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Public Campaign Finance, Vote Buying and Parties´ Policies on Government Spending

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Abstract
We study whether public campaign finance creates incentives for parties to adopt a moderate public spending policy (which maximizes the welfare of a majority of voters) or a polarized spending policy (which maximizes the welfare of a minority of voters). We find that symmetric campaign finance (all parties receive an equivalent amount) induces parties to converge in selecting a moderate policy on public spending while asymmetric campaign finance leads parties to diverge in their platforms and propose polarized policies with too much government spending. These findings suggest that public campaign finance has non-trivial effects on the degree of representation of voters' preferences into policies (an issue that is central for the adequate functioning of a democracy) and in the efficacy of government spending. In addition, we provide empirically verifiable tests of the effect of public campaign finance on the parties' strategic design of policy proposals on government spending.
Keywords: Elections, public goods, collective decision-making, performance of government, public campaign finance.
JEL Classification: D72; H4; D70; H11; D72.

Resumen
En este artículo estudiamos si el financiamiento público de campañas electorales crea incentivos a los partidos políticos para adoptar una plataforma de política de gasto público moderada (que maximiza el bienestar de una mayoría de votantes) o una política polarizante o extrema (que maximiza el bienestar de una coalición minoritaria de votantes). Encontramos que el financiamiento público a partidos es simétrico (todos los partidos reciben la misma cantidad) induce a los partidos a seleccionar una plataforma política de gasto moderada mientras que cuando el financiamiento público es asimétrico induce a los partidos a divergir en sus plataformas políticas y seleccionar una plataforma con un nivel de gasto público excesivo. Estos resultados sugieren las reglas que determinan el financiamiento público de campañas electorales de los partidos tienen efectos no triviales en el grado de representación de las preferencias de los votantes en las políticas públicas (un tema que es central para explicar un adecuado funcionamiento de una democracia) y en la eficacia del gasto de gobierno. Además, en este artículo, proveemos pruebas empíricamente verificables sobre el efecto de las plataformas de gasto público de los partidos del financiamiento público de campañas.
Palabras clave: Elecciones, bienes públicos, decisiones colectivas, desempeño de gobierno, financiamiento de campañas públicas.
Clasificación JEL: D72; H4; D70; H11; D72.

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Introduction

In many democracies, parties receive public funds to finance their political campaigns (see Falguera, Jones, and Ohman, 2014). Public campaign financing is then used by parties to pay for day to day activities such as rallies, meetings with voters and interest groups, informing voters over parties' proposals and propaganda among others. However, in some countries these funds are pervasively used to finance patronage in the form of vote buying. See Fox (1994) and Rigger (2000). A particular case of vote buying is when parties give direct material transfers to some voters with the objective of improving their supporters' turn out as well as the parties' chances of securing votes from their targeted constituency.

Vote buying cases significantly occur in developing economies (Kitschelt and Wilkinson, 2007) and are one of the main weaknesses in their democratic processes (Schaffer, 2007). For instance, according to the Americas Barometer survey, in 2010 the percentage of individuals that reported having been offered a material benefit in exchange for a vote, ranged from 5.5% in Chile, to 16.7% in Mexico with an upper bound of 22% in the Dominican Republic. However, these numbers may underestimate the situation if we consider the underreporting of vote buying cases as individuals know the "morally indefensible nature" of exchanging votes for material benefits (Çarkoğlu and Aytaç, 2015), also known as the social desirability bias.

Analyzing this phenomenon is relevant in several ways. For instance, it waters down electoral discipline that may reduce the impact of constitutional agreements (Finan and Schechter, 2012). It can also undermine elections of their policy content (Desposato, 2007; Stokes, 2007) and weaken their representation purpose and the role of accountability (Stokes, 2007a). Additionally, this practice compromises the time horizon of policies, undermining development and contributing to poverty traps (Magaloni, 2006). This makes the possibility of parties buying votes a topic of great concern in developing economies with high rates of poverty such as México, where approximately 43.6% and 7.6% of the population in are, respectively, poor and extremely poor.[1]

Given these distorsions and the economic vulnerability of individuals, we can infer that a succesful vote buying strategy not only affects electoral outcomes, but also the way parties represent voters' interests into public policies. Hence, the use of campaign finance into vote buying might be central in explaining not only the parties' chances of winning elections but also their policy platforms.

Furthermore, the source of campaign finance (private or public) matters for the analysis of vote buying and spending policy platforms. In terms of the literature, there are a considerable number of studies on the effect of private campaign contributions of voters and special interest groups on the parties' policy platforms (for a survey on this issue see Austen-Smith 1997). For instance, Welch (1980), Snyder (1990, 1993) and many others predict that parties change their policy positions to benefit donors of campaign contributions. Evidence also suggests that candidates that spend more in their campaign tend to increase their chances of winning the election (for a survey of international evidence on this issue, see Scarrow 2007).

Nonetheless, critics of the model of purely private campaign finance have emphasized the need of some form of regulation of money in politics such as bans on donations and changes in political institutions to foster the role of public campaign funding. In particular, advocates of public

[1] According to CONEVAL a person is poor if that person has an insufficient income to buy basic goods and services and the person meets at least one requirement of six indicators of basic needs. CONEVAL also considers a person to be extremely poor if that person satisfy three (out of six) indicators of basic needs and that person is below the line of minimum welfare (which is a threshold monetary value of a set of basic goods and services).

campaign finance argue that public funds constraint the influence of special interest groups, increase transparency of the candidates' finances and limit corruption. However, there is little research of the effect of public campaign finance on electoral competition and the policy platforms adopted by parties. A few exceptions include the analysis of Baron (1994), who shows that public campaign reduces the influence of interest groups and the parties' policies become less polarized, and Ortuño-Ortín and Schultz (2005) demonstrates that public campaign funds allocated according to the last election's share of the vote make the adoption of moderate policies more attractive (since getting more public campaign funding allows parties to buy more votes).

As mentioned previously, the analysis of public campaign funding on the parties' policy proposals has not received sufficient attention. To contribute to filling this gap in the literature, the objective of this paper is to analyze the impact of symmetric (all parties receive the same amount of public funds) and asymmetric public funds for campaign financing (parties receive different amount of public funds) on the parties' policy proposals. In particular, we seek to study the link between the distribution of public campaign finance among different parties and their incentives to adopt a moderate policy (which maximizes the welfare of a majority of voters in the electorate) versus the incentives of parties to adopt polarized or extreme policies (that seek to maximize the welfare of a minority coalition of voters). This discussion is of significant interest for students of political science and public economics because it sheds light on how electoral competition creates incentives to parties to represent the legitimate demands of voters into public policy, an issue that is central to explain both the quality of democracy and the efficacy of public policy.[2]

One distinction of our paper with respect to the literature is that we develop a probabilistic voting equilibrium model in which parties compete for votes by proposing policy platforms over the provision of a public good. In addition, parties compete for votes by using public campaign funds to buy votes to increase the parties' chances of winning the election. To the best of our knowledge, the analysis of campaign finance under probabilistic voting has not been studied. This is a relevant distinction because the predictions of the median voter model and those from probabilistic voting are quite different. In particular, while the recent literature suggests that public campaign funding induces moderate policies, our analysis identify conditions related with the (a)symmetric allocation of public funds that induce parties to adopt in some cases moderate and in others polarized policy platforms.

