

## Información adicional sobre Meta 3

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**Abstract.** Información adicional sobre Meta 3 de informe de proyecto interno UACJ 2018, *Métodos de Lie para campos vectoriales de dimensión tres*.

**Keyword:** Vector fields, Lorch differentiability, Generalized Cauchy Riemann equations

**MSC[2010]:** 37C10, 58C20, 53C22.

## Introduction

En el proyecto se plantea aplicar los métodos de Lie a campos vectoriales de dimensión tres para construir primeras integrales y para encontrar sus curvas integrales mediante la función inversa de rectificaciones. También se plantea dar seguimiento a la publicación de trabajos desarrollados en otros proyectos.

En el caso de campos vectoriales planares  $F = (f, g)$  que admitan una simetría infinitesimal  $G$  no trivial, esto es,  $dGF - dFG = \alpha F$ ,  $G \neq cF$ , para  $\alpha = \alpha(x, y)$  función escalar y para toda  $c \in \mathbb{R}$ , se tiene que el determinante  $H = \det(F, G) \neq 0$  de los campos vectoriales define un factor integrante inverso de  $F$ , esto significa que el campo vectorial  $K = (g, -f)/H$  es un campo conservativo, cuya función potencial  $h$  es una primera integral de  $F$ . De esta manera se considera que el sistema de ecuaciones diferenciales asociado al campo vectorial  $F$  está resuelto.

Una parte de este proyecto consistió en buscar resolver de manera análoga a cómo se hace para campos vectoriales en el plano, pero para campos vectoriales en el espacio. No se pudo avanzar en esta dirección ya que dado un campo vectorial  $F$  en el espacio, ya que para los candidatos a ser campos vectoriales conservativos  $G$  de tal manera que las funciones potenciales asociadas a estos  $G$  sean primeras integrales de  $F$ , no se logró mostrar mediante el software Mathematica que los  $G$  efectivamente son conservativos. La propuesta del proyecto fue porque esto sí se puede hacer para campos algebrizables, como se muestra en [7]. En base a esto se pensó que se podría generalizar esto a campos vectoriales más generales que campos algebrizables. Como resultado de este proyecto tampoco se puede decir que la idea falla. Más bien se puede

decir que no se pudo decidir si la idea se puede generalizar y tampoco el tipo de campo vectoriales para los que esto pudiera generalizar. El trabajo en esta dirección se considera terminado por los motivos mencionados. Se anexan los archivos:

- Candidato a campo conservativo.
- Candidato a campo a factor integrante inverso.
- Cuentas de Lie caso planar.

En el primer documento se hacen las cuentas buscando mostrar que el campo propuesto es campo conservativo. En el segundo documento se muestra la función propuesta a ser factor integrante, y en el tercer documento se muestran las cuentas básicas para el caso de campos vectoriales planares. De esta manera se comprueba que el trabajo de investigación se realizó sin los resultados esperados y sin decidir si la idea funciona en otros casos a los propuestos.

Sin embargo se avanzó en otra dirección, también basada en los métodos de Lie y que nos permite encontrar las curvas solución de una familia más amplia de campos vectoriales en el espacio. Se introdujo otro concepto de diferenciabilidad relacionada con álgebras en [9], la cual es una generalización de la derivada de Lorch (consultar en [6] y [7] y referencias dadas en estos). La diferenciación introducida en el trabajo [9] se le nombra  $\varphi\mathbb{A}$ -diferenciación. En los trabajos [6] y [7], se introducen rectificaciones para campos vectoriales algebrizables, mediante la inversa de estas funciones rectificadoras se pueden encontrar las curvas integrales, esta técnica pertenece a los métodos de Lie, pero en los trabajos [6] y [7] se dan estas rectificaciones en términos de una integral de línea del inverso multiplicativo con respecto al álgebra del campo vectorial algebrizable considerado. Para la familia de funciones  $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}^n$  con diferencial  $d\varphi$  con imagen en el conjunto de matrices ortogonales  $O(n, \mathbb{R})$ , se tiene que la  $\varphi\mathbb{A}$ -integral permite definir rectificaciones de estas familias de campos vectoriales de manera análoga a como se hizo en [6] y [7]. Esto da una manera de buscar las curvas integrales de estos campos vectoriales. Además en el trabajo [9] se introducen las ecuaciones  $\varphi\mathbb{A}$ -diferenciales, que es un tipo de ecuaciones diferenciales abstractas que se puede utilizar para resolver sistemas tanto ecuaciones diferenciales ordinarias (ver [9]), como sistemas de ecuaciones en derivadas parciales, como ya se trabaja en esta dirección en [5], pero no se considera parte de este proyecto.

El trabajo [9] se encuentra siendo evaluado para su posible publicación en el *Boletín de la Sociedad Matemática Mexicana*, la cual es una revista internacional indizada de arbitraje estricto. Un preprint se publicó en el sitio <https://arxiv.org/abs/1805.10524>, pero se consideró conveniente retirarlo para realizar algunas correcciones. También el trabajo [7] se ha enviado a *Electronic Journal of Differential Equations* para su posible publicación. Se cuenta con dos capítulos de libro aceptados por AVANZA para publicación.

## References

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In[4]= **F**[x1\_, x2\_, x3\_] = {f1[x1, x2, x3], f2[x1, x2, x3], f3[x1, x2, x3]}

Out[4]= {f1[x1, x2, x3], f2[x1, x2, x3], f3[x1, x2, x3]}

In[5]= **G**[x1\_, x2\_, x3\_] = {g1[x1, x2, x3], g2[x1, x2, x3], g3[x1, x2, x3]}

Out[5]= {g1[x1, x2, x3], g2[x1, x2, x3], g3[x1, x2, x3]}

In[6]= **K**[x1\_, x2\_, x3\_] = {k1[x1, x2, x3], k2[x1, x2, x3], k3[x1, x2, x3]}

Out[6]= {k1[x1, x2, x3], k2[x1, x2, x3], k3[x1, x2, x3]}

In[7]= **E1**[x1\_, x2\_, x3\_] = F[x1, x2, x3]

