That differs from traditional reliability analysis, which is performed using the IPLW and offers another perspective to fit the failure times to a more appropriate distribution. Besides, the AIC was calculated for the models under analysis, the results of this criteria showed that the IPLEWD describes in a better way the behavior, performance, and MTTF of SMEC than IPLW model used to the first instance.

The proposed model can be used for any application that showed a nonmonotone behavior such as ED, medical studies. A future work proposed for this model is to analyze the effects of the time-varying voltage (such as electrical harmonics situation) when the EWD describes the failure time of ED. Also, it can be possible to use the EWD to other reliability models such as temperature, vibration.

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APPENDIX A: OBSERVED FISHER MATRIX ELEMENTS

$$O_{kk} = -\frac{w}{k^2} - \left[(\theta - 1) \cdot \sum_{\zeta=1}^{w} \left\{ \frac{\alpha \cdot \left(kv_\zeta^n t_\zeta\right)^\alpha \cdot \left[\alpha \cdot e^{\left(kv_\zeta^n t_\zeta\right)^\alpha} \cdot \left(kv_\zeta^n t_\zeta\right)^\alpha - \alpha \cdot e^{\left(kv_\zeta^n t_\zeta\right)^\alpha} + e^{\left(kv_\zeta^n t_\zeta\right)^\alpha} + \alpha - 1 \right]}{k^2 \cdot \left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha} - 1 \right)^2} \right\} \right]$$

$$- \sum_{\zeta=1}^{w} \left[\frac{(\alpha - 1) \cdot \alpha \left(kv_\zeta^n t_\zeta\right)^\alpha}{k^2} \right] + \frac{1 - \alpha}{k^2}$$

$$O_{k\alpha}=\left[(\theta-1)\cdot\sum_{\zeta=1}^{w}\left\{\frac{\left(kv_\zeta^n t_\zeta\right)^\alpha\cdot\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}\cdot\alpha\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}\left(\left(kv_\zeta^n t_\zeta\right)^\alpha-1\right)+1\right)\cdot log\left(\left(kv_\zeta^n t_\zeta\right)^\alpha-1\right)\right)}{k\cdot\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1\right)^2}\right\}\right]$$

$$-\sum_{\zeta=1}^{w}\left[\frac{\left(kv_\zeta^n t_\zeta\right)^\alpha\cdot\left(\alpha\cdot log\left(kv_\zeta^n t_\zeta\right)+1\right)}{k}\right]+\frac{1}{k}$$

$$O_{k\theta}=\sum_{\zeta=1}^{w}\left\{\frac{\alpha\cdot\left(kv_\zeta^n t_\zeta\right)^\alpha}{k\cdot\left[e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1\right]}\right\}$$

$$O_{kn}=-\left[(\theta-1)\sum_{\zeta=1}^{w}\left\{\frac{\alpha^2\cdot log\left(v_\zeta\right)\cdot\left(kv_\zeta^n t_\zeta\right)^\alpha\cdot\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}\left(kv_\zeta^n t_\zeta\right)^\alpha-e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}+1\right)}{k\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1\right)^2}\right\}\right]$$

$$-\sum_{\zeta=1}^{w}\left\{\frac{\alpha^2\cdot log\left(v_\zeta\right)\cdot\left(kv_\zeta^n t_\zeta\right)^\alpha}{k}\right\}$$

$$O_{\alpha k}=O_{k\alpha}$$

$$O_{\alpha\alpha}=-\frac{w}{\alpha^2}-\left[\sum_{\zeta=1}^{w}\left\{\frac{\left(kv_\zeta^n t_\zeta\right)^\alpha\cdot\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}\left(kv_\zeta^n t_\zeta\right)^\alpha-e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}+1\right)\cdot log^2\left(kv_\zeta^n t_\zeta\right)}{\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1\right)^2}\right\}\right]$$

$$-\sum_{\zeta=1}^{w}\left\{\left(kv_\zeta^n t_\zeta\right)^\alpha log^2\left(kv_\zeta^n t_\zeta\right)\right\}$$

$$O_{\alpha\theta}=\sum_{\zeta=1}^{w}\left\{\frac{\left(kv_\zeta^n t_\zeta\right)^\alpha\cdot log\left(kv_\zeta^n t_\zeta\right)}{e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1}\right\}$$

$$O_{\alpha n}=\left[(\theta-1)\cdot\sum_{\zeta=1}^{w}\left\{\frac{log\left(v_j\right)\cdot\left(kv_\zeta^n t_\zeta\right)^\alpha\cdot\left(\alpha\cdot log\left(kv_\zeta^n t_\zeta\right)+1\right)}{e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1}-\frac{\alpha\cdot log\left(v_\zeta\right)\cdot e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}\left(kv_\zeta^n t_\zeta\right)^{2\alpha}\cdot log\left(kv_\zeta^n t_\zeta\right)}{\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1\right)^2}\right\}\right]$$

$$+\sum_{\zeta=1}^{w}\left\{log\left(v_\zeta\right)\left(kv_\zeta^n t_\zeta\right)^\alpha\cdot\left(\alpha\cdot log\left(kv_\zeta^n t_\zeta\right)+1\right)\right\}+\sum_{\zeta=1}^{w}\left\{log\left(v_\zeta\right)\right\}$$

$$O_{\theta k}=O_{k\theta}$$
$$O_{\theta\alpha}=O_{\alpha\theta}$$
$$O_{\theta\theta}=-\frac{\omega}{\theta^2}$$

$$O_{\theta n}=\sum_{\zeta=1}^{w}\left\{\frac{\alpha\cdot log\left(v_\zeta\right)\cdot kv_\zeta^n t_\zeta}{\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1\right)}\right\}$$

$$O_{nk}=O_{kn}$$
$$O_{n\alpha}=O_{\alpha n}$$
$$O_{n\theta}=O_{\theta n}$$

$$O_{nn}=-\left[(\theta-1)\cdot\sum_{\zeta=1}^{w}\left\{\frac{\alpha^2\cdot log^2\left(v_\zeta\right)\cdot\left(kv_\zeta^n t_\zeta\right)^\alpha\cdot\left(\left(kv_\zeta^n t_\zeta\right)^\alpha-e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}+1\right)}{\left(e^{\left(kv_\zeta^n t_\zeta\right)^\alpha}-1\right)^2}\right\}\right]$$
$$-\sum_{\zeta=1}^{w}\left\{\alpha^2\cdot log^2\left(v_\zeta\right)\cdot\left(kv_\zeta^n t_\zeta\right)^\alpha\right\}$$