The main results of our study are the following: *first*, if the electoral authority allocates public campaign funds symmetrically in a two party system in which parties select their policies to maximize their share of the vote in the election then parties converge in selecting moderate policies that maximize the net fiscal incidence of government spending for the society as a whole. This outcome is, from the perspective of the society, the optimal level of government spending since this policy maximizes a symmetric utilitarian social welfare function.[3]

Second, if the electoral authority allocates campaign public funds asymmetrically (for instance, if the electoral authority provides funds according to the share of the vote of each party in the last election and the share of the vote is not the same for all parties), then parties diverge and

[2] If parties select a moderate policy that maximizes the wellbeing of the average voter then the resulting policy also maximizes the gains from the net fiscal incidence of government intervention for the society as a whole. If, in contrast, parties select a polarized policy that maximizes the welfare of a minority then social welfare will be lower compared with the social wellbeing that could be achieved if parties select a policy that maximizes the society's gains from the net fiscal incidence of government spending.

[3] The net fiscal incidence of government spending reflects the following tradeoff: On the one hand, an increase of government spending increases the welfare of individuals. On the other hand, parties need to increase the income tax to finance public spending which reduces the amount of resources available for households to consume private goods and their welfare.

select a polarized policy that maximizes the welfare of a minority coalition of voters. In our economy, parties have incentives to adopt polarized policies because:

i) Parties have incentives to weigh (and discount) more heavily the demands for public spending of voters with a high (low) marginal probability of voting for the party. In other words, parties will discriminate the demands for public spending of different groups of voters. The demand for public spending of those groups of voters with a high marginal probability of voting for the party are more important that the corresponding demand of those group of voters with a low marginal probability of voting for the party. This in turn might lead to a process of preference aggregation in which a minority in the electorate might have a high electoral influence over parties. In this case, parties design policy platforms that seek to maximize the well-being of a minority coalition of voters in the electorate (instead of designing a policy that maximizes the utility of a majority of voters such as the median voter) and therefore the process of electoral competition leads to a suboptimal allocation of resources with too much or too little government spending.

In our analysis, we identify conditions in which a polarized government spending from parties leads to either too much or too little government spending relative to the socially optimal size of government spending. For instance, parties can maximize their share of the vote by increasing the size of public spending above the socially optimum level of government spending when they face a distribution of voters in which the covariance between the voters' marginal probability of the vote and their income is positive. This is the case because voters with higher than average income have a higher demand for public spending than the corresponding demand of public spending of the average voter. Moreover, if the covariance between the marginal probability of the vote and their income is positive parties weigh more heavily the demand for public spending of voters with higher than average income and, as a result, parties choose the size of public spending well above the socially optimum level of government spending. If the covariance between the voters' marginal probability of the vote and their income is negative then parties choose a suboptimal amount of spending with too little government spending.

ii) Parties also have incentives to adopt polarized policies because they use campaign funds to make transfers to some targeted voters and therefore parties induce a positive income effect on targeted voters that changes the aggregate demand for public spending. Hence, the parties' strategies of using campaign funds to buy votes induce a bias towards a bigger size of government which might explain why, in some cases, if some party receives more public campaign funds than its competitor then this party has incentives to propose a higher level of public spending than its competitor.

In this paper we also provide empirically verifiable tests on the effect of the distribution of public campaign finance on the parties' policy platforms. Our theory suggests that if public campaign funds are allocated asymmetrically then an increase in the party's own campaign funds induces the party to propose a higher, equal or lower amount of government spending depending on an electoral benefit-cost measure of the transfer and tax policies adopted by those parties. Our theory also predicts that an increase in the funds allocated to the party's competitor induce the party to propose a lower size of public spending. In contrast, if public campaign funds are allocated symmetrically then an increase in the party's own campaign funds and an increase in the funds of the party's competitor induce both parties to propose a lower size of government spending.

The structure of the paper is as follows: section 2 presents the models with symmetric and asymmetric public campaign finance. In this section we present the main results of the theory. Section 3 includes a discussion of our results and section 4 concludes.

The Model with Symmetric and Asymmetric Public Campaign Finance

In this section we develop a probabilistic voting model to characterize the policies chosen by parties to win public office. In our economy these strategies are constituted by a policy platform on government spending and a strategy of a monetary transfer to buy votes. Both strategies are aimed to maximize the expected share of votes of parties that seek to win an election. We are also interested in studying the parties' policies on government spending to have a better understanding of the efficiency of public spending. To do so, we focus our analysis in studying conditions in which parties might adopt a moderate policy on government spending (which maximizes the welfare of a majority of voters in the electorate) and why parties might adopt polarized or extreme policies that seek to maximize the welfare of a minority coalition of voters in the electorate.

In our economy, conflicts of voters over the size of government arise because of heterogeneous preferences and endowments of voters that explain why the demand of voters for public spending is also heterogeneous (there are voters who want low government spending while others want high public spending). The institution of collective choice that solves the conflicting views of voters over the size of government is the delegation of the design of public spending to an elected policy maker.

We analyze a model of electoral competition for an economy with a majority two party electoral system. We assume two homogeneous parties, denoted by z and $-z$, competing in a nationwide election.[4,5] Candidates put forward a platform constituted by an income Lump sum tax that finances government spending.[6] To characterize the election, we consider a dynamic game of perfect information in which the timing of the model is as follows: in the first stage, nature reveals the size of public funds to be allocated to each party by the electoral authority. In the second stage, candidates announce binding policies regarding the size of an income tax that finance the provision of a public good. In the third stage, voters vote sincerely for the policies that make them better off. At the fourth stage, the candidate with the highest relative plurality of the votes in the election is elected, forms the government, and implements the platform of his or her party on public spending.

As we just mentioned before, in the first stage, nature reveals the size of public funds to be allocated to each party by the electoral authority. The movement of nature is common knowledge. The electoral authority allocates public funds according to the constraint $tH = R^z + R^{-z}$ where tH is the tax revenue of the electoral authority and t is an income tax applied to all individuals in the society. In our model there are $h = 1,2 \ldots H$ individuals in the economy. Since our interest is to analyze the impact of the distribution of public campaign finance on the parties' strategies of policy platforms and monetary transfers, we assume that the tax t that finances the political campaign of parties is already established in the constitution and therefore it is not a policy choice

[4] Two parties are homogeneous if they don't have a distinctive feature (such as a partisan preference) that gives one of them an electoral advantage over the other.

[5] In this paper we analyze a model for an economy with a majoritarian electoral system with a two party system. However, Ponce-Rodriguez (2010) identifies conditions in which the parties' policy platforms in a two party system are equivalent to the parties' policy platforms for economies with a proportional representation electoral system and multi-party electoral competition.

[6] We take as given that the government uses income taxation and provides a public good and we don't model which taxes (whether direct or indirect taxation) should finance the government's spending since our main interest is to develop a model to explain the influence of public campaign finance in electoral competition and the size of government spending.

for parties. Since H is exogenously then tH is also exogenous.[7] Moreover, R^z, R^{-z} are the public funds allocated to parties z and $-z$ to finance their campaigns and R^z, and R^{-z} are also given. Moreover, we assume that the electoral regulation allows only public funds to parties.[8]

In the second stage, candidates announce binding policies regarding the size of an income tax $\tau^z\ \forall z$ that finance the provision of a public good $g^z\ \forall z$. Parties design tax and spending policies to maximize their share of the vote in the election. Moreover, parties use available public campaign funds to make transfers to some targeted voters to influence the way they vote. In the third stage, voters vote sincerely for the policies that make them better off. All voters vote. As we mentioned before, at the fourth stage, the candidate with the highest relative plurality of the votes in the election is elected, forms the government, and implements the platform of his or her party on public spending.