Out[7]= {f1[x1, x2, x3], f2[x1, x2, x3], f3[x1, x2, x3]}

In[8]= **E2**[x1\_, x2\_, x3\_] = {-p7 f2[x1, x2, x3] - p8 f3[x1, x2, x3],  
f1[x1, x2, x3] - p1 f2[x1, x2, x3] - p3 f3[x1, x2, x3],  
-p2 f2[x1, x2, x3] - p4 f3[x1, x2, x3]}

Out[8]= {-p7 f2[x1, x2, x3] - p8 f3[x1, x2, x3],  
f1[x1, x2, x3] - p1 f2[x1, x2, x3] - p3 f3[x1, x2, x3], -p2 f2[x1, x2, x3] - p4 f3[x1, x2, x3]}

In[9]= **E3**[x1\_, x2\_, x3\_] = {-p8 f2[x1, x2, x3] - p9 f3[x1, x2, x3],  
-p3 f2[x1, x2, x3] - p5 f3[x1, x2, x3], f1[x1, x2, x3] - p4 f2[x1, x2, x3] - p6 f3[x1, x2, x3]}

Out[9]= {-p8 f2[x1, x2, x3] - p9 f3[x1, x2, x3], -p3 f2[x1, x2, x3] - p5 f3[x1, x2, x3],  
f1[x1, x2, x3] - p4 f2[x1, x2, x3] - p6 f3[x1, x2, x3]}

In[10]= **RF**[x1\_, x2\_, x3\_] = {{f1[x1, x2, x3], f2[x1, x2, x3], f3[x1, x2, x3]},  
{g1[x1, x2, x3], g2[x1, x2, x3], g3[x1, x2, x3]},  
{k1[x1, x2, x3], k2[x1, x2, x3], k3[x1, x2, x3]}}

Out[10]= {{f1[x1, x2, x3], f2[x1, x2, x3], f3[x1, x2, x3]},  
{g1[x1, x2, x3], g2[x1, x2, x3], g3[x1, x2, x3]},  
{k1[x1, x2, x3], k2[x1, x2, x3], k3[x1, x2, x3]}}

In[11]= **MatrixForm**[RF[x1, x2, x3]]

[\[forma de matriz\]](#)

Out[11]/MatrixForm=

$$\begin{pmatrix} f1[x1, x2, x3] & f2[x1, x2, x3] & f3[x1, x2, x3] \\ g1[x1, x2, x3] & g2[x1, x2, x3] & g3[x1, x2, x3] \\ k1[x1, x2, x3] & k2[x1, x2, x3] & k3[x1, x2, x3] \end{pmatrix}$$

