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Hesitant Fuzzy Linguistic Term and TOPSIS to Assess Lean Performance

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Abstract: Manufacturing companies usually expect strategic improvements to focus on reducing both waste and variability in processes, whereas markets demand greater flexibility and low product costs. To deal with this issue, lean manufacturing (LM) emerged as a solution; however, it is often challenging to evaluate its true effect on corporate performance. This challenge can be overcome, nonetheless, by treating it as a multi-criteria problem using the Hesitant Fuzzy linguistic and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. In fact, the hesitant fuzzy linguistic term sets (HFLTS) is vastly employed in decision-making problems. The main contribution of this work is a method to assess the performance of LM applications in the manufacturing industry using the hesitant fuzzy set and TOPSIS to deal with criteria and attitudes from decision makers regarding such LM applications. At the end of the paper, we present a reasonable study to analyze the obtained results.

Keywords: hesitant linsguistic fuzzy term sets; TOPSIS; lean manufacturing; KPI

1. Introduction

Lean manufacturing (LM) combines a wide rage of management practices, such as just in time (JIT), quality systems, work teams, cellular manufacturing, and supply chain management (SCM) in a whole system [1]. The LM method aims at saving costs by reducing waste in the manufacturing system, thereby dealing with economic aspects [2]. Nowadays, LM covers the multiple stages of a product's life cycle, from its development and manufacturing to its delivery [3]; however, LM is also a challenge amid mass production practices, especially as quality products, and customer satisfaction are prioritized, inventory, time to market and manufacturing space, and everything that adds no value to a product is systematically categorized as waste [4]. LM is often discussed with respect to key performance indicators (KPIs) [5,6]. In addition, Kan et al. [7] affirm the KPI parameters have an association with LM performance. In fact, research evidence has found that LM practices have a positive impact on operational performance [8,9], yet it is often challenging to assess company performance with respect to LM implementation [10,11] and, according to [12,13], it is an attractive and hot topic for exploration through multi-criteria decision-making (MCDM) methodologies.

MCDM has recently gained relevance, especially in engineering [14,15]; however, when an MCDM problem involves objective and subjective information, experts discuss the classical hybrid MCDM method with the fuzzy sets theory [16,17]. In this sense, motivated by the hesitant fuzzy set, Rodríguez et al. [18] introduced the hesitant fuzzy linguistic term sets (HFLTS), which allows decision makers to elicit several linguistic terms for the same linguistic variable [17,19]. Nowadays, the HFLTS is a popular effective tool for representing hesitant qualitative judgments from decision makers; consequently, multiple HFLTS-based decision-making methods have been developed [20]. For instance, in their work, Hwang and Yoon [21] introduced the Technique of Order of Preference by Similarity to Ideal Solution (TOPSIS) method. TOPSIS method is denoted like a significant research issue, which has received a prodigious deal of attention from academics [22–25]. Additionally, there is HFLTS of TOPSIS proposed by [26,27].

The two main contributions of this work can be stated as follows: first, we propose an HFLTS-based data handling procedure to deal with lean manufacturing performance assessments. The procedure can handle KPI matrices of arbitrary preferences in decision-making situations. Second, we propose a systematic solution to measure the LM performance with respect to a series of criteria. The remainder of this paper is organized in five sections. Section 2 introduces a series of basic definitions of HFLTS and TOPSIS, whereas in Section 3 we present materials and methods which describes details about our application. Next, whereas Section 4 presents a numerical example to illustrate our approach to multi-attribute decision making, in Section 5, we describe the result analysis and discussions related to our method. Finally, research conclusions are proposed in Section 6.

2. Preliminaries

This section introduces basic definitions related to HFLTSs and TOPSIS, as they will be necessary to better understand subsequent sections.

2.1. Hesitant Fuzzy Linguistic Term Sets (HFLTSs)

An HFLTS is a very operative and flexible method that emphasizes one explicit type of complex language term, i.e., reasonable linguistic terms. An HFLTS is a successive, ordered, and finite subset of a specified linguistic term set [17].

Definition 1 ([28])**.** *Let us assume that Z_H is a fixed set of grammar and $T = \{L_{\varsigma 0}, \varsigma_1, \ldots, \varsigma_\rho\}$ depict a hesitant linguistic fuzzy set (HLFS) on Z using a function that when applied to Z encompasses a subset of [0,1]. At the same time, per convenience, the description of the grammar will be called H_ς. Then, the grammar set is presented, as follows: $\varsigma = (\varsigma_0 : nil; \varsigma_1 : insignificant; \varsigma_2 : medium–insignificant; \varsigma_3 : unbiased; \varsigma_4 : middle–good; \varsigma_5 : fine; \varsigma_6 : strong; \varsigma_7 : very–strong; \varsigma_8 : excellent)$,*

$$H_\varsigma = \{z_i, h_\varsigma(z) > |z_i \in Z\}. \tag{1}$$

Definition 2 ([18])**.** *Given an HFLTS H_ς as in Equation (2), its envelope, denoted by $env(H_\varsigma)$, is defined by an uncertain linguistic terms (ULT) [29] whose limits are the upper and lower bounds of H_ς, i.e.,*

$$Env(H_\varsigma) = \{H_\varsigma^-, H_\varsigma^+\}, \tag{2}$$

where $H^- = min(\varsigma_x)$ and $H^+ = max(\varsigma_x)$, $\forall \varsigma_z \in H_\varsigma$, $Z \in i, i+1, \ldots, j$.

Definition 3 ([30])**.** *Assuming that $\varsigma = \{\breve{\xi}_0, \breve{\xi}_1, \ldots, \breve{\xi}_\sigma\}$ represents a linguistic term set (LTS), HFLTS H_ς, is an ordered and finite subset of consecutive linguistic term of $\ddot{\xi}$.*

Definition 4 ([31])**.** *The score function is presented as follows:*

$$\lambda(H_\varsigma) = \frac{1}{n} \sum_{g=1}^{n} \chi_g, for\ g = 1, \ldots n. \tag{3}$$

Definition 5 (Distance [26,28,31,32]). *Assuming that H_1 and H_2 are two HFLTS and env $(H_S^1) = [\varsigma_\alpha, \varsigma_\beta]$ and env$(H_\varsigma^2) = [\varsigma_{\hat{\alpha}}, \varsigma_{\hat{\beta}}]$*

$$d(H_1, H_2) = |\hat{\beta} - \beta| + |\hat{\alpha} - \alpha|\,, \tag{4}$$

where β represents a higher element from H_1 and $\hat{\beta}$ depicts the maximum or higher element from H_2. Thus, α denotes a low element from H_1 and $\hat{\alpha}$ depicts the minimal or low element from H_2:

$$d(H_1, H_2) = \frac{1}{6\rho}\left(\left|I(H_\varsigma^{1+}) - I(H_\varsigma^{2+})\right| + \left|I(H_\varsigma^{1-}) - I(H_\varsigma^{2-})\right| + \left|v(H_\varsigma^1) - v(H_\varsigma^2)\right|\right), \tag{5}$$

where ρ depicts # of the elements of the set ς and, $I(\varsigma_i)$ stands for the subfix of linguistic term ς_i

Definition 6 ([30]). *Let $\varsigma = \{\varsigma_0, \ldots, \varsigma_\tau\}$ be a linguistic term set. A hesitant fuzzy linguistic term sets (HFLTS), H_ς, is an ordered and finite subset of the consecutive linguistic terms of S.*

In addition, an HFLTS can be used to elicit several linguistic values for a linguistic term, yet it is still not comparable to human thinking and reasoning processes. Thus, Rodríguez et al. [18] further presented a context-free grammar to describe linguistic terms that are more parallel to the human expressions and can be simply denoted by means of HFLTSs. In addition, according to [20], the grammar H_ς is used to express the linguistic term and transformed to HFLEs by using function $h_\varsigma(x_i)$:

$$h_\varsigma(x_i) = \{\varsigma_{\tau l}(x_i) | \varsigma_{\tau l}(x_i) \in \varsigma, l = 1, 2, \ldots \rho\}\,. \tag{6}$$

2.2. TOPSIS in Conventional Version

In this section, the conventional manner of TOPSIS is presented

Step 1. Establish the final decisión matrix.