Preferences and budget constraint of a voter type $e^h\ \ \forall h = 1,2 \dots H$ are given by:

$$\mu^{zh} = \mu^{hz}(x^{zh}, g^z) = x^{hz} g^z \quad s.t: \quad x^{zh} = e^h - t - \tau^z + T^z \tag{1}$$

Where $\mu^{zh}(x^{zh}, g^z)$ is the preference relation over a private good x^{zh} and a public good g^z provided by some party z. The budget constraint of a voter type e^h says that x^{zh} is bought with the voter's resource constraint $e^h - t - \tau^z + T^z$ where e^h is an endowment which belongs to a distribution of voters' endowments given by $e^h \in [\underline{e}, \overline{e}]$. Recall that t is a tax that finances the political campaign of parties and it is not a policy choice for parties, τ^z is an income tax that finance the provision of the public good and T^z is a monetary transfer proposed by party z.

Voters recognize that the government's budget constraint is $g^z = \tau^z H\ \ \forall z, -z$ where the size of the public good $g^z\ \forall z$ is financed by the income tax $\tau^z\ \forall z$. Hence, the indirect utility of a voter type e^h is given by

$$v^{zh}(e^h, t, g^z, T^z) = \left(e^h - t - \frac{g^z}{H} + T^z\right) g^z \qquad \forall e^h, \forall z \tag{2}$$

Under the fiscal platforms of parties z and $-z$ the welfare of a voter type e^h is given by $v^{zh}(e^h, t, g^z, T^z)$ and $v^{-zh}(e^h, t, g^{-z}, T^{-z})$ where $v^{zh}(e^h, t, g^z, T^z)$ is the welfare of voter type e^h under the policies of party z g^z, T^z. A similar interpretation is given to $v^{-zh}(e^h, t, g^{-z}, T^{-z})$. Sincere voting implies that a voter type e^h votes for party z if the voter's welfare calculus function, $\chi^{zh}(e^h)$, is positive where $\chi^{zh}(e^h) = v^{zh}(e^h, t, g^z, T^z) - v^{-zh}(e^h, t, g^{-z}, T^{-z})$, if $\chi^{zh}(e^h) < 0$ he votes for party $-z$, and the voter flips a coin if $\chi^{zh} = 0$.

For the fourth stage, we define $\boldsymbol{\Omega}$ as a non-decreasing cumulative distribution of the sequence $\{\chi^{zh}(e^h)\}_{\forall e^h}$ with $\boldsymbol{\Omega} \in [0,1]$. The function $\boldsymbol{\Omega}$ represents the proportion of the electorate that votes for some party. Therefore, if there exists a majority of voters $\forall\ e^h \in [\underline{e}, \overline{e}]: \chi^{zh}(e^h) > 0$ (voting in favor of party z) then $\boldsymbol{\Omega}\left(\forall\ e^h \in [\underline{e}, \overline{e}]: \chi^{zh}(e^h) > 0\right) > 1/2$ and party z wins the election and implements g^z. In contrast, if a majority of voters prefers the policies of party $-z$

[7] We assume that the total amount of funds devoted to finance the parties' campaigns tH is exogenous for mathematical simplicity. A complete analysis of the design of the size of campaign funds and its distribution by an electoral authority is beyond the scope of this paper and left for future research.
[8] For recent analysis of private campaign financing see Grossman, and E. Helpman (2001), Austen-Smith (1997) among many others. In this paper we are interested in public campaign finance and the parties' policy platforms and vote-buying strategies.

then $\boldsymbol{\Omega}\big(\forall\, e^h \in [\underline{e}, \overline{e}]: \chi^{zh}(e^h) < 0\big) > 1/2$ which implies that party $-z$ wins the election and implements g^{-z}.

Given the set of policies g^z, T^z, g^{-z}, T^{-z}, parties have a system of beliefs on how voters vote. This system of beliefs is characterized by the probability that a voter type e^h votes for party z, $F^{zh}(\chi^{zh}) = \int_{-\infty}^{\tilde{\chi}^{zh}} f^{zh}(\chi^{zh})\, d\chi^{zh}$ evaluated at some feasible value χ^{zh}, where $F^{zh}(\chi^{zh})$ is a continuous cumulative distribution over χ^{zh} and $f^{zh}(\chi^{zh}) = dF^{zh}(\chi^{zh})/d\chi^{zh}$ is the probability distribution function. The share of the vote of party z in the election is

$$s^Z = \sum_{h=1}^{H} F^{zh}(\chi^{zh}) \tag{3}$$

We follow the literature by assuming that s^Z is a strictly concave function of g^z, T^z (for an in-depth analysis of this issue see Coughlin 1992).

In the real world, parties use public funds to finance day to day activities such as political rallies, advertising, transfers to voters, etc. To simplify the analysis, we assume parties use public funds only to provide transfers T^z to obtain the vote from targeted voters. Hence, the parties' operational budget constraint is that the sum of money for "buying" votes must be equal to the available campaign public funds of the party, hence $R^z = T^z J^z$ where T^z is a per-capita uniform transfer and J^z is the number of voters that will receive the transfer. To simplify the analysis, we assume that only voters with the lowest levels of endowments receive the transfer T^{*z}. In particular, we assume that voters $e^1, e^2 \ldots\ldots\ldots e^{J^z}$ are the voters who receive the transfer from party z.[9,10]

For this reason, voters with endowments $e^1, e^2 \ldots\ldots\ldots e^{J^z}$ will receive a monetary transfer from parties and will have the following indirect utility $v^{zj^z}(e^{j^z}, t, g^{*z}, T^{*z}) = \left(e^{j^z} - t - \frac{g^z}{H} + T^z\right) g^z \;\forall j^z = 1 \ldots\ldots J^z$ while voters with endowments $e^h > e^{j^z}$ do not receive a monetary transfer from parties. In this last case the indirect utility of voters who do not receive transfers are given by $v^{zh}(e^h, t, g^{*z}, T^{*z}) = \left(e^h - t - \frac{g^z}{H}\right) g^z \quad \forall h = J^z + 1 \ldots\ldots\ldots H$.[11]

It follows that the problem of policy design for parties z and $-z$ is to maximize the parties' share of the vote by designing policy platforms g^Z and tax τ^z and in addition parties make use of public campaign funds to attract the vote from certain coalitions of voters through monetary transfers T^z. Formally, the problem of parties is:

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[9] In other words, consider a sequence of voters with endowments $\{e^{J^z}\}_{j=1}^{J^z}: \{e^{1^z} < e^{2^z} < e^{3^z} \ldots\ldots < e^{J^z}\}$. This set of voters receive the per-capita transfer from party z. Voters with $e^h > e^{J^z}$ do not receive the transfer.