**H = G1**

**Clear**[H]

[\[borra\]](#)

```
In[12]= H[x1_, x2_, x3_] =
  1 / Det[RF[x1, x2, x3]] {f1[x1, x2, x3]^2 - (p1 + p4) f1[x1, x2, x3] f2[x1, x2, x3] -
    |determinante
    (p3 + p6) f1[x1, x2, x3] f3[x1, x2, x3] + (p1 p4 - p2 p3) f2[x1, x2, x3]^2 +
    (p1 p6 - p2 p5) f2[x1, x2, x3] f3[x1, x2, x3] + (p3 p6 - p4 p5) f3[x1, x2, x3]^2,
    p7 f1[x1, x2, x3] f2[x1, x2, x3] + p8 f1[x1, x2, x3] f3[x1, x2, x3] +
    (p2 p8 - p4 p7) f2[x1, x2, x3]^2 + (p2 p9 - p6 p7) f2[x1, x2, x3] f3[x1, x2, x3] +
    (p5 p7 - p3 p8) f3[x1, x2, x3]^2, p8 f1[x1, x2, x3] f2[x1, x2, x3] +
    p9 f1[x1, x2, x3] f3[x1, x2, x3] + (p2 p9 - p4 p8) f2[x1, x2, x3]^2 +
    (p5 p7 - p1 p9) f2[x1, x2, x3] f3[x1, x2, x3] + (p5 p8 - p3 p9) f3[x1, x2, x3]^2}
```

```
Out[12]= { (f1[x1, x2, x3]^2 - (p1 + p4) f1[x1, x2, x3] f2[x1, x2, x3] +
  (-p2 p3 + p1 p4) f2[x1, x2, x3]^2 - (p3 + p6) f1[x1, x2, x3] f3[x1, x2, x3] +
  (-p2 p5 + p1 p6) f2[x1, x2, x3] f3[x1, x2, x3] + (-p4 p5 + p3 p6) f3[x1, x2, x3]^2) /
  (-f3[x1, x2, x3] g2[x1, x2, x3] k1[x1, x2, x3] + f2[x1, x2, x3] g3[x1, x2, x3] k1[x1, x2, x3] +
  f3[x1, x2, x3] g1[x1, x2, x3] k2[x1, x2, x3] -
  f1[x1, x2, x3] g3[x1, x2, x3] k2[x1, x2, x3] - f2[x1, x2, x3] g1[x1, x2, x3] k3[x1, x2, x3] +
  f1[x1, x2, x3] g2[x1, x2, x3] k3[x1, x2, x3]),
  (p7 f1[x1, x2, x3] f2[x1, x2, x3] + (-p4 p7 + p2 p8) f2[x1, x2, x3]^2 +
  p8 f1[x1, x2, x3] f3[x1, x2, x3] + (-p6 p7 + p2 p9) f2[x1, x2, x3] f3[x1, x2, x3] +
  (p5 p7 - p3 p8) f3[x1, x2, x3]^2) / (-f3[x1, x2, x3] g2[x1, x2, x3] k1[x1, x2, x3] +
  f2[x1, x2, x3] g3[x1, x2, x3] k1[x1, x2, x3] + f3[x1, x2, x3] g1[x1, x2, x3] k2[x1, x2, x3] -
  f1[x1, x2, x3] g3[x1, x2, x3] k2[x1, x2, x3] - f2[x1, x2, x3] g1[x1, x2, x3] k3[x1, x2, x3] +
  f1[x1, x2, x3] g2[x1, x2, x3] k3[x1, x2, x3]),
  (p8 f1[x1, x2, x3] f2[x1, x2, x3] + (-p4 p8 + p2 p9) f2[x1, x2, x3]^2 +
  p9 f1[x1, x2, x3] f3[x1, x2, x3] + (p5 p7 - p1 p9) f2[x1, x2, x3] f3[x1, x2, x3] +
  (p5 p8 - p3 p9) f3[x1, x2, x3]^2) / (-f3[x1, x2, x3] g2[x1, x2, x3] k1[x1, x2, x3] +
  f2[x1, x2, x3] g3[x1, x2, x3] k1[x1, x2, x3] + f3[x1, x2, x3] g1[x1, x2, x3] k2[x1, x2, x3] -
  f1[x1, x2, x3] g3[x1, x2, x3] k2[x1, x2, x3] - f2[x1, x2, x3] g1[x1, x2, x3] k3[x1, x2, x3] +
  f1[x1, x2, x3] g2[x1, x2, x3] k3[x1, x2, x3]) }
```

```
In[13]= DF[x1_, x2_, x3_] =
  Transpose[{D[F[x1, x2, x3], x1], D[F[x1, x2, x3], x2], D[F[x1, x2, x3], x3]}]
  |transposición |deriva |deriva |deriva
```

```
DF[x1, x2, x3] // MatrixForm
|forma de matriz
```

```
Out[13]= { {f1^(1,0,0)[x1, x2, x3], f1^(0,1,0)[x1, x2, x3], f1^(0,0,1)[x1, x2, x3]},
  {f2^(1,0,0)[x1, x2, x3], f2^(0,1,0)[x1, x2, x3], f2^(0,0,1)[x1, x2, x3]},
  {f3^(1,0,0)[x1, x2, x3], f3^(0,1,0)[x1, x2, x3], f3^(0,0,1)[x1, x2, x3]} }
```

```
Out[14]//MatrixForm=
```

```
( f1^(1,0,0)[x1, x2, x3] f1^(0,1,0)[x1, x2, x3] f1^(0,0,1)[x1, x2, x3]
  f2^(1,0,0)[x1, x2, x3] f2^(0,1,0)[x1, x2, x3] f2^(0,0,1)[x1, x2, x3]
  f3^(1,0,0)[x1, x2, x3] f3^(0,1,0)[x1, x2, x3] f3^(0,0,1)[x1, x2, x3] )
```

```
In[15]= DG[x1_, x2_, x3_] =
  Transpose[{D[G[x1, x2, x3], x1], D[G[x1, x2, x3], x2], D[G[x1, x2, x3], x3]}]
  |transposición |deriva |deriva |deriva
  DG[x1, x2, x3] // MatrixForm
  |forma de matriz
```

```
Out[15]= {{g1^(1,0,0)[x1, x2, x3], g1^(0,1,0)[x1, x2, x3], g1^(0,0,1)[x1, x2, x3]},
  {g2^(1,0,0)[x1, x2, x3], g2^(0,1,0)[x1, x2, x3], g2^(0,0,1)[x1, x2, x3]},
  {g3^(1,0,0)[x1, x2, x3], g3^(0,1,0)[x1, x2, x3], g3^(0,0,1)[x1, x2, x3]}}
```

```
Out[16]//MatrixForm=
  ( g1^(1,0,0)[x1, x2, x3] g1^(0,1,0)[x1, x2, x3] g1^(0,0,1)[x1, x2, x3]
    g2^(1,0,0)[x1, x2, x3] g2^(0,1,0)[x1, x2, x3] g2^(0,0,1)[x1, x2, x3]
    g3^(1,0,0)[x1, x2, x3] g3^(0,1,0)[x1, x2, x3] g3^(0,0,1)[x1, x2, x3] )
```

```
In[17]= DK[x1_, x2_, x3_] =
  Transpose[{D[K[x1, x2, x3], x1], D[K[x1, x2, x3], x2], D[K[x1, x2, x3], x3]}]
  |transposición |deriva |deriva |deriva
  DK[x1, x2, x3] // MatrixForm
  |forma de matriz
```

```
Out[17]= {{k1^(1,0,0)[x1, x2, x3], k1^(0,1,0)[x1, x2, x3], k1^(0,0,1)[x1, x2, x3]},
  {k2^(1,0,0)[x1, x2, x3], k2^(0,1,0)[x1, x2, x3], k2^(0,0,1)[x1, x2, x3]},
  {k3^(1,0,0)[x1, x2, x3], k3^(0,1,0)[x1, x2, x3], k3^(0,0,1)[x1, x2, x3]}}
```

```
Out[18]//MatrixForm=
  ( k1^(1,0,0)[x1, x2, x3] k1^(0,1,0)[x1, x2, x3] k1^(0,0,1)[x1, x2, x3]
    k2^(1,0,0)[x1, x2, x3] k2^(0,1,0)[x1, x2, x3] k2^(0,0,1)[x1, x2, x3]
    k3^(1,0,0)[x1, x2, x3] k3^(0,1,0)[x1, x2, x3] k3^(0,0,1)[x1, x2, x3] )
```

```
In[19]= EcuacionFG[x1, x2, x3] =
  DG[x1, x2, x3].F[x1, x2, x3] - DF[x1, x2, x3].G[x1, x2, x3] - beta F[x1, x2, x3]
```

```
Out[19]= {-beta f1[x1, x2, x3] -
  g3[x1, x2, x3] f1^(0,0,1)[x1, x2, x3] + f3[x1, x2, x3] g1^(0,0,1)[x1, x2, x3] -
  g2[x1, x2, x3] f1^(0,1,0)[x1, x2, x3] + f2[x1, x2, x3] g1^(0,1,0)[x1, x2, x3] -
  g1[x1, x2, x3] f1^(1,0,0)[x1, x2, x3] + f1[x1, x2, x3] g1^(1,0,0)[x1, x2, x3],
  -beta f2[x1, x2, x3] - g3[x1, x2, x3] f2^(0,0,1)[x1, x2, x3] +
  f3[x1, x2, x3] g2^(0,0,1)[x1, x2, x3] - g2[x1, x2, x3] f2^(0,1,0)[x1, x2, x3] +
  f2[x1, x2, x3] g2^(0,1,0)[x1, x2, x3] - g1[x1, x2, x3] f2^(1,0,0)[x1, x2, x3] +
  f1[x1, x2, x3] g2^(1,0,0)[x1, x2, x3], -beta f3[x1, x2, x3] -
  g3[x1, x2, x3] f3^(0,0,1)[x1, x2, x3] + f3[x1, x2, x3] g3^(0,0,1)[x1, x2, x3] -
  g2[x1, x2, x3] f3^(0,1,0)[x1, x2, x3] + f2[x1, x2, x3] g3^(0,1,0)[x1, x2, x3] -
  g1[x1, x2, x3] f3^(1,0,0)[x1, x2, x3] + f1[x1, x2, x3] g3^(1,0,0)[x1, x2, x3]}
```

```
MatrixForm[EcuacionFG[x1, x2, x3]]
```

```
|forma de matriz
```

```
( -beta f1[x1, x2, x3] - g3[x1, x2, x3] f1^(0,0,1)[x1, x2, x3] + f3[x1, x2, x3] g1^(0,0,1)[x1, x2, x3] -
  -beta f2[x1, x2, x3] - g3[x1, x2, x3] f2^(0,0,1)[x1, x2, x3] + f3[x1, x2, x3] g2^(0,0,1)[x1, x2, x3] -
  -beta f3[x1, x2, x3] - g3[x1, x2, x3] f3^(0,0,1)[x1, x2, x3] + f3[x1, x2, x3] g3^(0,0,1)[x1, x2, x3] -
```