Table 1 shows the set of the alternatives $A_i(A_1, A_2, \ldots A_m)$ and $C_j(C_1, C_2 \ldots C_n)$ be a finite set of criteria involved in the MCDM problem.

Table 1. Final decision matrix.

Alternatives	C_1	C_2	...	C_n
A_1	ϕ_{11}	ϕ_{12}	...	ϕ_{1n}
A_2	ϕ_{12}	ϕ_{22}	...	ϕ_{2n}
...	...	...	...	...
A_m	ϕ_{m1}	ϕ_{m2}	...	ϕ_{mn}

Step 2. Normalize the final decision matrix using Equation (6):

$$\xi_{ij} = \frac{\phi_{ij}}{\sqrt{\sum_{j=1}^{n} \phi_{ij}^2}}, \tag{7}$$

where $i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n.$

Step 3. Construct the aggregate matrix

$$\hat{R}_{ij} = w_z * \xi_{ij}, \tag{8}$$

where $i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n$

and w_z represents the weight vector of the criteria $C_j(j = 1, \ldots n)$

Step 4. Establish the vector ideal positive A^+ and the vector anti-ideal negative A^- by means of Equations (9) and (10):

$$\hat{R}_j^+ = \left\{(max(\hat{R}_{ij})|j \in \delta), (min(\hat{R}_{ij})|j \in \delta')\right\}, where\ A^+ = \{\hat{R}_1^+, \ldots \hat{R}_n^+\}, \quad (9)$$

$$\hat{R}_j^- = \left\{min(\hat{R}_{ij})|j \in \delta), (max(\hat{R}_{ij})|j \in \delta')\right\}, where\ A^- = \{\hat{R}_1^-, \ldots \hat{R}_n^-\}, \quad (10)$$

where δ depicts the sets of benefit criteria and δ' represents the sets of cost criteria.

Step 5. Compute the S_i^+ and S_i^-

$$S_i^+ = \sqrt{\sum_{j=1}^{n}(\hat{R}_{ij} - \hat{R}_j^+)^2}\ j = 1, 2 \ldots m, \quad (11)$$

$$S_i^+ = \sqrt{\sum_{j=1}^{n}(\hat{R}_{ij} - \hat{R}_j^-)^2}\ j = 1, 2 \ldots m. \quad (12)$$

Step 6. Ranking of the alternatives

$$K_i = \frac{S_i^-}{S_i^+ + S_i^-}. \quad (13)$$

3. Materials and Methods

This section presents the material and method used in the investigation. We introduce the procedure of HFLTs and TOPSIS for the Lean Improvement Assessment. At the same time, in order to explain the proposed method, Figure 1 shows the flowcharts of the different steps about it.

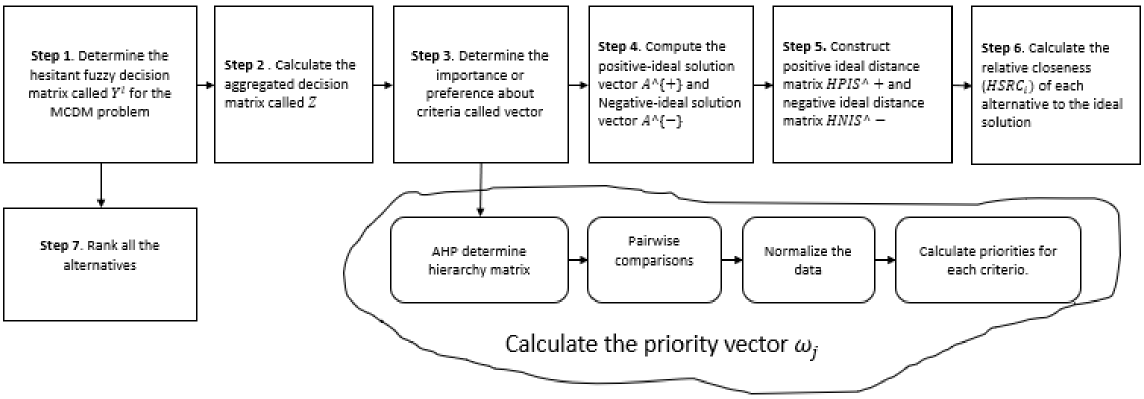

Figure 1. Flowcharts of the algorithms to assess the Lean Manufacturing Performance.

Hesitant Fuzzy Linguistic Term and TOPSIS to Assess Lean Performance

In this section, we introduce an algorithm through a Hesitant Fuzzy Linguistic Term and TOPSIS in order to be applied to Lean Improvement Assessment. The method is described in the following steps:

Step 1. Determine the hesitant fuzzy decision matrix called $Y^l = [\rho_{ij}^l]_{mxn}$ for the MCDM problem. Appraisal the alternative with respect to DM preferences and the criteria.

Step 2. Calculate the aggregated decision matrix called Z. This process requires the aggregation of the preferences of the DMs $(Y^1, Y^2, \ldots, Y^k)$ through Equations (14) and (15).

Then, $Z = [z_{ij}]$, where $z_{ij} = [\varsigma_{p_{ij}}, \varsigma_{q_{ij}}]$,

$$\varsigma_{p_{ij}} = min\left\{\min_{l=1}^{k}(max\ \rho_{ij}^{l}), \max_{l=1}^{k}(min\ \rho_{ij}^{l})\right\}, \tag{14}$$

$$\varsigma_{q_{ij}} = max\left\{\min_{l=1}^{k}(max\ \rho_{ij}^{l}),\ \max_{l=1}^{k}(min\ \rho_{ij}^{l})\right\}. \tag{15}$$

Step 3. Determine the importance or preference about criteria called vector ω_j, for the MCDM problem via Analytic Hierarchy Process (AHP) method proposed by [33]. Appraise the criteria with respect to DM preferences.