[10] The strategy of providing transfers only to voters with the lowest level of endowments is a dominant strategy compared to the alternative of providing a universal transfer to all voters since the former strategy provides parties with more expected share of votes per dollar spent due to the fact that the lower the voter's endowment the higher is his or her marginal utility gain from a monetary transfer from parties and the higher is the cumulative mass of expected share of votes for parties.

[11] Here we are just trying to show that the indirect utility of voters is different depending on whether voters receive transfers or not from parties. Recall that voters with endowments $e^1, e^2 \ldots\ldots\ldots e^{J^z}$ will receive a transfer from parties and voters with endowments $e^{j^z+1}, e^{j^z+2} \ldots\ldots\ldots e^H$ will not receive a transfer from parties.

$$Max_{\{g^Z, T^z\}} \quad s^Z = \sum_{h=1}^{H} F^{zh}(\chi^{zh}) \tag{4}$$

$$\text{s.t:} \quad i) \quad R^z = T^z J^z \quad \forall\, z, -z \tag{5}$$

On what follows, we define the subgame perfect Nash equilibrium for our electoral-economic game.

Definition. *The electoral equilibrium for our economy is characterized by the parties' policy choices* g^{*Z} *and* T^{*z} $\forall\, e^j \in [e^1, e^{J^z}], \forall\, z, -z$ *and voting choices for voters type* e^h *such that* [12]

I) In the first stage, the electoral authority allocates public funds R^z *and* R^{-z}

II) In the second stage, parties z and -z select g^{*Z} *and* T^{*z}*, and* g^{*Z-} *and* T^{*-z}*. In particular, for the case of party z, the problem of policy design is to select* [13]

$$g^{*z}, T^{*z} \in argmax \;\; s^z = \sum_{h=1}^{H} F^{zh}(\chi^{zh})$$
$$s.t\colon \; i) \;\; R^z = T^{*z} J^{*z} \quad \forall\, z$$

III) In the third stage, voters type e^h *observe the parties' policies and vote for:*
Party z if $\chi^{zh}(e^h) = v^{zh}(e^h, t, g^{*z}, T^{*z}) - v^{-zh}(e^h, t, g^{*-z}, T^{*-z}) > 0$
If $\chi^{zh} < 0$*, he or she votes for party* $-z$

IV) In the fourth stage, if there exists a majority of voters $\forall\, e^h \in [\underline{e^h}, \overline{e}^h]\colon \chi^{zh}(e^h) > 0$ *then it is satisfied*

$$\mathbf{\Omega}\big(\forall\, e^h \in [\underline{e}, \overline{e}]\colon \chi^{zh}(e^h) > 0\big) > 1/2$$

In this case party z wins the election and implements g^{*z} *and* T^{*z}*. In contrast, if*

$$\mathbf{\Omega}\big(\forall\, e^h \in [\underline{e}, \overline{e}]\colon \chi^{zh}(e^h) < 0\big) > 1/2$$

then party $-z$ *wins the election and implements* g^{*-z} *and* T^{*-z}*.*

We proceed to develop the main results of this section. In proposition 1 we characterize the optimal design of policy platforms g^{*z} and monetary transfers T^{*z} $\forall\, z, -z$

Proposition 1. *Party z proposes* g^{*z} *and provide transfers* T^{*z} $\forall\, e^j \in [e^1, e^{J^z}], \forall\, z, -z$ *satisfying*

$$g^{*z} = \frac{\sigma^z[f^{zh}, e^h]}{2E[f^{zh}]} + \frac{E[e^h]}{2} + \frac{R^z}{2}\left\{\frac{E[f^{zJ^z}]}{E[f^{zh}]} - 1\right\} - \frac{R^{-z}}{2} \qquad \forall z, -z \tag{6}$$

And

$$T^{*z} = \frac{R^z \lambda^{*z}}{\sum_{j=1}^{J^z} f^{zh} \frac{\partial \mu^{zh}}{\partial x^{zh}}} = \frac{R^z \lambda^{*z}}{g^{*z} \sum_{j=1}^{J^z} f^{zj}} \qquad \forall\, z, -z \tag{7}$$

[12] Once we determine g^{*z}, the equilibrium value of τ^{*z} is found with the government's budget constraint $\tau^{*z} = g^{*z}/H$.
[13] A similar problem of policy design is solved by party -z.

Where $\sigma^z[f^{zh}, e^h]$ is the covariance between the voter's marginal propensity to vote for party z, f^{zh}, and the voter's endowment e^h. Moreover, $E[f^{zh}]$ is the average marginal probability of the vote for party z and $E\left[f^{zJ^z}\right]$ is the average marginal probability of the vote from voters who receive a transfer such that

$$E[f^{zh}] = \sum_{h=1}^{H} f^{zh}/H \quad and \quad E\left[f^{zJ^z}\right] = \sum_{j=1}^{J^z} f^{zj}/J^{*z} \tag{8}$$

The average endowment in the economy is

$$E[e^h] = \sum_{h=1}^{H} e^h/H \tag{9}$$

And λ^{*z} is the marginal share of votes that the party could obtain if we relax the constraint of campaign funding of the party.

Proof

See the appendix

Proposition 1 says that the parties' proposals on public spending depend positively on the covariance $\sigma^z[f^{zh}, e^h]$ between the voter's marginal propensity to vote for party z, f^{zh}, and the voter's endowment e^h. Assuming $\sigma^z[f^{zh}, e^h] > 0$, parties can increase their share of the vote by increasing the size of g^{*z} since they expect that voters with higher than average marginal probabilities to vote for party z, f^{zh}, also have higher than average endowments e^h. An increase in the economy's average endowment $E[e^h]$ also increases g^{*z}. Since the public good is a normal good, the higher the aggregate endowment of the economy the higher is the demand of voters for the public good and the equilibrium level of g^{*z}.

An increase in the size of the party's public campaign funds R^z can induce parties to propose an increase or reduction in g^{*z} depending on whether $\left\{\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} - 1\right\}$ is positive or negative. To explain further this outcome, note that $\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]}$ is the ratio between the average of the marginal probability of voting for party z from voters who receive a monetary transfer T^{*z}, $E\left[f^{zJ^z}\right]$, and the average of the marginal probability of voting for party z from the whole electorate $E[f^{zh}]$. The expression $E\left[f^{zJ^z}\right]$ is the expected marginal increase in the party's' share of the vote when the party transfers \$1 to some voters through T^{*z} while $E[f^{zh}]$ is the expected marginal share of the votes lost when the party takes away \$1 through income taxes (to finance the public good). Hence the ratio $\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]}$ is a relative benefit-cost measure of the transfer to buy votes-tax policies of parties.

It is relevant to note that changes in public financing of the campaign of parties have two effects in their policy platforms on public spending: *The first effect:* an increase in $\frac{R^z}{2}\left\{\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]}\right\}$ induces more government spending because a higher amount of campaign funds leads parties to devote more resources (more transfers) to buy votes. By making these transfers, parties increase the full income of some voters, their demand for the public good and the size of g^{*z}. *The second effect*: an increase in R^z also induces the electoral authority to increase the per-capita tax t given by $t = \frac{R^z + R^{-z}}{H}$. This tax reduces the net value of the voters' endowments and since the public good is a normal good then the voters' demand for public spending falls. Parties respond to this effect by reducing the equilibrium level of g^{*z}. Therefore, if $\left\{\frac{E\left[f^{zJ^z}\right]}{E\left[f^{zh}\right]} - 1\right\} > 0$, the *first effect* dominates the *second effect* and the equilibrium level of spending goes up. If $\left\{\frac{E\left[f^{zJ^z}\right]}{E\left[f^{zh}\right]} - 1\right\} < 0$ the *second effect* dominates the *first effect* and the equilibrium level of spending goes down.