In[20]= **EcuacionFK[x1, x2, x3] =**  
**DK[x1, x2, x3].F[x1, x2, x3] - DF[x1, x2, x3].K[x1, x2, x3] - gamma F[x1, x2, x3]**

Out[20]= 
$$\left\{ \begin{aligned} & -\text{gamma } f_1[x_1, x_2, x_3] - \\ & k_3[x_1, x_2, x_3] f_1^{(0,0,1)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] k_1^{(0,0,1)}[x_1, x_2, x_3] - \\ & k_2[x_1, x_2, x_3] f_1^{(0,1,0)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] k_1^{(0,1,0)}[x_1, x_2, x_3] - \\ & k_1[x_1, x_2, x_3] f_1^{(1,0,0)}[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] k_1^{(1,0,0)}[x_1, x_2, x_3], \\ & -\text{gamma } f_2[x_1, x_2, x_3] - k_3[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] + \\ & f_3[x_1, x_2, x_3] k_2^{(0,0,1)}[x_1, x_2, x_3] - k_2[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] + \\ & f_2[x_1, x_2, x_3] k_2^{(0,1,0)}[x_1, x_2, x_3] - k_1[x_1, x_2, x_3] f_2^{(1,0,0)}[x_1, x_2, x_3] + \\ & f_1[x_1, x_2, x_3] k_2^{(1,0,0)}[x_1, x_2, x_3], -\text{gamma } f_3[x_1, x_2, x_3] - \\ & k_3[x_1, x_2, x_3] f_3^{(0,0,1)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] k_3^{(0,0,1)}[x_1, x_2, x_3] - \\ & k_2[x_1, x_2, x_3] f_3^{(0,1,0)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] k_3^{(0,1,0)}[x_1, x_2, x_3] - \\ & k_1[x_1, x_2, x_3] f_3^{(1,0,0)}[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] k_3^{(1,0,0)}[x_1, x_2, x_3] \end{aligned} \right\}$$

**MatrixForm[EcuacionFK[x1, x2, x3]]**

[\[forma de matriz\]](#)

$$\begin{pmatrix} -\text{gamma } f_1[x_1, x_2, x_3] - k_3[x_1, x_2, x_3] f_1^{(0,0,1)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] k_1^{(0,0,1)}[x_1, x_2, x_3] \cdot \\ -\text{gamma } f_2[x_1, x_2, x_3] - k_3[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] k_2^{(0,0,1)}[x_1, x_2, x_3] \cdot \\ -\text{gamma } f_3[x_1, x_2, x_3] - k_3[x_1, x_2, x_3] f_3^{(0,0,1)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] k_3^{(0,0,1)}[x_1, x_2, x_3] \cdot \end{pmatrix}$$


In[2]= **Needs["VectorAnalysis`"]**

[\[necesita\]](#)

In[21]= **Cur1[H[x1, x2, x3], {x1, x2, x3}]**

[\[rotacional\]](#)

Out[21]= 
$$\left\{ \begin{aligned} & - \left( (p_7 f_2[x_1, x_2, x_3] f_1^{(0,0,1)}[x_1, x_2, x_3] + p_8 f_3[x_1, x_2, x_3] f_1^{(0,0,1)}[x_1, x_2, x_3] + \right. \\ & p_7 f_1[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] + 2(-p_4 p_7 + p_2 p_8) f_2[x_1, x_2, x_3] \\ & f_2^{(0,0,1)}[x_1, x_2, x_3] + (-p_6 p_7 + p_2 p_9) f_3[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] + \\ & p_8 f_1[x_1, x_2, x_3] f_3^{(0,0,1)}[x_1, x_2, x_3] + (-p_6 p_7 + p_2 p_9) f_2[x_1, x_2, x_3] \\ & f_3^{(0,0,1)}[x_1, x_2, x_3] + 2(p_5 p_7 - p_3 p_8) f_3[x_1, x_2, x_3] f_3^{(0,0,1)}[x_1, x_2, x_3]) / \\ & (-f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \\ & k_1[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\ & f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\ & k_3[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3]) + \\ & ((p_7 f_1[x_1, x_2, x_3] f_2[x_1, x_2, x_3] + (-p_4 p_7 + p_2 p_8) f_2[x_1, x_2, x_3]^2 + \\ & p_8 f_1[x_1, x_2, x_3] f_3[x_1, x_2, x_3] + \\ & (-p_6 p_7 + p_2 p_9) f_2[x_1, x_2, x_3] f_3[x_1, x_2, x_3] + (p_5 p_7 - p_3 p_8) f_3[x_1, x_2, x_3]^2) \\ & (-g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] f_1^{(0,0,1)}[x_1, x_2, x_3] + g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \\ & f_1^{(0,0,1)}[x_1, x_2, x_3] + g_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] - \\ & g_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] - g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] \\ & f_3^{(0,0,1)}[x_1, x_2, x_3] + g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] f_3^{(0,0,1)}[x_1, x_2, x_3] + \\ & f_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] g_1^{(0,0,1)}[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \\ & g_1^{(0,0,1)}[x_1, x_2, x_3] - f_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] g_2^{(0,0,1)}[x_1, x_2, x_3] + \\ & f_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] g_2^{(0,0,1)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] \\ & g_3^{(0,0,1)}[x_1, x_2, x_3] - f_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] g_3^{(0,0,1)}[x_1, x_2, x_3] - \\ & f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1^{(0,0,1)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \\ & k_1^{(0,0,1)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2^{(0,0,1)}[x_1, x_2, x_3] - \\ & f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2^{(0,0,1)}[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \end{aligned} \right\}$$