Step 4. Compute the positive-ideal solution vector (A^+) and Negative-ideal solution vector (A^-). At this mode, the evaluation of alternative A_i by mean of criterion C_j is symbolized as z_{ij} using an aggregated matrix Z. Thus, θ_B depicts a set of benefit criteria and represents the greater preference of the criterion C_j and θ_C depicts a set of cost criteria and describes the smaller preference of the criterion C_j:

$$\dot{A}^+ = \left[\left(\max_{l=1}^{k}\left(\max_{i}\ \rho_{ij}^{l}\right)\right)|j \in \theta_B,\ \left(\min_{l=1}^{k}\left(\min_{i}\ \rho_{ij}^{l}\right)\right)|j \in \theta_C\right], where\ i = 1, \ldots, m. \tag{16}$$

Then, $A^+ = \left[\dot{R}_1^+, \dot{R}_2^+ \ldots, \dot{R}_n^+\right]; \dot{R}_j^+ = [\varsigma_{p_{ij}}, \varsigma_{q_{ij}}](j = 1, \ldots n)$,

$$\dot{A}^- = \left[\left(\min_{l=1}^{k}\left(\min_{i}\ \rho_{ij}^{l}\right)\right)|j \in \theta_B,\ \left(\max_{l=1}^{k}\left(\max_{i}\ \rho_{ij}^{l}\right)\right)|j \in \theta_C\right], where\ i = 1, \ldots, m, \tag{17}$$

thus $A^- = \left[\dot{R}_1^-, \dot{R}_2^- \ldots, \dot{R}_n^-\right]; \dot{R}_j^- = [\varsigma_{p_{ij}}, \varsigma_{q_{ij}}](j = 1, \ldots n)$.

Step 5. Construct positive ideal distance matrix $(HPIS^+)$ and negative ideal distance matrix $(HNIS^-)$, which are denoted as follows:

$$HPIS^+ = \begin{pmatrix} \omega_1 d(z_{11}, R_1^+) & + & \ldots & + & \omega_n d(z_{1n}, R_n^+) \\ \omega_2 d(z_{21}, R_1^+) & + & \ldots & + & \omega_1 d(z_{2n}, R_n^+) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega_j d(z_{m1}, R_1^+) & + & \ldots & + & \omega_j d(z_{mn}, R_2^+) \end{pmatrix}, \tag{18}$$

$$HNIS^- = \begin{pmatrix} \omega_1 d(z_{11}, R_1^-) & + & \ldots & + & \omega_n d(z_{1n}, R_n^-) \\ \omega_2 d(z_{21}, R_1^-) & + & \ldots & + & \omega_n d(z_{2n}, R_n^-) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega_j d(z_{m1}, R_1^-) & + & \ldots & + & \omega_n d(z_{mn}, R_n^-) \end{pmatrix}. \tag{19}$$

Step 6. Calculate the relative closeness $(HSRC_i)$ of each alternative to the ideal solution as follows:

$$HSRC_i = \frac{HNIS^-}{HNIS^- + HPIS^+}, \tag{20}$$

where

$$HPIS^+ = \sum_{j=1}^{n} \omega_j d(z_{ij}, R_j^+),$$

and

$$HNIS^- = \sum_{j=1}^{n}\ \omega_j d(z_{ij}, R_j^-).$$

Step 7. Rank all the alternatives

4. Numerical Example

This section introduces a real-life example, which was applied in an automotive company based in Ciudad Juárez, Chihuahua, Mexico. The company works under an LM methodology and focuses on minimizing operational waste; thus, managers are particularly interested in assessing the real impact of the LM methodology. To this end, a group of experts first assessed the company's LM implementation improvement metrics. Simultaneously, we described the set of criteria and the KPIs depicted like alternatives as follows: C_1: Defects, C_2: Productivity, C_3: Lead time, C_4: Customer, C_5: Demand satisfaction, C_6: Cycle time, C_7: Tack time, C_8: Effectiveness, C_9: Levels of inventory C_{10}: Suppliers. Additionally, during the evaluation of lean projects, nineteen alternatives to be considered are summarized: A1: Sales, A2: Markeshare, A3: Maintenance, A4: OEE, A5: On-time delivery, A6: 5,S, A7: KAIZEN, A8: Bottleneck removal, A9: Cross-functional work force, A10: Focused factory production, A11: JIT/continuous flow production, A12: Lot size reductions, A13: Maintenance optimization, A14: Process capability measurements, A15: Kanban, A16: Quick changeover, A17: Total quality management, A18: Self-directed work teams, A19: Safety improvement programs.

Step 1. Determine the hesitant fuzzy decision matrix called ρ_{ij} for the MCDM problem. Appraise the alternative with respect to DM preferences and the criteria. Establish the final decision matrix. Let $Y^l = [\rho_{ij}^l]_{mxn}$ be a fuzzy decision matrix for the MCDM problem, and the following notations are used to depict the considered problems. At the same time, the matrices (Tables 2 and 3) describe the preferences DM_1,DM_2, DM_3, DM_4, DM_5 and DM_6.

Table 2. Decision matrix Y^1 with respect to decision makers 1, 2, and 3.