In addition, the marginal effect of R^{-z} on g^{*z} is negative. The explanation of this outcome is as follows: an increase in the campaign funds allocated to party $-z$, R^{-z}, induces the electoral authority to increase the per-capita tax t which, in turn, leads to a fall in the net value of endowments of voters. In this case, the demand of voters for the public good falls and parties respond by reducing the supply of g^{*z}.

We now turn to the analysis of the parties' strategic use of transfers to buy votes. Proposition 1 says that the size of the per-capita transfer T^{*z} depends positively on the amount available of public campaign funds for party z, R^z and λ^{*z} which is the marginal share of votes that the party could obtain if we relax the constraint of campaign funding of the party. Furthermore, the per-capita transfer T^{*z} depends negatively on the expected proportion of votes associated with the parties' monetary transfers $\sum_{j=1}^{J^z} f^{zj} \frac{\partial \mu^{zj}}{\partial x^{zj}} = g^{*z} \sum_{j=1}^{J^z} f^{zj}$ because the higher is $\sum_{j=1}^{J^z} f^{zj} \frac{\partial \mu^{zj}}{\partial x^{zj}}$ the higher is the number J^{*z} of targeted voters that will receive a transfer from parties. Since $T^{*z} = R^z / J^{*z}$ the higher is J^{*z} the lower is the per-capita amount received by each of these voters.

In what follows, we analyze the parties' policy platforms when public financing is constituted by symmetric and asymmetric public campaign funds. Proposition 2 characterizes the parties' platforms under symmetric public campaign funds $R^z = R^{-z}$ while proposition 3 shows the parties' platforms under asymmetric campaign funds.

Proposition 2. *Symmetric public campaign finance* $\hat{R}^z = \hat{R}^{-z} = \hat{R}$ *leads parties to converge in proposing "moderate policies"* $\hat{g}^{*z} = \hat{g}^{*-z} = \hat{g}^*$ *and* $\hat{T}^{*z} = \hat{T}^{*-z} = \hat{T}^*$ $\forall\, e^j \in \left[\underline{e^1}, e^{J^z}\right], \forall\, z, -z$ *satisfying*

$$\hat{g}^* = \frac{E[e^h] - \hat{R}}{2} \tag{10}$$

And

$$\hat{T}^* = \frac{\hat{R}\hat{\lambda}^*}{\hat{g}^* \sum_{j=1}^{J^z} f^{zj}(0)} \tag{11}$$

Proof

See the appendix

Proposition 2 says that if parties receive the same amount in public campaign funds then parties converge in selecting a moderate policy of government spending. In particular, it is simple to verify that $\hat{g}^*$, in proposition 2, is equivalent to a spending policy that maximizes the welfare of the average voter in the economy. Therefore, this policy maximizes the society's net fiscal incidence of government spending and any deviation of this level of government spending entails a reduction of the maximum welfare for the society that could be achieved with the provision of the public good.[14]

Proposition 2 says that $\hat{g}^*$ depends positively on, $E[e^h]$, the average endowment of the economy since the public good is a normal good and negatively on $\hat{R}$. This last outcome is explained as follows: On the one hand, campaign financing involves a fall in income of voters because of the income tax t that finance the parties' campaigns. This is a negative income effect that reduces the voters' demand for the public good and $\hat{g}^*$. On the other hand, party z uses $\hat{R}$ to create transfers. However, note that $\hat{R}$ is just half of the resources collected by the electoral authority to finance the parties' campaigns (since $R^z = R^{-z} = \hat{R}$ and $\hat{R} = tH/2$). Because party z does not take into account how party –z might affect the voter's budget constraint by providing a transfer T^{*-z} then, from the perspective of party z, the structure of campaign financing (that is to say, the tax t and transfer T^{*z} altogether) entails a net negative aggregate income effect on households given by the amount $-tH + \hat{R} < 0$. As a result of this aggregate net negative income effect, the demand of the public good falls and party z responds by reducing $\hat{g}^*$.

Proposition 2 also says that the size of the per-capita transfer $\hat{T}^*$ depends positively on the amount available of public campaign funds $\hat{R}$, and the marginal share of votes that the party could obtain if we relax the constraint of the parties' campaign fund $\hat{\lambda}^*$. The per-capita transfer $\hat{T}^*$ also depends negatively on the expected proportion of votes associated with the parties' monetary transfers $\sum_{j=1}^{J^z} f^{zh} \frac{\partial \mu^{zj}}{\partial x^{zj}} = \hat{g}^* \sum_{j=1}^{J^z} f^{zj}(0)$. Because the higher is $\hat{g}^* \sum_{j=1}^{J^z} f^{zj}(0)$ the higher is the amount $J^z = \hat{J}^*$ of voters that are targeted by parties to receive such transfers, the per-capita amount received by each of these voters fall since $\hat{T}^* = \hat{R}/\hat{J}^*$.[15]

Proposition 3 shows that asymmetric campaign funds induces parties to diverge in proposing polarized policies. Proposition 3 also identifies sufficient conditions in which all parties propose too much government spending relative the size of public spending that maximizes the net fiscal incidence of the public good for the society, that is $g^{*z} > \hat{g}^*$ and $g^{*-z} > \hat{g}^*$.

Proposition 3. A*symmetric public campaign funding $R^z \neq R^{-z}$ leads parties to diverge in their policies $\hat{g}^{*z} \neq \hat{g}^{*-z} \neq \hat{g}^*$ and $\hat{T}^{*z} \neq \hat{T}^{*-z} \neq \hat{T}^*$ $\forall\, e^j \in \left[\underline{e^1}, e^{J^z}\right]$ $\forall\, z, -z$.*

Moreover, if

3.1) $\sigma^z[f^{zh}, e^h] > 0$ *and*

3.2) $R^z < R^{-z}$ and $E\left[f^{zJ^z}\right] > E[f^{zh}]$: $\frac{E\left[f^{zJ^z}\right]}{E\left[f^{zh}\right]} > \frac{\hat{R}}{R^z}$ $\forall z, -z$

[14] In the literature of public economics, it is well know that, a policy that maximizes the welfare of the average voter is also a policy that maximizes the welfare of a utilitarian symmetric social welfare function.

[15] For a full derivation of $\hat{J}^*$ and $\hat{T}^*$ see proposition2 in the appendix.

Then both parties select polarized policies satisfying

$$g^{*z} > \hat{g}^{*} \ \ and \ \ g^{*-z} > \hat{g}^{*}$$

Proof

See the appendix

Proposition 3 says that if the electoral authority allocates public funds between parties z and $-z$ asymmetrically then the parties' spending policies diverge $\hat{g}^{*z} \neq \hat{g}^{*-z}$ and $\hat{T}^{*z} \neq \hat{T}^{*-z}$. Proposition 3 also says that if parties expect that voters with higher than average marginal probabilities to vote for party z, f^{zh}, also have higher than average endowments e^h, that is if $\sigma^z[f^{zh}, e^h] > 0$. Moreover, if the expected increase in the parties' share of the vote when the party transfers $1 to voters is sufficiently high in relation to the expected share of the votes lost when parties take away $1 through income taxes (that is condition 3.2 in proposition 3) then the size of government spending of equilibrium g^{*z} of both parties is higher than the size of government spending that maximizes the net fiscal incidence of the society $\hat{g}^{*}$.