$$\begin{aligned}
& \left. \left( k_3^{(0,0,1)}[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3^{(0,0,1)}[x_1, x_2, x_3] \right) \right) / \\
& \left( -f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \right. \\
& \quad k_1[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad \left. k_3[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \right)^2 + \\
& \left( p_8 f_2[x_1, x_2, x_3] f_1^{(0,1,0)}[x_1, x_2, x_3] + p_9 f_3[x_1, x_2, x_3] f_1^{(0,1,0)}[x_1, x_2, x_3] + \right. \\
& \quad p_8 f_1[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad 2(-p_4 p_8 + p_2 p_9) f_2[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad (p_5 p_7 - p_1 p_9) f_3[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad p_9 f_1[x_1, x_2, x_3] f_3^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad (p_5 p_7 - p_1 p_9) f_2[x_1, x_2, x_3] f_3^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad \left. 2(p_5 p_8 - p_3 p_9) f_3[x_1, x_2, x_3] f_3^{(0,1,0)}[x_1, x_2, x_3] \right) / \\
& \left( -f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + \right. \\
& \quad f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + \\
& \quad f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] + \\
& \quad \left. f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \right) - \\
& \left( (p_8 f_1[x_1, x_2, x_3] f_2[x_1, x_2, x_3] + (-p_4 p_8 + p_2 p_9) f_2[x_1, x_2, x_3]^2 + \right. \\
& \quad p_9 f_1[x_1, x_2, x_3] f_3[x_1, x_2, x_3] + \\
& \quad (p_5 p_7 - p_1 p_9) f_2[x_1, x_2, x_3] f_3[x_1, x_2, x_3] + (p_5 p_8 - p_3 p_9) f_3[x_1, x_2, x_3]^2) \\
& \quad \left. (-g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] f_1^{(0,1,0)}[x_1, x_2, x_3] + g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \right. \\
& \quad f_1^{(0,1,0)}[x_1, x_2, x_3] + g_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] - \\
& \quad g_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] - g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] \\
& \quad f_3^{(0,1,0)}[x_1, x_2, x_3] + g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] f_3^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad f_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] g_1^{(0,1,0)}[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \\
& \quad g_1^{(0,1,0)}[x_1, x_2, x_3] - f_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] g_2^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad f_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] g_2^{(0,1,0)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] \\
& \quad g_3^{(0,1,0)}[x_1, x_2, x_3] - f_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] g_3^{(0,1,0)}[x_1, x_2, x_3] - \\
& \quad f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1^{(0,1,0)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \\
& \quad k_1^{(0,1,0)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2^{(0,1,0)}[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2^{(0,1,0)}[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad \left. k_3^{(0,1,0)}[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3^{(0,1,0)}[x_1, x_2, x_3] \right) \Big) / \\
& \left( -f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \right. \\
& \quad k_1[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad \left. k_3[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \right)^2, \\
& \left( 2 f_1[x_1, x_2, x_3] f_1^{(0,0,1)}[x_1, x_2, x_3] - (p_1 + p_4) f_2[x_1, x_2, x_3] f_1^{(0,0,1)}[x_1, x_2, x_3] - \right. \\
& \quad (p_3 + p_6) f_3[x_1, x_2, x_3] f_1^{(0,0,1)}[x_1, x_2, x_3] - \\
& \quad (p_1 + p_4) f_1[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] + \\
& \quad 2(-p_2 p_3 + p_1 p_4) f_2[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] + \\
& \quad (-p_2 p_5 + p_1 p_6) f_3[x_1, x_2, x_3] f_2^{(0,0,1)}[x_1, x_2, x_3] - \\
& \quad (p_3 + p_6) f_1[x_1, x_2, x_3] f_3^{(0,0,1)}[x_1, x_2, x_3] + \\
& \quad (-p_2 p_5 + p_1 p_6) f_2[x_1, x_2, x_3] f_3^{(0,0,1)}[x_1, x_2, x_3] + \\
& \quad \left. 2(-p_4 p_5 + p_3 p_6) f_3[x_1, x_2, x_3] f_3^{(0,0,1)}[x_1, x_2, x_3] \right) / \\
& \left( -f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + \right. \\
& \quad f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + \\
& \quad f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad \left. f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \right.
\end{aligned}$$