Item	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A_1	$\{\varsigma_3,\varsigma_8\}$	$\{\varsigma_4,\varsigma_5,\varsigma_7\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_1,\varsigma_3,\varsigma_5\}$	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_0,\varsigma_3\}$	$\{\varsigma_4,\varsigma_6,\varsigma_7\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_3,\varsigma_4,\varsigma_8\}$
A_2	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_0,\varsigma_3,\varsigma_5\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_2,\varsigma_4,\varsigma\varsigma_6\}$	$\{\varsigma_2,\varsigma_3,\varsigma_5\}$	$\{\varsigma_0,\varsigma_2\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_0,\varsigma_3,\varsigma_4\}$
A_3	$\{\varsigma_2,\varsigma_4,\varsigma_6\}$	$\{\varsigma_2,\varsigma_3,\varsigma_4\}$	$\{\varsigma^6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_1,\varsigma_2,\varsigma_3\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7$	$\{\varsigma_4,\varsigma_6,\varsigma_7\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_7\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1\}$
A_4	$\{\varsigma_3,\varsigma_7,\varsigma_8\}$	$\{\varsigma_5,\varsigma_7\}$	$\{\varsigma_1,\varsigma_3,\varsigma_4\}$	$\{\varsigma_1,\varsigma_3,\varsigma_4\}$	$\{\varsigma_0,\varsigma_3\}$	$\{\varsigma_5,\varsigma_6,\varsigma_8\}$	$\{\varsigma_4,\varsigma_6,\varsigma_7\}$	$\{\varsigma_4,\varsigma_5,\varsigma_7\}$	$\{\varsigma_6,\varsigma_8\}$	$\{\varsigma_2,\varsigma_3,\varsigma_4\}$
A_5	$\{\varsigma_1,\varsigma_2\}$	$\{\varsigma_1,\varsigma_4\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_4,\varsigma_5,\varsigma_8\}$	$\{\varsigma_3,\varsigma_4,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_4,\varsigma_6\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$
A_6	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_1,\varsigma_3,\varsigma_4\}$	$\{\varsigma_0,\varsigma_2,\varsigma_3\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_4,\varsigma_5,\varsigma_7\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_2,\varsigma_5\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$
A_7	$\{\varsigma_0,\varsigma_3\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_1,\varsigma_3,\varsigma_4\}$	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_1,\varsigma_3,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_4,\varsigma_5,\varsigma_8\}$	$\{\varsigma_6,\varsigma_8\}$
A_8	$\{\varsigma_6,\varsigma_8\}$	$\{\varsigma_3,\varsigma_4,\varsigma_5\}$	$\{\varsigma_1,\varsigma_2,\varsigma_8\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_3,\varsigma_4,\varsigma_6\}$	$\{\varsigma_2,\varsigma_3,\varsigma_7\}$	$\{\varsigma_1,\varsigma_2,\varsigma_3\}$	$\{\varsigma_4,\varsigma_6,\varsigma_7\}$	$\{\varsigma_3,\varsigma_4,\varsigma_5\}$
A_9	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_6,\varsigma_8\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_0,\varsigma_1,\varsigma_4\}$	$\{\varsigma_5,\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_3\}$	$\{\varsigma_2,\varsigma_4\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_3,\varsigma_5,\varsigma_8\}$
A_{10}	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_1,\varsigma_6,\varsigma_7\}$	$\{\varsigma_5,\varsigma_8\}$	$\{\varsigma_3,\varsigma_7\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_2,S_4,\varsigma_6\}$	$\{\varsigma_2,\varsigma_5,\varsigma_8\}$	$\{\varsigma_2,\varsigma_4,\varsigma_6\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1\}$
A_{11}	$\{\varsigma_4,\varsigma_5,\varsigma_7\}$	$\{\varsigma_0,\varsigma_2,\varsigma_3\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_2,\varsigma_4,\varsigma_6\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_2,\varsigma_3,\varsigma_4\}$	$\{\varsigma_4,\varsigma_5,\varsigma_7\}$	$\{\varsigma_2,\varsigma_5,\varsigma_6\}$	$\{\varsigma_4,\varsigma_5,\varsigma_8\}$	$\{\varsigma_6,\varsigma_7\}$
A_{12}	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_1,\varsigma_4,\varsigma_5\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_4,\varsigma_5\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,S_4,\varsigma_6\}$	$\{\varsigma_1,\varsigma_2,\varsigma_7\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_5,\varsigma_8\}$	$\{\varsigma_2,\varsigma_3,\varsigma_5\}$
A_{13}	$\{\varsigma_6,\varsigma_8\}$	$\{\varsigma_5,\varsigma_7\}$	$\{\varsigma_3,\varsigma_4,\varsigma_5\}$	$\{\varsigma_4,\varsigma_6,\varsigma_8\}$	$\{\varsigma_0,\varsigma_2,\varsigma_4\}$	$\{\varsigma_3,\varsigma_4,\varsigma_6\}$	$\{\varsigma_0,\varsigma_2,\varsigma_4\}$	$\{\varsigma_0,\varsigma_5\}$	$\{\varsigma_4,\varsigma_6,\varsigma_8\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$
A_{14}	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_5,\varsigma_8\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6,\varsigma_8\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_1,\varsigma_3,\varsigma_4\}$
A_{15}	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_3,\varsigma_5,\varsigma_6\}$	$\{\varsigma_0,\varsigma_2,\varsigma_5\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_2,\varsigma_4\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_4,\varsigma_6\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_3,\varsigma_4,\varsigma_6\}$
A_{16}	$\{\varsigma_1,\varsigma_2,\varsigma_5\}$	$\{\varsigma_2,\varsigma_6\}$	$\{\varsigma_1,\varsigma_6\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_4,\varsigma_6\}$	$\{\varsigma_4,\varsigma_7\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_0,\varsigma_1,\varsigma_4\}$
A_{17}	$\{\varsigma_3,\varsigma_5\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_5,\varsigma_6,\varsigma_8\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_1,\varsigma_2\}$	$\{\varsigma_0,\varsigma_1,\varsigma_4\}$	$\{\varsigma_6,\varsigma_8\}$	$\{\varsigma_2,\varsigma_3,\varsigma_4\}$	$\{\varsigma_5,\varsigma_7\}$	$\{\varsigma_2,\varsigma_3\}$
A_{18}	$\{\varsigma_3,\varsigma_5,\varsigma_6\}$	$\{\varsigma_2,\varsigma_3,\varsigma_5\}$	$\{\varsigma_4,\varsigma_7\}$	$\{\varsigma_4,\varsigma_6,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_6,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_1,\varsigma_2,\varsigma_8\}$
A_{19}	$\{\varsigma_4,\varsigma_7\}$	$\{\varsigma_1,\varsigma_2\}$	$\{\varsigma_6,\varsigma_8\}$	$\{\varsigma_2,\varsigma_5,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_0,\varsigma_2\}$	$\{\varsigma_5,\varsigma_7\}$	$\{\varsigma_1,\varsigma_3,\varsigma_5\}$	$\{\varsigma_6,\varsigma_8\}$

Table 3. Decision matrix Y^2 with respect to decision makers 4, 5, and 6.