In this case, the spending policies of parties z and $-z$ are considered as polarized policies since g^{*z} does not seek to maximize the wellbeing of the average voter but to maximize the welfare of a minority coalition of voters in the electorate. In this case, the higher the difference between $g^{*z} - \hat{g}^{*} > 0$, the larger is the fall in the society's welfare (relative the policy that maximizes the society's wellbeing) associated with the net fiscal incidence of public spending.

In our economy, parties have incentives to adopt a polarized policy that maximizes the welfare of a minority of voters because of the following reasons: *I)* Parties have incentives to weigh (and discount) more heavily the demands for public spending of voters with a high (low) marginal probability of voting for the party. This in turn might lead to a process of preference aggregation in which some minority in the electorate might have a high electoral influence over parties.

II) Because parties use campaign funds to make transfers to some voters, parties create a positive income effect on voters that changes the distribution and the aggregate demand for public spending. Hence, if parties use campaign funds to buy votes then this strategy induces a bias towards a bigger size of government which explains why if party z receives more public campaign funds than its competitor, that is if $R^z > R^{-z}$, then the equilibrium policies are given by $g^{*z} > g^{*-z}$ (this follows from proposition 1).

In what follows, proposition 4 characterizes how changes in the distribution of funds allocated to parties z and $-z$ by the electoral authority affect the parties' proposals on public spending.

Proposition 4. *The (a)symmetric distribution of public funds affects the parties' proposals on public spending. In particular, if public funds are allocated asymmetrically then the following is satisfied*

$$4.1)\ if\ \left\{\frac{E\left[f^{zJ^z}\right]}{E\left[f^{zh}\right]} - 1\right\} \underset{<}{\overset{>}{=}} 0\ \ then\ \ \frac{\partial g^{*z}}{\partial R^z} \underset{<}{\overset{>}{=}} 0\ \ and\ \ \frac{\partial g^{*z}}{\partial R^{-z}} < 0 \quad \forall z, -z$$

If public funds are allocated symmetrically $R^z = R^{-z} = \hat{R}$ *then*

4.1) $\frac{\partial g^{*z}}{\partial R^{z}} = \frac{\partial g^{*z}}{\partial R^{-z}} < 0 \quad \forall z, -z$

Proof

The results follow from the equilibrium conditions (6) and (10) of propositions 1 and 2.

Proposition 4 provides empirically verifiable tests on how changes in the distribution of funds allocated to parties z and $-z$ by the electoral authority affect the parties' proposals on public spending. Our theory suggests that if public funds are allocated asymmetrically then an increase in the amount of campaign funds that a party receives induces the party to propose a higher, equal, or lower amount of public spending depending on whether $\left\{\frac{E\left[f^{zJ^{z}}\right]}{E\left[f^{zh}\right]} - 1\right\} \gtreqless 0$, while an increase in the funds allocated to the party's competitor induce the party to propose a lower size of public spending. However, if public funds are allocated symmetrically then an increase in the average amount of public funds induce both parties to propose a lower size of public spending.

Discussion

Our analysis shows that the rule for distributing public campaign funds among parties is relevant to explain why parties have incentives to diverge or converge in their policy positions and to explain whether parties select moderate or polarized policies. Our results are not the same as those predicted by the median voter model that would suggest that parties always converge in selecting a moderate policy, that is the ideal policy of the median voter, and the distribution of campaign funds are only relevant to explain the ideal size of government spending for the median voter and the differences in the probability that parties win the election.[16]

One interesting extension is to consider that parties have preferences for policies (see Wittman 1973). In this case, parties do not seek to design policy to win elections but to design policies to maximize the wellbeing of a minority coalition of voters that control each party (see also Roemer 2001). This type of model predicts that, if parties cannot credibly commit to their proposals in the first stage, then after winning the election parties choose the ideal policy of the coalition of voters controlling each party and therefore parties are likely to choose divergent polarized policies (because the heterogeneity of preferences and endowments of voters that control each party explain why parties have different ideal policy platforms). In an economy with electoral competition a lá Wittman, the symmetric and asymmetric distribution of public campaign finance determine the ideal size of government spending of each party and the differences in the probability of winning of parties.

[16] Our results are different to the median voter outcome because we make, the realistic assumption, that parties have imperfect information on the distribution of preferences of voters and on how voters vote while the median voter model assumes parties have perfect information. As a result of these differences, in our model the parties' proposals are determined by the whole distribution of preferences and endowments of the electorate while in the median voter, model public spending is determined by the preferences and endowment of the median voter. For more analysis of the differences between these two models see Mueller (2003) and Roemer (2001).

Our main results also hold if we relax the assumption that parties use a Lump Sum income tax to finance the public good. In the case of distortionary taxation, the determinants of public spending change because in the latter economy parties need to consider the elasticity of the supply of labor with respect the income tax. However, our main results remain unchanged.

Conclusion

In this paper we analyze the effect of symmetric and asymmetric public campaign finance on the parties' proposals on the provision of a public good. We study whether public campaign finance creates incentives for parties to adopt a moderate policy (which maximizes the welfare of a majority of voters in the electorate) versus a polarized policy (which maximizes the welfare of a minority coalition of voters). This discussion is of significant interest for students of political science and public economics because it sheds light on the impact of public campaign finance on how parties represent the legitimate demands of voters into public policy, an issue that is central to explain the quality of democracy and the efficacy of public policy.

The main results of our paper are the following: *first*, if the electoral authority allocates public funds symmetrically in a two party system and parties select their policy platforms to maximize their share of the vote in the election, then parties converge in selecting moderate policies that maximize the net fiscal incidence of government spending for the society as a whole. *Second*, if the electoral authority allocates public funds asymmetrically, then parties diverge and select a polarized policy that maximizes the welfare of a minority coalition of voters.

These findings suggest that the allocation of public funds to parties have non trivial effects on the welfare properties of electoral competition and the efficacy of the government's public policy since symmetric campaign finance leads parties to propose a spending policy that maximizes the welfare of the society while asymmetric campaign finance entails a welfare loss for the society (relative the maximum wellbeing that could be achieved) due to parties propose a suboptimal allocation with too much or too little government spending.

Finally, our paper also provides empirically verifiable tests on the distribution of campaign finance and the parties' policy platforms. Our theory suggests that if public funds are allocated asymmetrically then an increase in the amount of campaign funds that a party receives induces the party to propose an increment or reduction of the amount of public spending depending on whether an electoral benefit-cost measure of the transfers used to buy votes and the tax policies of the party is positive or negative. Moreover, an increase in the funds allocated to the party's competitor induce the party to propose a reduction in the size of public spending. However, if public campaign funds are allocated symmetrically then an increase in the party's own campaign funds and an increase in the funds of the party's competitor induce both parties to propose a lower size of spending on the public good.