$$\begin{aligned}
& f2[x1, x2, x3] g1[x1, x2, x3] k3[x1, x2, x3] + \\
& f1[x1, x2, x3] g2[x1, x2, x3] k3[x1, x2, x3] ) - \\
& ( (f1[x1, x2, x3]^2 - (p1 + p4) f1[x1, x2, x3] f2[x1, x2, x3] + \\
& (-p2 p3 + p1 p4) f2[x1, x2, x3]^2 - (p3 + p6) f1[x1, x2, x3] f3[x1, x2, x3] + \\
& (-p2 p5 + p1 p6) f2[x1, x2, x3] f3[x1, x2, x3] + (-p4 p5 + p3 p6) f3[x1, x2, x3]^2) \\
& (-g3[x1, x2, x3] k2[x1, x2, x3] f1^{(0,0,1)}[x1, x2, x3] + g2[x1, x2, x3] k3[x1, x2, x3] \\
& f1^{(0,0,1)}[x1, x2, x3] + g3[x1, x2, x3] k1[x1, x2, x3] f2^{(0,0,1)}[x1, x2, x3] - \\
& g1[x1, x2, x3] k3[x1, x2, x3] f2^{(0,0,1)}[x1, x2, x3] - g2[x1, x2, x3] k1[x1, x2, x3] \\
& f3^{(0,0,1)}[x1, x2, x3] + g1[x1, x2, x3] k2[x1, x2, x3] f3^{(0,0,1)}[x1, x2, x3] + \\
& f3[x1, x2, x3] k2[x1, x2, x3] g1^{(0,0,1)}[x1, x2, x3] - f2[x1, x2, x3] k3[x1, x2, x3] \\
& g1^{(0,0,1)}[x1, x2, x3] - f3[x1, x2, x3] k1[x1, x2, x3] g2^{(0,0,1)}[x1, x2, x3] + \\
& f1[x1, x2, x3] k3[x1, x2, x3] g2^{(0,0,1)}[x1, x2, x3] + f2[x1, x2, x3] k1[x1, x2, x3] \\
& g3^{(0,0,1)}[x1, x2, x3] - f1[x1, x2, x3] k2[x1, x2, x3] g3^{(0,0,1)}[x1, x2, x3] - \\
& f3[x1, x2, x3] g2[x1, x2, x3] k1^{(0,0,1)}[x1, x2, x3] + f2[x1, x2, x3] g3[x1, x2, x3] \\
& k1^{(0,0,1)}[x1, x2, x3] + f3[x1, x2, x3] g1[x1, x2, x3] k2^{(0,0,1)}[x1, x2, x3] - \\
& f1[x1, x2, x3] g3[x1, x2, x3] k2^{(0,0,1)}[x1, x2, x3] - f2[x1, x2, x3] g1[x1, x2, x3] \\
& k3^{(0,0,1)}[x1, x2, x3] + f1[x1, x2, x3] g2[x1, x2, x3] k3^{(0,0,1)}[x1, x2, x3] ) ) / \\
& (-f3[x1, x2, x3] g2[x1, x2, x3] k1[x1, x2, x3] + f2[x1, x2, x3] g3[x1, x2, x3] \\
& k1[x1, x2, x3] + f3[x1, x2, x3] g1[x1, x2, x3] k2[x1, x2, x3] - \\
& f1[x1, x2, x3] g3[x1, x2, x3] k2[x1, x2, x3] - f2[x1, x2, x3] g1[x1, x2, x3] \\
& k3[x1, x2, x3] + f1[x1, x2, x3] g2[x1, x2, x3] k3[x1, x2, x3])^2 - \\
& (p8 f2[x1, x2, x3] f1^{(1,0,0)}[x1, x2, x3] + p9 f3[x1, x2, x3] f1^{(1,0,0)}[x1, x2, x3] + \\
& p8 f1[x1, x2, x3] f2^{(1,0,0)}[x1, x2, x3] + \\
& 2(-p4 p8 + p2 p9) f2[x1, x2, x3] f2^{(1,0,0)}[x1, x2, x3] + \\
& (p5 p7 - p1 p9) f3[x1, x2, x3] f2^{(1,0,0)}[x1, x2, x3] + \\
& p9 f1[x1, x2, x3] f3^{(1,0,0)}[x1, x2, x3] + \\
& (p5 p7 - p1 p9) f2[x1, x2, x3] f3^{(1,0,0)}[x1, x2, x3] + \\
& 2(p5 p8 - p3 p9) f3[x1, x2, x3] f3^{(1,0,0)}[x1, x2, x3] ) / \\
& (-f3[x1, x2, x3] g2[x1, x2, x3] k1[x1, x2, x3] + \\
& f2[x1, x2, x3] g3[x1, x2, x3] k1[x1, x2, x3] + \\
& f3[x1, x2, x3] g1[x1, x2, x3] k2[x1, x2, x3] - \\
& f1[x1, x2, x3] g3[x1, x2, x3] k2[x1, x2, x3] - \\
& f2[x1, x2, x3] g1[x1, x2, x3] k3[x1, x2, x3] + \\
& f1[x1, x2, x3] g2[x1, x2, x3] k3[x1, x2, x3] ) + \\
& ( (p8 f1[x1, x2, x3] f2[x1, x2, x3] + (-p4 p8 + p2 p9) f2[x1, x2, x3]^2 + \\
& p9 f1[x1, x2, x3] f3[x1, x2, x3] + \\
& (p5 p7 - p1 p9) f2[x1, x2, x3] f3[x1, x2, x3] + (p5 p8 - p3 p9) f3[x1, x2, x3]^2) \\
& (-g3[x1, x2, x3] k2[x1, x2, x3] f1^{(1,0,0)}[x1, x2, x3] + g2[x1, x2, x3] k3[x1, x2, x3] \\
& f1^{(1,0,0)}[x1, x2, x3] + g3[x1, x2, x3] k1[x1, x2, x3] f2^{(1,0,0)}[x1, x2, x3] - \\
& g1[x1, x2, x3] k3[x1, x2, x3] f2^{(1,0,0)}[x1, x2, x3] - g2[x1, x2, x3] k1[x1, x2, x3] \\
& f3^{(1,0,0)}[x1, x2, x3] + g1[x1, x2, x3] k2[x1, x2, x3] f3^{(1,0,0)}[x1, x2, x3] + \\
& f3[x1, x2, x3] k2[x1, x2, x3] g1^{(1,0,0)}[x1, x2, x3] - f2[x1, x2, x3] k3[x1, x2, x3] \\
& g1^{(1,0,0)}[x1, x2, x3] - f3[x1, x2, x3] k1[x1, x2, x3] g2^{(1,0,0)}[x1, x2, x3] + \\
& f1[x1, x2, x3] k3[x1, x2, x3] g2^{(1,0,0)}[x1, x2, x3] + f2[x1, x2, x3] k1[x1, x2, x3] \\
& g3^{(1,0,0)}[x1, x2, x3] - f1[x1, x2, x3] k2[x1, x2, x3] g3^{(1,0,0)}[x1, x2, x3] - \\
& f3[x1, x2, x3] g2[x1, x2, x3] k1^{(1,0,0)}[x1, x2, x3] + f2[x1, x2, x3] g3[x1, x2, x3] \\
& k1^{(1,0,0)}[x1, x2, x3] + f3[x1, x2, x3] g1[x1, x2, x3] k2^{(1,0,0)}[x1, x2, x3] - \\
& f1[x1, x2, x3] g3[x1, x2, x3] k2^{(1,0,0)}[x1, x2, x3] - f2[x1, x2, x3] g1[x1, x2, x3] \\
& k3^{(1,0,0)}[x1, x2, x3] + f1[x1, x2, x3] g2[x1, x2, x3] k3^{(1,0,0)}[x1, x2, x3] ) ) /
\end{aligned}$$