Item	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A_1	$\{\varsigma_2,\varsigma_5\}$	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_2,\varsigma_7\}$	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_0,\varsigma_2,\varsigma_3\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_1,\varsigma_2,\varsigma_3\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_1,\varsigma_5\}$	$\{\varsigma_1,\varsigma_2,\varsigma_3\}$
A_2	$\{\varsigma_1,\varsigma_4\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_2,\varsigma_4\}$	$\{\varsigma_1,\varsigma_3,\varsigma_4\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_0,\varsigma_2\}$	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_2,\varsigma_4,\varsigma_5\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_5,\varsigma_6\}$
A_3	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_1,\varsigma_2,\varsigma_4\}$	$\{\varsigma_3,\varsigma_6\}$	$\{\varsigma_0,\varsigma_3\}$	$\{\varsigma_1,\varsigma_4\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_2,\varsigma_3,\varsigma_5\}$	$\{\varsigma_2,\varsigma_3,\varsigma_4\}$	$\{\varsigma_2,\varsigma_3\}$
A_4	$\{\varsigma_2,\varsigma_3,\varsigma_4\}$	$\{\varsigma_2,\varsigma_4\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_0,\varsigma_2,\varsigma_4\}$	$\{\varsigma_1,\varsigma_2,\varsigma_7\}$	$\{\varsigma_0,\varsigma_4\}$	$\{\varsigma_0,\varsigma_1,\varsigma_8\}$	$\{\varsigma_6,\varsigma_7\}$
A_5	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_2,\varsigma_5,\varsigma_7\}$	$\{\varsigma_1,\varsigma_2,\varsigma_5\}$	$\{\varsigma_7,S_8\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\varsigma_1,\varsigma_2,\varsigma_3\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_2,\varsigma_3\}$
A_6	$\{\varsigma_2,\varsigma_4\}$	$\{\varsigma_1,\varsigma_3\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_4,\varsigma_6\}$	$\{\varsigma_2,\varsigma_4,\varsigma_5\}$	$\{S_1,\varsigma_6\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_5,\varsigma_7,\varsigma_8\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_2,\varsigma_4,\varsigma_6\}$
A_7	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_4,\varsigma_7\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_3,\varsigma_5,\varsigma_6\}$	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_4,\varsigma_5,\varsigma_7\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_3,\varsigma_4\}$	$\{S_0,\varsigma_2,\varsigma_3\}$	$\{\varsigma_2,\varsigma_3\}$
A_8	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_4,\varsigma_7\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1\varsigma_2\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_1,\varsigma_4\}$	$\{\varsigma_1,\varsigma_2,\varsigma_3\}$
A_9	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_2,\varsigma_5\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_4,\varsigma_7\}$	$\{\varsigma_2,\varsigma_3,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$
A_{10}	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_8\}$	$\{\varsigma_3,\varsigma_4,\varsigma_5\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_5,\varsigma_6,\varsigma_8\}$	$\{\varsigma_6,S_7\}$	$\{\varsigma_2,\varsigma_4,\varsigma_5\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_2,\varsigma_3,\varsigma_7\}$	$\{\varsigma_5,\varsigma_7\}$
A_{11}	$\{\varsigma_1,\varsigma_2,\varsigma_3\}$	$\{\varsigma_2,\varsigma_4,\varsigma_6\}$	$\{\varsigma_4,\varsigma_6\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_1,\varsigma_4,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_2,\varsigma_3,\varsigma_7\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_2,\varsigma_4,\varsigma_5\}$
A_{12}	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_5,\varsigma_6\}$	$\{\varsigma_0,\varsigma_5\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_1,\varsigma_2,\varsigma_5\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_1,\varsigma_2,\varsigma_3\}$	$\{\varsigma_0,\varsigma_1\}$
A_{13}	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_5,\varsigma_6,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_0,S_1,\varsigma_3\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_8\}$
A_{14}	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_0,\varsigma_2,\varsigma_3\}$	$\{\varsigma_1,\varsigma_2\}$	$\{\varsigma_4,\varsigma_5,\varsigma_6\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_0,\varsigma_2,\varsigma_5\}$	$\{\varsigma_5,\varsigma_6\}$
A_{15}	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_2,\varsigma_4,\varsigma_5\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_5,\varsigma_7,\varsigma_8\}$	$\{\varsigma_1,\varsigma_2,\varsigma_7\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_5,\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_2\}$
A_{16}	$\{\varsigma_7,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_6,\varsigma_7\}$	$\{\varsigma_1,\varsigma_3,\varsigma_4\}$	$\{\varsigma_0,\varsigma_2,\varsigma_4\}$	$\{\varsigma_0,S_1,\varsigma_2\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_6,\varsigma_7,\varsigma_8\}$	$\{\varsigma_4,\varsigma_7\}$
A_{17}	$\{\varsigma_8\}$	$\{\varsigma_6,\varsigma_8\}$	$\{\varsigma_1,\varsigma_2\}$	$\{\varsigma_0,\varsigma_1,\varsigma_6\}$	$\{\varsigma_2,\varsigma_3,\varsigma_6\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_0,\varsigma_5\}$	$\{\varsigma_4,\varsigma_5\}$	$\{\varsigma_1,\varsigma_4,\varsigma_5\}$	$\{\varsigma_4,\varsigma_5\}$
A_{18}	$\{\varsigma_0,\varsigma_1,\varsigma_2\}$	$\{\varsigma_5,\varsigma_7\}$	$\{\varsigma_1,\varsigma_3,\varsigma_4\}$	$\{\varsigma_3,\varsigma_4\}$	$\{\varsigma_3,\varsigma_4,\varsigma_8\}$	$\{\varsigma_0,\varsigma_1,\varsigma_3\}$	$\{\varsigma_4,\varsigma_5,\varsigma_7\}$	$\{\varsigma_1,\varsigma_2,\varsigma_3\}$	$\{\varsigma_2,\varsigma_3,\varsigma_5\}$	$\{\varsigma_0,\varsigma_1\}$
A_{19}	$\{\varsigma_1,\varsigma_2\}$	$\{\varsigma_3,\varsigma_4,\varsigma_6\}$	$\{\varsigma_2,\varsigma_5\}$	$\{\varsigma_0,\varsigma_1\}$	$\{\varsigma_5,\varsigma_6,\varsigma_7\}$	$\{\varsigma_2,\varsigma_4,\varsigma_6\}$	$\{\varsigma_3,\varsigma_6\}$	$\{\varsigma_2,\varsigma_3,\varsigma_4\}$	$\{\varsigma_2,\varsigma_3\}$	$\{\varsigma_0,\varsigma_2,\varsigma_4\}$

Step 2. Calculate the aggregated decision matrix called Z. This process requires the aggregation of the preferences of the DMs using the matrices $(Y^1 and Y^2)$ through Equations (14) and (15). Table 4 shows the hesitant aggregated matrix called Z.

Table 4. Decision hesitant aggregated matrix Z.

Item	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**
A_1	$\{\hat{\varsigma}_2,\hat{\varsigma}_8\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_3\}$
A_2	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$
A_3	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_2\}$
A_4	$\{\hat{\varsigma}_2,\hat{\varsigma}_8\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_6\}$
A_5	$\{\hat{\varsigma}_1,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$
A_6	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_7,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_6\}$
A_7	$\{\hat{\varsigma}_3,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_6\}$
A_8	$\{\hat{\varsigma}_2,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_3\}$
A_9	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_3\}$
A_{10}	$\{\hat{\varsigma}_4,\hat{\varsigma}^6\}$	$\{\hat{\varsigma}_7,\hat{\varsigma}_8\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_7,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_5\}$
A_{11}	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$
A_{12}	$\{\hat{\varsigma}_3,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_1\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_2\}$
A_{13}	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_8\}$
A_{14}	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$
A_{15}	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_1\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_3\}$
A_{16}	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_6,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$
A_{17}	$\{\hat{\varsigma}_5,\hat{\varsigma}_8\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_8\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_4\}$
A_{18}	$\{\hat{\varsigma}_2,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_7\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_3,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_1\}$
A_{19}	$\{\hat{\varsigma}_2,\hat{\varsigma}_4\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_5,\hat{\varsigma}_6\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_1,\hat{\varsigma}_2\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_5\}$	$\{\hat{\varsigma}_2,\hat{\varsigma}_3\}$	$\{\hat{\varsigma}_4,\hat{\varsigma}_6\}$
	min	max	min	max	max	max	max	max	max	max

Step 3. Determine the importance or preference about criteria called vector ω_j for the MCDM problem via the Analytic Hierarchy Process (AHP) method. Appraise the criteria with respect to DM preferences. Table 5 depict the preferences of the criteria in order to obtain the vector ω_j.