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Appendix

Proposition 1. Parties propose g^{*z} and provide transfers $T^{*z}\ \forall\, e^{j} \in \left[\underline{e^{h}}, e^{J^{z}}\right], \forall\, z, -z$ satisfying

$$g^{*z} = \frac{\sigma^{z}[f^{zh}, e^{h}]}{2E[f^{zh}]} + \frac{E[e^{h}]}{2} + \frac{R^{z}}{2}\left\{\frac{E\left[f^{zJ^{z}}\right]}{E[f^{zh}]} - 1\right\} - \frac{R^{-z}}{2} \qquad \forall z, -z \tag{6}$$

And

$$T^{*z} = \frac{R^{z}\lambda^{*z}}{\sum_{j=1}^{J^{z}} f^{zh}\frac{\partial \mu^{h}}{\partial x^{h}}} = \frac{R^{z}\lambda^{*z}}{g^{*z}\sum_{j=1}^{J^{z}} f^{zj}} \qquad \forall\, z, -z \tag{7}$$

Where $E[e^{h}]$ is the average endowment in the economy

$$E[e^{h}] = \sum_{h=1}^{H} e^{h}/H \tag{8}$$

$E[f^{zh}]$ is the average marginal probability of the vote for party z, and $E\left[f^{zJ^{z}}\right]$ is the average marginal probability of the vote from voters who receive a transfer such that

$$E[f^{zh}] = \sum_{h=1}^{H} f^{zh}/H \quad \text{and} \quad E\left[f^{zJ^{z}}\right] = \sum_{j=1}^{J^{z}} f^{zj}/J^{*z} \tag{9}$$

Proof

State the problem of spending and transfer design for parties through the following Lagrangian:

$$\delta^{z} = \sum_{j=1}^{J^{z}} F^{zj}\left(\chi^{zj}(e^{h})\right) + \sum_{h=J^{z}+1}^{H} F^{zh}(\chi^{zh}(e^{h})) + \lambda^{z}\{R^{z} - T^{z}J^{z}\} \tag{A.1.1}$$

Where

$$\chi^{zj}(e^{h}) = \nu^{zj}\left(e^{j}, t, g^{z}, T^{z}\right) - \nu^{-zj}\left(e^{j}, t, g^{-z}, T^{-z}\right) \qquad \forall j = 1 \ldots\ldots J \tag{A.1.2}$$

And

$$\nu^{zj}(e^{h}, t, g^{*z}, T^{*z}) = \left(e^{h} - t - \frac{g^{z}}{H} + T^{z}\right) g^{z} \qquad \forall j = 1 \ldots\ldots J^{z}, \forall\, z \tag{A.1.3}$$

Moreover, there is the set of voters who don't receive a transfer. For them, relative welfare of the parties' policies is given by:

$$\chi^{zh}(e^{h}) = \nu^{zh}(e^{h}, t, g^{z}) - \nu^{-zh}(e^{h}, t, g^{-z}) \qquad \forall h = J^{z} + 1 \ldots\ldots H \tag{A.1.4}$$

Where

$$\nu^{zh}(e^{h}, t, g^{*z}, T^{*z}) = \left(e^{h} - t - \frac{g^{z}}{H}\right) g^{z} \qquad \forall h = J^{z} + 1 \ldots\ldots\ldots H, \forall\, z \tag{A.1.5}$$

The first order conditions are given by:

$$\frac{\partial \delta^{z}}{\partial g^{z}} = \frac{-1}{H}\sum_{h=1}^{H} f^{zh}\frac{\partial \mu^{zh}}{\partial x^{h}} + \sum_{h=1}^{H} f^{zh}\frac{\partial \mu^{zh}}{\partial g^{z}} = 0 \quad \forall\, g^{*z} > 0 \tag{A.1.6}$$

In the last condition we use the fact that $\frac{\partial \nu^{zh}}{\partial x^{h}} = \frac{\partial \mu^{zh}}{\partial x^{h}}$ and $\frac{\partial \nu^{zh}}{\partial g^{z}} = \frac{\partial \mu^{zh}}{\partial g^{z}}$

$$\frac{\partial \delta^{z}}{\partial T^{z}} = \sum_{j=1}^{J^{z}} f^{zj}\frac{\partial \mu^{zj}}{\partial x^{zj}} - \lambda^{z}J^{*z} = 0 \quad \forall\, T^{*z} > 0 \tag{A.1.7}$$

And

$$\frac{\partial \delta^{z}}{\partial \lambda^{z}} = R^{z} - T^{*z}J^{*z} = 0 \quad \forall\, \lambda^{*z} > 0 \tag{A.1.8}$$

State (A.1.6) as follows

$$\frac{\partial\delta^z}{\partial g^z} = \frac{-2g^{*z}}{H}\sum_{h=1}^{H} f^{zh} + \sum_{h=1}^{H} f^{zh}(e^h - t) + T^{*z}\sum_{j=1}^{J^z} f^{zj} = 0 \qquad (A.1.9)$$

Define $E[f^{zh}] = \sum_{h=1}^{H} f^{zh}/H$ and $E\left[f^{zJ^z}\right] = \sum_{j=1}^{J^z} f^{zj}/J^{*z}$ and use $R^z = T^{*z}J^{*z}$ to re-arrange terms and show that

$$g^{*z} = \frac{\sum_{h=1}^{H} f^{zh}e^h}{2E[f^{zh}]} - \frac{tH}{2} + \frac{R^z}{2}\left\{\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]}\right\} \qquad (A.1.10)$$

From the definition of the covariance between A and B it is satisfied that $\sigma[A,B] = E[AB] - E[A]E[B]$. Name $A = f^{zh}$ and $B = e^h$ to state the following

$$\sum_{h=1}^{H} f^{zh}e^h = \sigma^z[f^{zh}, e^h] + \frac{\sum_{h=1}^{H} f^{zh}}{H}\frac{\sum_{h=1}^{H} e^h}{H} \qquad (A.1.11)$$

Use the former condition into A.9 to show

$$g^{*z} = \frac{\sigma^z[f^{zh}, e^h]}{2E[f^{zh}]} + \frac{E[e^h]}{2} - \frac{tH}{2} + \frac{R^z}{2}\left\{\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]}\right\} \qquad (A.1.12)$$

Where $E[e^h] = \sum_{h=1}^{H} e^h/H$.

Use the fact that $tH = R^z + R^{-z}$ into (A.12) to show

$$g^{*z} = \frac{\sigma^z[f^{zh}, e^h]}{2E[f^{zh}]} + \frac{E[e^h]}{2} + \frac{R^z}{2}\left\{\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} - 1\right\} - \frac{R^{-z}}{2} \quad \forall z, -z \qquad (A.1.13)$$

Now we turn to the analysis of transfers T^{*z}. We start by considering condition (A.1.7) and show that this condition is equivalent to

$$J^{*z} = \frac{\sum_{j=1}^{J^z} f^{zj}\frac{\partial\mu^{zj}}{\partial x^{zj}}}{\lambda^{*z}} = \frac{g^{*z}\sum_{j=1}^{J^z} f^{zj}}{\lambda^{*z}} \quad \forall z, -z \qquad (A.1.14)$$

From the parties' constraint on campaign funds

$$T^{*z} = \frac{R^z}{J^{*z}} = \frac{R^z\lambda^{*z}}{g^{*z}\sum_{j=1}^{J^z} f^{zj}} \quad \forall z, -z \qquad (A.1.15)$$