$$\begin{aligned}
& (-f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \\
& \quad k_1[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad k_3[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3])^2, \\
& - \left( (2 f_1[x_1, x_2, x_3] f_1^{(0,1,0)}[x_1, x_2, x_3] - (p_1 + p_4) f_2[x_1, x_2, x_3] f_1^{(0,1,0)}[x_1, x_2, x_3] - \right. \\
& \quad (p_3 + p_6) f_3[x_1, x_2, x_3] f_1^{(0,1,0)}[x_1, x_2, x_3] - (p_1 + p_4) f_1[x_1, x_2, x_3] \\
& \quad f_2^{(0,1,0)}[x_1, x_2, x_3] + 2(-p_2 p_3 + p_1 p_4) f_2[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad (-p_2 p_5 + p_1 p_6) f_3[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] - (p_3 + p_6) f_1[x_1, x_2, x_3] \\
& \quad f_3^{(0,1,0)}[x_1, x_2, x_3] + (-p_2 p_5 + p_1 p_6) f_2[x_1, x_2, x_3] f_3^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad \left. 2(-p_4 p_5 + p_3 p_6) f_3[x_1, x_2, x_3] f_3^{(0,1,0)}[x_1, x_2, x_3] \right) / \\
& (-f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \\
& \quad k_1[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad k_3[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3]) + \\
& \left( (f_1[x_1, x_2, x_3]^2 - (p_1 + p_4) f_1[x_1, x_2, x_3] f_2[x_1, x_2, x_3] + \right. \\
& \quad (-p_2 p_3 + p_1 p_4) f_2[x_1, x_2, x_3]^2 - (p_3 + p_6) f_1[x_1, x_2, x_3] f_3[x_1, x_2, x_3] + \\
& \quad (-p_2 p_5 + p_1 p_6) f_2[x_1, x_2, x_3] f_3[x_1, x_2, x_3] + (-p_4 p_5 + p_3 p_6) f_3[x_1, x_2, x_3]^2) \\
& \quad (-g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] f_1^{(0,1,0)}[x_1, x_2, x_3] + g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \\
& \quad f_1^{(0,1,0)}[x_1, x_2, x_3] + g_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] - \\
& \quad g_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] f_2^{(0,1,0)}[x_1, x_2, x_3] - g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] \\
& \quad f_3^{(0,1,0)}[x_1, x_2, x_3] + g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] f_3^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad f_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] g_1^{(0,1,0)}[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \\
& \quad g_1^{(0,1,0)}[x_1, x_2, x_3] - f_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] g_2^{(0,1,0)}[x_1, x_2, x_3] + \\
& \quad f_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] g_2^{(0,1,0)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] \\
& \quad g_3^{(0,1,0)}[x_1, x_2, x_3] - f_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] g_3^{(0,1,0)}[x_1, x_2, x_3] - \\
& \quad f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1^{(0,1,0)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \\
& \quad k_1^{(0,1,0)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2^{(0,1,0)}[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2^{(0,1,0)}[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad \left. k_3^{(0,1,0)}[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3^{(0,1,0)}[x_1, x_2, x_3] \right) / \\
& (-f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \\
& \quad k_1[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad k_3[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3])^2 + \\
& (p_7 f_2[x_1, x_2, x_3] f_1^{(1,0,0)}[x_1, x_2, x_3] + p_8 f_3[x_1, x_2, x_3] f_1^{(1,0,0)}[x_1, x_2, x_3] + \\
& \quad p_7 f_1[x_1, x_2, x_3] f_2^{(1,0,0)}[x_1, x_2, x_3] + \\
& \quad 2(-p_4 p_7 + p_2 p_8) f_2[x_1, x_2, x_3] f_2^{(1,0,0)}[x_1, x_2, x_3] + \\
& \quad (-p_6 p_7 + p_2 p_9) f_3[x_1, x_2, x_3] f_2^{(1,0,0)}[x_1, x_2, x_3] + \\
& \quad p_8 f_1[x_1, x_2, x_3] f_3^{(1,0,0)}[x_1, x_2, x_3] + \\
& \quad (-p_6 p_7 + p_2 p_9) f_2[x_1, x_2, x_3] f_3^{(1,0,0)}[x_1, x_2, x_3] + \\
& \quad \left. 2(p_5 p_7 - p_3 p_8) f_3[x_1, x_2, x_3] f_3^{(1,0,0)}[x_1, x_2, x_3] \right) / \\
& (-f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + \\
& \quad f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + \\
& \quad f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] + \\
& \quad f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3]) - \\
& \left( (p_7 f_1[x_1, x_2, x_3] f_2[x_1, x_2, x_3] + (-p_4 p_7 + p_2 p_8) f_2[x_1, x_2, x_3]^2 + \right. \\
& \quad p_8 f_1[x_1, x_2, x_3] f_3[x_1, x_2, x_3] + \\
& \quad \left. (-p_6 p_7 + p_2 p_9) f_2[x_1, x_2, x_3] f_3[x_1, x_2, x_3] + (p_5 p_7 - p_3 p_8) f_3[x_1, x_2, x_3]^2) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] f_1^{(1,0,0)}[x_1, x_2, x_3] + g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \right. \\
& \quad f_1^{(1,0,0)}[x_1, x_2, x_3] + g_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] f_2^{(1,0,0)}[x_1, x_2, x_3] - \\
& \quad g_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] f_2^{(1,0,0)}[x_1, x_2, x_3] - g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] \\
& \quad f_3^{(1,0,0)}[x_1, x_2, x_3] + g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] f_3^{(1,0,0)}[x_1, x_2, x_3] + \\
& \quad f_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] g_1^{(1,0,0)}[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \\
& \quad g_1^{(1,0,0)}[x_1, x_2, x_3] - f_3[x_1, x_2, x_3] k_1[x_1, x_2, x_3] g_2^{(1,0,0)}[x_1, x_2, x_3] + \\
& \quad f_1[x_1, x_2, x_3] k_3[x_1, x_2, x_3] g_2^{(1,0,0)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] \\
& \quad g_3^{(1,0,0)}[x_1, x_2, x_3] - f_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] g_3^{(1,0,0)}[x_1, x_2, x_3] - \\
& \quad f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1^{(1,0,0)}[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \\
& \quad k_1^{(1,0,0)}[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2^{(1,0,0)}[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2^{(1,0,0)}[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad \left. \left. k_3^{(1,0,0)}[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3^{(1,0,0)}[x_1, x_2, x_3] \right) \right) / \\
& \left( -f_3[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_1[x_1, x_2, x_3] + f_2[x_1, x_2, x_3] g_3[x_1, x_2, x_3] \right. \\
& \quad k_1[x_1, x_2, x_3] + f_3[x_1, x_2, x_3] g_1[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - \\
& \quad f_1[x_1, x_2, x_3] g_3[x_1, x_2, x_3] k_2[x_1, x_2, x_3] - f_2[x_1, x_2, x_3] g_1[x_1, x_2, x_3] \\
& \quad \left. \left. k_3[x_1, x_2, x_3] + f_1[x_1, x_2, x_3] g_2[x_1, x_2, x_3] k_3[x_1, x_2, x_3] \right)^2 \right\}
\end{aligned}$$