Table 5. Analytic Hierarchy Process (AHP) matrix.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**
C1	1	8	3	3	2	4	3	5	3	2
C2	1/8	1	3	4	3	2	5	4	2	3
C3	1/3	1/3	1	3	2	5	6	2	3	2
C4	1/3	1/4	1/3	1	2	3	4	5	2	3
C5	1/2	1/3	1/2	1/2	1	3	2	3	3	2
C6	1/4	1/2	1/5	1/3	1/3	1	4	2	2	4
C7	1/3	1/5	1/6	1/4	1/2	1/4	1	3	4	3
C8	1/5	1/4	1/2	1/5	1/3	1/2	1/3	1	2	2
C9	1/3	1/2	1/3	1/2	1/3	1/2	1/4	1/2	1	5
C10	1/2	1/3	1/3	1/3	1/2	1/4	1/3	1/2	1/5	1

$$\omega_i = \{0.238, 0.164, 0.139, 0.109, 0.089, 0.070, 0.064, 0.041, 0.051, 0.035\}^T.$$

Step 4. Compute the positive-ideal solution vector $(\dot{A}^+)$ and Negative-ideal solution vector$(\dot{A}^-)$:

$$\dot{A}^+ = (\{S0, S2\}, \{S6, S8\}, \{S0, S2\}, \{S7, S8\}, \{S7, S8\}, \{S7, S8\}, \{S7, S8\}, \{S7, S8\}, \{S7, S8\}, \{S6, S8\}),$$

$$\dot{A}^- = (\{S7, S8\}, \{S0, S2\}, \{S7, S8\}, \{S0, S1\}, \{S0, S1\}, \{S0, S1\}, \{S0, S1\}, \{S0, S1\}, \{S0, S1\}, \{S0, S1\}).$$

Step 5. Construct positive ideal distance matrix $(HPIS^+)$ and negative ideal distance matrix $(HNIS^-)$, which are denoted as follows:

$$HPIS^+ = \begin{pmatrix}
1.902 & + & 0.986 & + & 0.278 & + & 0.761 & + & 0.709 & + & 0.632 & + & 0.511 & + & 0.328 & & 0.154 & + & 0.280 \\
0.476 & + & 0.657 & + & 0.834 & + & 0.652 & + & 0.976 & + & 0.772 & + & 0.639 & + & 0.205 & + & 0.103 & + & 0.175 \\
0.951 & + & 0.986 & + & 0.973 & + & 0.978 & + & 0.621 & + & 0.491 & + & 0.639 & + & 0.123 & + & 0.257 & + & 0.385 \\
1.902 & + & 0.657 & + & 1.112 & + & 0.435 & + & 0.532 & + & 0.421 & + & 0.447 & + & 0.287 & + & 0.360 & + & 0.140 \\
0.951 & + & 1.150 & + & 0.556 & + & 1.196 & + & 0.621 & + & 0.140 & + & 0.447 & + & 0.328 & + & 0.360 & + & 0.140 \\
1.427 & + & 1.150 & + & 1.251 & + & 0.869 & + & 0.355 & + & 0.562 & + & 0.064 & + & 0.164 & + & 0.154 & + & 0.070 \\
0.951 & + & 1.315 & + & 0.834 & + & 1.087 & + & 0.621 & + & 0.491 & + & 0.192 & + & 0.246 & + & 0.411 & + & 0.175 \\
0.476 & + & 0.657 & + & 1.251 & + & 0.326 & + & 0.443 & + & 0.702 & + & 0.767 & + & 0.246 & + & 0.360 & + & 0.280 \\
2.615 & + & 0.657 & + & 1.390 & + & 0.652 & + & 0.532 & + & 0.491 & + & 0.511 & + & 0.369 & + & 0.360 & + & 0.280 \\
1.902 & + & 0.164 & + & 1.112 & + & 1.087 & + & 0.709 & + & 0.211 & + & 0.511 & + & 0.123 & + & 0.051 & + & 0.280 \\
1.189 & + & 1.150 & + & 0.695 & + & 0.217 & + & 0.266 & + & 0.421 & + & 0.256 & + & 0.082 & + & 0.257 & + & 0.105 \\
1.664 & + & 0.657 & + & 1.251 & + & 0.435 & + & 0.355 & + & 0.140 & + & 0.831 & + & 0.328 & + & 0.360 & + & 0.385 \\
2.140 & + & 0.657 & + & 0.556 & + & 0.978 & + & 0.532 & + & 0.632 & + & 0.319 & + & 0.123 & + & 0.514 & + & 0.035 \\
1.189 & + & 0.986 & + & 0.695 & + & 0.543 & + & 0.709 & + & 0.562 & + & 0.447 & + & 0.287 & + & 0.257 & + & 0.175 \\
2.615 & + & 0.493 & + & 1.529 & + & 0.543 & + & 1.153 & + & 0.772 & + & 0.703 & + & 0.082 & + & 0.360 & + & 0.315 \\
2.140 & + & 1.643 & + & 1.390 & + & 0.543 & + & 0.621 & + & 0.632 & + & 0.767 & + & 0.205 & + & 0.411 & + & 0.210 \\
2.615 & + & 0.657 & + & 0.695 & + & 0.652 & + & 0.976 & + & 0.421 & + & 0.256 & + & 0.287 & + & 0.257 & + & 0.245 \\
0.713 & + & 0.657 & + & 0.834 & + & 0.761 & + & 0.976 & + & 0.351 & + & 0.447 & + & 0.246 & + & 0.617 & + & 0.420 \\
1.427 & + & 1.479 & + & 1.251 & + & 1.304 & + & 0.709 & + & 0.842 & + & 0.639 & + & 0.246 & + & 0.514 & + & 0.140
\end{pmatrix} = \begin{pmatrix}
6.542 \\ 5.489 \\ 6.405 \\ 6.294 \\ 5.889 \\ 6.066 \\ 6.323 \\ 5.508 \\ 7.859 \\ 6.151 \\ 4.638 \\ 6.406 \\ 6.487 \\ 5.851 \\ 8.566 \\ 8.562 \\ 7.061 \\ 6.022 \\ 8.552
\end{pmatrix},$$