Proposition 2. *Symmetric public campaign funding leads parties to converge in proposing* $\hat{g}^{*z} = \hat{g}^{*-z}$ *and* $T^{*z} = T^{*-z}\ \forall\, e^j \in \left[\underline{e^1}, e^{J^z}\right], \forall\, z, -z$ *satisfying*

$$\hat{g}^* = \frac{E[e^h] - \hat{R}}{2} \qquad (10)$$

And

$$\hat{T}^* = \frac{\hat{R}\hat{\lambda}^*}{\hat{g}^*\sum_{j=1}^{J^z} f^z(0)} \qquad (11)$$

Proof

It is well known that if parties maximize a share of the vote that is continuous, strictly concave and if parties face the same constraints then the parties' policies converge (for formal proofs see Coughlin 1992). In our economy, parties satisfy these assumptions therefore $\hat{g}^{*z} = \hat{g}^{*-z}$ and $T^{*z} = T^{*-z}\ \forall\, e^j \in \left[\underline{e^1}, e^{J^z}\right], \forall\, z, -z$.This outcome means that

$$\chi^{zj}(e^j) = \nu^{zj}(e^j, t, g^{*z}, T^{*z}) - \nu^{-zj}(e^j, t, g^{*-z}, T^{*-z}) = 0 \quad \forall j = 1 \dots J^z \quad (A.2.1)$$

And

$$\chi^{zh}(e^h) = \nu^{zh}(e^h, t, g^{*z}) - \nu^{-zh}(e^h, t, g^{*-z}) = 0 \quad \forall\, h = J^z + 1 \dots\dots H \quad (A.2.2)$$

Which in turn means that

$$f^{zj} = f^{zj}(0) \quad \forall j = 1 \dots. J^z \quad \text{and} \quad f^{zh} = f^{zh}(0) \quad \forall h = J^z + 1 \dots\dots H \quad (A.2.3)$$

Which implies

$$\sigma^z[f^{zh}, e^h] = \sigma^z[f^z(0), e^h] = 0 \quad (A.2.4)$$

$$E[f^{zh}] = \sum_{h=1}^{H} f^{zh}/H = \frac{Hf^{zh}(0)}{H} = f^z(0) \quad (A.2.5)$$

$$E\left[f^{zJ^z}\right] = \sum_{j=1}^{J^z} f^{zj}/J^{*z} = \frac{J^{*z}f^{zj}(0)}{J^{*z}} = f^z(0) \quad (A.2.6)$$

$$\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} = \frac{f^{zj}(0)}{f^{zh}(0)} = 1 \quad (A.2.7)$$

Use these outcomes in the equilibrium levels of government spending and transfers of parties (conditions 6 and 7 of proposition 1) to show that symmetric public campaign funding $\hat{R}^z = \hat{R}^{-z} = \hat{R}$ leads parties to converge in proposing $\hat{g}^{*z} = \hat{g}^{*-z} = \hat{g}^*$ and $\hat{T}^{*z} = \hat{T}^{*-z} = \hat{T}^* \; \forall\, e^j \in \left[\underline{e^1}, e^{J^z}\right], \forall\, z, -z$ and $\hat{\lambda}^{*z} = \hat{\lambda}^{*-z} = \hat{\lambda}^*$ satisfying

$$\hat{g}^* = \frac{E[e^h] - \hat{R}}{2} \quad (A.2.8)$$

And from the first order conditions

$$\hat{J}^* = \frac{\sum_{j=1}^{J^z} f^{zj}(0)\hat{g}^*}{\hat{\lambda}^*} \quad (A.2.9)$$

Implying that $\hat{T}^*$ is given by

$$\hat{T}^* = \frac{\hat{R}\hat{\lambda}^*}{\hat{g}^* \sum_{j=1}^{J^z} f^{zj}(0)} \quad (A.2.10)$$

Proposition 3. A*symmetric public campaign funding $\hat{R}^z \neq \hat{R}^{-z}$ leads parties to diverge in their policies $\hat{g}^{*z} \neq \hat{g}^{*-z} = \hat{g}^*$ and $\hat{T}^{*z} \neq \hat{T}^{*-z} \; \forall\, e^j \in \left[\underline{e^1}, e^{J^z}\right], \forall\, z, -z$. If*

3.1) $\sigma^z[f^{zh}, e^h] > 0$ *and*

3.2) $R^z < R^{-z}$ and $E\left[f^{zJ^z}\right] > E[f^{zh}]: \frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} > \frac{\hat{R}}{R^z} \;\; \forall z, -z$

Then both parties select polarized policies satisfying

$$g^{*z} > \hat{g}^* \quad \text{and } g^{*-z} > \hat{g}^*$$

Proof

From first order conditions of g^{*z} when public campaign financing is asymmetric, it follows

$$g^{*z} = \frac{\sigma^z[f^{zh}, e^h]}{2E[f^{zh}]} + \frac{E[e^h]}{2} - \frac{tH}{2} + \frac{R^z}{2}\left\{\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]}\right\} \quad \forall z, -z \qquad (A.3.1)$$

Since $\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} = \frac{f^{zj}(0)}{f^{zh}(0)} = 1$ and $\sigma^z[f^{zh}, e^h] = \sigma^z[f^{zh}(0), e^h] = 0.$ Hence $\hat{g}^*$ is given by

$$\hat{g}^* = \frac{E[e^h]}{2} - \frac{tH}{2} + \frac{\hat{R}}{2} \quad \forall z, -z \qquad (A.3.2)$$

Therefore $g^{*z} - \hat{g}^*$ satisfies the following

$$g^{*z} - \hat{g}^* = \frac{\sigma^z[f^{zh}, e^h]}{2E[f^{zh}]} + \frac{1}{2}\left\{R^z \frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} - \hat{R}\right\} \qquad (A..3.3)$$

Without loss of generality consider $R^z = min\{R^z, R^{-z}\}$. If $E\left[f^{zJ^z}\right] > E[f^{zh}]$: $\frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} > \frac{\hat{R}}{R^z}$ then $\left\{R^z \frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} - \hat{R}\right\} > 0$ and since $R^{-z} > R^z = min\{R^z, R^{-z}\}$ then $\left\{R^{-z} \frac{E\left[f^{zJ^z}\right]}{E[f^{zh}]} - \hat{R}\right\} > 0$. By assumption $\sigma^z[f^{zh}, e^h] > 0$ and $E[f^{zh}] > 0 \Longrightarrow \frac{\sigma^z[f^{zh}, e^h]}{2E[f^{zh}]} > 0$ therefore $g^{*z} > \hat{g}^* \ \forall z, -z$.

To complete the analysis, now consider

$$T^{*z} - \hat{T}^* = \frac{R^z \lambda^{*z}}{g^{*z} \sum_{j=1}^{J^z} f^{zj}} - \frac{\hat{R}\hat{\lambda}^*}{\hat{g}^* \sum_{j=1}^{J} f^z(0)} \underset{<}{\overset{>}{=}} 0 \qquad (A..3.4)$$

Since $R^z > \hat{R} > R^{-z}$, $g^{*z} > \hat{g}^* \ \forall z, -z$ and $\sum_{j=1}^{J} f^{zj} \underset{<}{\overset{>}{=}} \sum_{j=1}^{J} f^{zj}(0)$ therefore $T^{*z} \underset{<}{\overset{>}{=}} \hat{T}^* \ \forall z, -z$.