In[1]:= **F** = {f1, f2, f3}

Out[1]= {f1, f2, f3}

In[2]:= **E1** = **F**

Out[2]= {f1, f2, f3}

In[3]:= **E2** = {-p7 f2 - p8 f3,  
f1 - p1 f2 - p3 f3,  
-p2 f2 - p4 f3}

Out[3]= {-f2 p7 - f3 p8, f1 - f2 p1 - f3 p3, -f2 p2 - f3 p4}

In[4]:= **E3** = {-p8 f2 - p9 f3, -p3 f2 - p5 f3, f1 - p4 f2 - p6 f3}

Out[4]= {-f2 p8 - f3 p9, -f2 p3 - f3 p5, f1 - f2 p4 - f3 p6}

In[5]:= **RF** = {{f1, -p7 f2 - p8 f3, -p8 f2 - p9 f3}, {f2, f1 - p1 f2 - p3 f3, -p3 f2 - p5 f3},  
{f3, -p2 f2 - p4 f3, f1 - p4 f2 - p6 f3}}

Out[5]= {{f1, -f2 p7 - f3 p8, -f2 p8 - f3 p9},  
{f2, f1 - f2 p1 - f3 p3, -f2 p3 - f3 p5}, {f3, -f2 p2 - f3 p4, f1 - f2 p4 - f3 p6}}

In[6]:= **MatrixForm**[**RF**]

Out[6]//MatrixForm=

$$\begin{pmatrix} f1 & -f2 p7 - f3 p8 & -f2 p8 - f3 p9 \\ f2 & f1 - f2 p1 - f3 p3 & -f2 p3 - f3 p5 \\ f3 & -f2 p2 - f3 p4 & f1 - f2 p4 - f3 p6 \end{pmatrix}$$

**H** = **G1**

In[7]:= **H** = 1 / **Det**[**RF**] {f1^2 - (p1 + p4) f1 f2 - (p3 + p6) f1 f3 +  
(p1 p4 - p2 p3) f2^2 + (p1 p6 - p2 p5) f2 f3 + (p3 p6 - p4 p5) f3^2 t,  
p7 f1 f2 + p8 f1 f3 + (p2 p8 - p4 p7) f2^2 + (p2 p9 - p6 p7) f2 f3 + (p5 p7 - p3 p8) f3^2,  
p8 f1 f2 + p9 f1 f3 + (p2 p9 - p4 p8) f2^2 + (p5 p7 - p1 p9) f2 f3 + (p5 p8 - p3 p9) f3^2}

Out[7]= { (f1^2 - f1 f2 (p1 + p4) + f2^2 (-p2 p3 + p1 p4) - f1 f3 (p3 + p6) + f2 f3 (-p2 p5 + p1 p6) +  
f3^2 (-p4 p5 + p3 p6) t) / ((f1 - f2 p4 - f3 p6) (f1^2 - f1 f2 p1 - f1 f3 p3 + f2^2 p7 + f2 f3 p8) -  
(-f2 p2 - f3 p4) (-f1 f2 p3 - f1 f3 p5 + f2^2 p8 + f2 f3 p9) +  
f3 (f2^2 p3 p7 + f2 f3 p5 p7 + f1 f2 p8 - f2^2 p1 p8 + f3^2 p5 p8 + f1 f3 p9 - f2 f3 p1 p9 - f3^2 p3 p9)),  
(f1 f2 p7 + f1 f3 p8 + f2^2 (-p4 p7 + p2 p8) + f3^2 (p5 p7 - p3 p8) + f2 f3 (-p6 p7 + p2 p9)) /  
((f1 - f2 p4 - f3 p6) (f1^2 - f1 f2 p1 - f1 f3 p3 + f2^2 p7 + f2 f3 p8) -  
(-f2 p2 - f3 p4) (-f1 f2 p3 - f1 f3 p5 + f2^2 p8 + f2 f3 p9) +  
f3 (f2^2 p3 p7 + f2 f3 p5 p7 + f1 f2 p8 - f2^2 p1 p8 + f3^2 p5 p8 + f1 f3 p9 - f2 f3 p1 p9 - f3^2 p3 p9)),  
(f1 f2 p8 + f1 f3 p9 + f2 f3 (p5 p7 - p1 p9) + f2^2 (-p4 p8 + p2 p9) + f3^2 (p5 p8 - p3 p9)) /  
((f1 - f2 p4 - f3 p6) (f1^2 - f1 f2 p1 - f1 f3 p3 + f2^2 p7 + f2 f3 p8) -  
(-f2 p2 - f3 p4) (-f1 f2 p3 - f1 f3 p5 + f2^2 p8 + f2 f3 p9) +  
f3 (f2^2 p3 p7 + f2 f3 p5 p7 + f1 f2 p8 - f2^2 p1 p8 + f3^2 p5 p8 + f1 f3 p9 - f2 f3 p1 p9 - f3^2 p3 p9)) }