$$HNIS^- = \begin{pmatrix}
1.189 & + & 0.986 & + & 1.529 & + & 0.761 & + & 0.532 & + & 0.351 & + & 0.383 & + & 0.246 & + & 0.566 & + & 0.175 \\
1.664 & + & 1.315 & + & 0.973 & + & 0.869 & + & 0.266 & + & 0.211 & + & 0.256 & + & 0.369 & + & 0.617 & + & 0.280 \\
2.615 & + & 0.986 & + & 0.834 & + & 0.543 & + & 0.621 & + & 0.491 & + & 0.256 & + & 0.451 & + & 0.463 & + & 0.070 \\
1.189 & + & 1.315 & + & 0.695 & + & 1.087 & + & 0.709 & + & 0.562 & + & 0.447 & + & 0.287 & + & 0.360 & + & 0.315 \\
2.140 & + & 0.822 & + & 1.251 & + & 0.326 & + & 0.621 & + & 0.842 & + & 0.447 & + & 0.246 & + & 0.360 & + & 0.315 \\
1.664 & + & 0.822 & + & 0.556 & + & 0.652 & + & 0.887 & + & 0.421 & + & 0.831 & + & 0.410 & + & 0.566 & + & 0.385 \\
2.140 & + & 0.657 & + & 0.973 & + & 0.435 & + & 0.621 & + & 0.491 & + & 0.703 & + & 0.328 & + & 0.309 & + & 0.280 \\
1.664 & + & 1.315 & + & 0.556 & + & 1.196 & + & 0.798 & + & 0.281 & + & 0.128 & + & 0.328 & + & 0.360 & + & 0.175 \\
0.476 & + & 1.315 & + & 0.417 & + & 0.869 & + & 0.709 & + & 0.491 & + & 0.383 & + & 0.205 & + & 0.360 & + & 0.175 \\
1.189 & + & 2.136 & + & 0.695 & + & 0.435 & + & 0.532 & + & 0.772 & + & 0.383 & + & 0.451 & + & 0.669 & + & 0.175 \\
1.902 & + & 0.822 & + & 1.112 & + & 1.304 & + & 0.976 & + & 0.562 & + & 0.639 & + & 0.492 & + & 0.463 & + & 0.350 \\
1.427 & + & 1.315 & + & 0.556 & + & 1.087 & + & 0.887 & + & 0.842 & + & 0.064 & + & 0.246 & + & 0.360 & + & 0.070 \\
0.951 & + & 1.315 & + & 1.251 & + & 0.543 & + & 0.709 & + & 0.351 & + & 0.575 & + & 0.451 & + & 0.206 & + & 0.420 \\
1.902 & + & 0.986 & + & 1.112 & + & 0.978 & + & 0.532 & + & 0.421 & + & 0.447 & + & 0.287 & + & 0.463 & + & 0.280 \\
0.476 & + & 1.479 & + & 0.278 & + & 0.978 & + & 0.089 & + & 0.211 & + & 0.192 & + & 0.492 & + & 0.360 & + & 0.140 \\
0.951 & + & 0.329 & + & 0.417 & + & 0.978 & + & 0.621 & + & 0.351 & + & 0.128 & + & 0.369 & + & 0.309 & + & 0.245 \\
0.476 & + & 1.315 & + & 1.112 & + & 0.869 & + & 0.266 & + & 0.562 & + & 0.639 & + & 0.287 & + & 0.463 & + & 0.210 \\
2.378 & + & 1.315 & + & 0.973 & + & 0.761 & + & 0.266 & + & 0.632 & + & 0.447 & + & 0.328 & + & 0.103 & + & 0.035 \\
2.140 & + & 0.493 & + & 0.556 & + & 0.217 & + & 0.532 & + & 0.140 & + & 0.256 & + & 0.328 & + & 0.206 & + & 0.315
\end{pmatrix} = \begin{pmatrix}
6.718 \\ 6.820 \\ 7.331 \\ 6.966 \\ 7.370 \\ 7.194 \\ 6.937 \\ 6.800 \\ 5.401 \\ 7.437 \\ 8.621 \\ 6.853 \\ 6.772 \\ 7.409 \\ 4.694 \\ 4.697 \\ 6.198 \\ 7.237 \\ 5.183
\end{pmatrix}.$$

Step 6. Calculate the relative closeness $(HSRC_i)$ of each alternative to the ideal solution as follows:

Table 6. Relative closeness $(HSRC_i)$.

Item	HPIS$^+$	HNIS$^-$	HSRC$_i$	Ranking
$A1$	6.542	6.718	0.507	14
$A2$	5.489	6.820	0.554	4
$A3$	6.405	7.331	0.534	9
$A4$	6.294	6.966	0.525	10
$A5$	5.889	7.370	0.556	3
$A6$	6.066	7.194	0.543	8
$A7$	6.323	6.937	0.523	11
$A8$	5.508	6.800	0.552	5
$A9$	7.859	5.401	0.407	16
$A10$	6.151	7.437	0.547	6
$A11$	4.638	8.621	0.650	1
$A12$	6.406	6.853	0.517	12
$A13$	6.487	6.772	0.511	13
$A14$	5.851	7.409	0.559	2
$A15$	8.566	4.694	0.354	19
$A16$	8.562	4.697	0.354	18
$A17$	7.061	6.198	0.467	15
$A18$	6.022	7.237	0.546	7
$A19$	8.552	5.183	0.377	17

Table 6 depict the hesitant relative closeness index called $HSRC_i$

Step 7. Ranking of the alternatives.

$$A_{11} \succ A_{14} \succ A_5 \succ A_2 \succ A_8 \succ A_{10} \succ A_{18} \succ A_6 \succ A_3 \succ A_4 \succ A_7 \succ A_{12} \succ A_{13} \succ A_1 \succ A_{17} \succ A_9 \succ A_{19} \succ A_{16} \succ A_{15}.$$

5. Result Analysis and Discussions

The method proposed by [26] present a weakness to determine the position of the alternatives due to duplicate ranking of the closeness coefficients values. The information shown in Table 7 depicts a comparison that reports this kind of duplicate issue. However, there is the alternative A_{11} as a best option identified by both analyses.

Table 7. Comparisons of the closeness $(HSRC_i)$.

	Proposed by [26]				**Proposed by [28]**				**Our Proposed**			
Item	$HPIS^+$	$HNIS^-$	$HSRC_i$	**Ranking**	$HPIS^+$	$HNIS^-$	$HSRC_i$	**Ranking**	$HPIS^+$	$HNIS^-$	$HSRC_i$	**Ranking**
A_1	67	68	0.504	11	0.179	0.292	1.630	15	6.542	6.718	0.507	14
A_2	62	69	0.527	8	0.179	0.297	1.658	11	5.489	6.820	0.554	4
A_3	69	68	0.496	12	0.179	0.295	1.646	12	6.405	7.331	0.534	9
A_4	61	77	0.558	4	0.170	0.316	1.859	4	6.294	6.966	0.525	10
A_5	61	74	0.548	6	0.176	0.297	1.688	10	5.889	7.370	0.556	3
A_6	52	83	0.615	2	0.166	0.316	1.904	3	6.066	7.194	0.543	8
A_7	64	71	0.526	9	0.176	0.289	1.642	14	6.323	6.937	0.523	11
A_8	66	65	0.496	13	0.175	0.304	1.734	7	5.508	6.800	0.552	5
A_9	76	59	0.437	16	0.176	0.289	1.645	13	7.859	5.401	0.407	16
A_{10}	58	79	0.577	3	0.174	0.331	1.905	2	6.151	7.437	0.547	6
A_{11}	42	93	0.689	1	0.157	0.342	2.176	1	4.638	8.621	0.650	1
A_{12}	69	66	0.489	14	0.172	0.310	1.803	5	6.406	6.853	0.517	12
A_{13}	60	75	0.556	5	0.176	0.304	1.729	8	6.487	6.772	0.511	13
A_{14}	61	74	0.548	6	0.174	0.306	1.763	6	5.851	7.409	0.559	2
A_{15}	83	52	0.385	18	0.184	0.269	1.463	17	8.566	4.694	0.354	19
A_{16}	81	54	0.400	17	0.178	0.256	1.438	18	8.562	4.697	0.354	18
A_{17}	66	69	0.511	10	0.175	0.298	1.705	9	7.061	6.198	0.467	15
A_{18}	73	62	0.459	15	0.182	0.294	1.611	16	6.022	7.237	0.546	7
A_{19}	86	51	0.372	19	0.188	0.243	1.291	19	8.552	5.183	0.377	17

Normally, the manufacturing company handles a high standard of the KPIs to monitor the best performances of LM. At this sense, our method offers the initiative to appraise the key performance indicators (KPIs).

Table 8 introduces the correlation between the three methods by taking into account their results. As can be observed, there is a significant correspondence between our approach and the two MCDM approaches proposed by [26] and [28], respectively.

Table 8. Correlation matrix.

	Proposed by [26]	Proposed by [28]	Our Method
Proposed by [26]	1.000	0.820	0.677
Proposed by [28]	0.820	1.000	0.630
Our method	0.677	0.630	1.000

Similarly, Table 9 lists the residual covariances between the methods.

Table 9. Covariance matrix.

	Proposed by [26]	Proposed by [28]	Our Method
Proposed by [26]	32.053	26.111	21.556
Proposed by [28]	26.111	31.667	19.944
Our method	21.556	19.944	31.667

On the other side, Table 10 lists the statistical parameters of the case studies. As can be observed, the mean and standard deviation values are similar in the three methods. In fact, the results can be interpreted with minimal error in the three case studies.

Table 10. Analysis of statistical parameters.

Variable	Count	Mean	StDev
Proposed by [26]	19	9.947	5.662
Proposed by [28]	19	10.000	5.627
Our method	19	10.000	5.627
Total	19	29.947	15.186

Finally, Table 11 lists the internal consistency values as expressed by the Cronbach's alpha coefficient. Our study reported an overall Cronbach's alpha value of 0.9008, which is considerably higher than 0.7, the usual threshold. This confirms the reliability of the results, since higher values of Cronbach's alpha imply greater internal data consistency.

Table 11. Correlation between our method's final ranking and other MCDM techniques.

	Adj. Total Mean	Adj. Total StDev	Item-Adj. Total Corr	Cronbach's Alpha
Proposed by [26]	20.00	10.16	0.8287	0.7729
Proposed by [28]	19.95	10.34	0.7918	0.8071
Our method	19.95	10.77	0.6849	0.9008

To perform an error analysis on the ranking results, we employed a neural network. In this sense, Figure 2 indicates that almost 78 epochs are found below the minimal error.

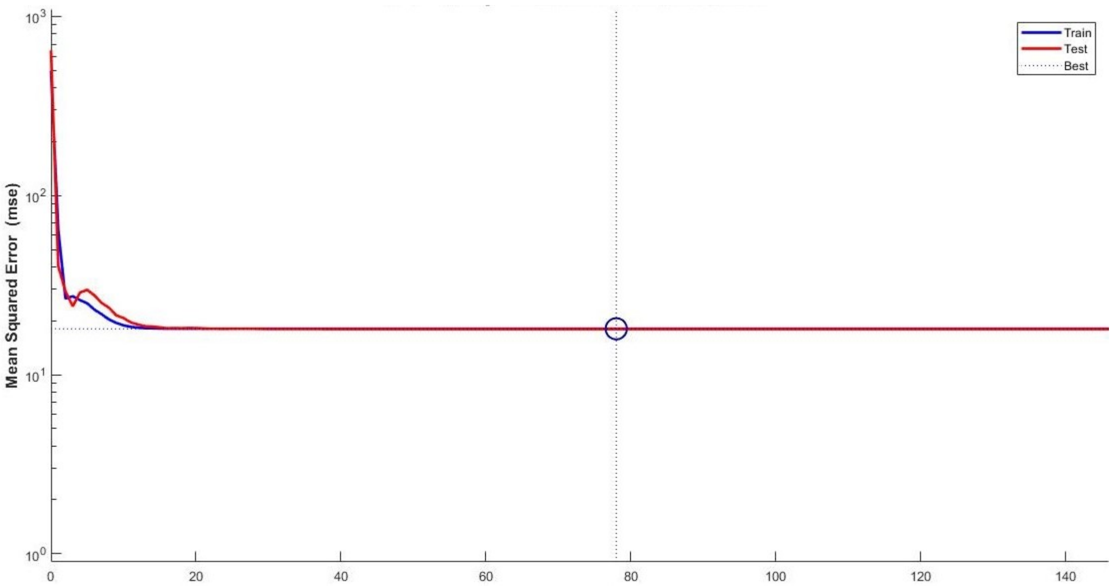

Figure 2. Error analysis neural network.

The results from the neural network indicate that the major contribution of the LM methodology is offered by JIT/continuous production flow. In this sense, a productivity bonus shares for the workers based on the top 10 metrics classified using the Hesitant Fuzzy Linguistic Term and the TOPSIS method. Similarly, we plan to develop a waste minimization project to take into account the ranking results obtained from the assessments. Additionally, a sensitivity analysis was planted, which implies the comparisons with other methods in order to check the stability of our application and the results are shown in Figure 3.

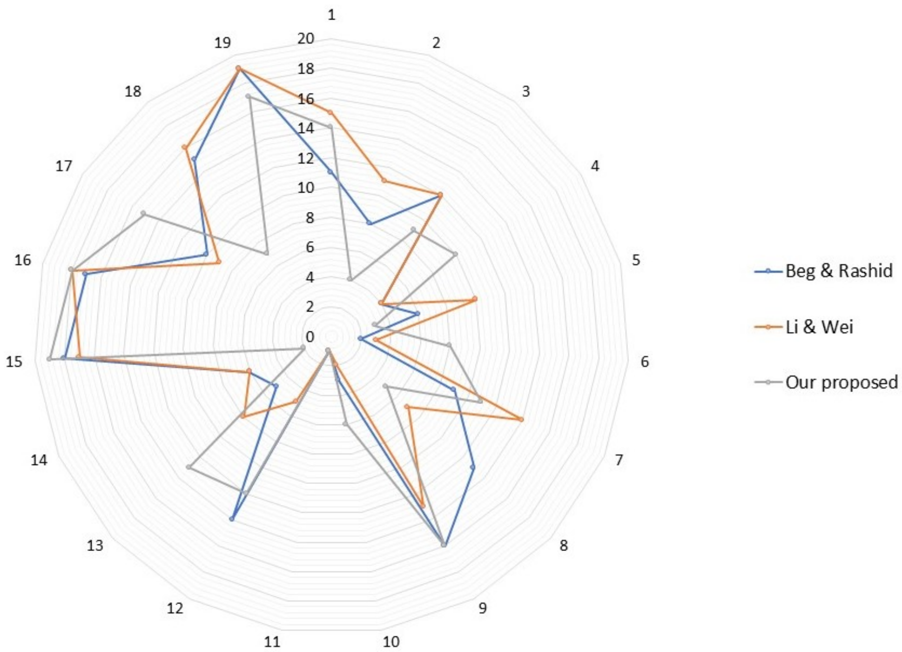

Figure 3. Sensitivity analysis using other methods.

Observing in Figure 3, we can notice the stability of the gained results. In addition, two different methods were applied and the ranking of the best position does not change. Finally, we demonstrated that there is a significant correspondence between our approach and the two approaches compared.

6. Conclusions

In this research, we propose an operative method for dealing with hesitant assessments in lean manufacturing problems. TOPSIS and HFLTS are a useful tool for managers who wish to assess the KPI's performance of the LM projects. In this research, we propose a multi-criteria decision-making method to find the desirable alternatives. Likewise, the results from our proposed can be used to design an action plan. Normally, developing cost minimization projects in a manufacturing environment is challenging, yet HFLTS and TOPSIS offer a systematic method for establishing priorities, thereby helping managers determine what key performance indicators (KPIs) have a low performance. Finally, the results represent a robust solution to deal with KPI assessments and provides visibility in terms of how lean manufacturing projects impact corporate performance. In addition, we present the use of AHP in order to determine the weights of criteria. There are some guidelines for future research where MCDM problems exist within the context of HSFLT situations—for example, evaluating the Lean Six Sigma projects, appraising performance of supply chains, among others. In addition, the consideration of the comparisons with other methods of MCDM.

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