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Analysis of mechanical and electrical imperfect contacts in piezoelectric composites

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ABSTRACT

The problem of a heterogeneous piezoelectric composite with mechanical and electrical imperfect contact conditions is studied by means of the asymptotic homogenization method. As an illustrative example, in order to provide a benchmark for the numerical validation of theoretical models, a one-dimensional two-phase periodic laminate piezoelectric composite is considered with imperfect adhesion. Numerical examples show the effect of interface imperfections on the effective properties. Reductions in the effective Young's modulus, the piezoelectric coupling and the dielectric permittivity under the effect of the imperfect contact conditions are reported herein.

1. Introduction

Piezoelectric composites are of higher sensitivity and lower mechanical losses than single phase materials. Their properties are highly dependent on the constituent's interfaces. Piezoelectric composites are the key component of many types of devices as sensors and/or actuators. The study of contact phenomena and the modeling of solids interfaces are gaining special importance in the composite effective property estimation by means of continuum mechanics. Its applications can be found in different fields like medical science [1], and piezoelectric sensors [2], among others.

Multi-scale approaches are associated with composites where perfect interface conditions are assumed. However, many results show that local or partial debonding at interfaces is a rule rather than the exception in materials such as reinforced composites [3–5]. The existence of a stiff region is only an idealization of the complex phenomenon which occurs at the interface where a transition zone (interphase) between both components is a common occurrence. This third phase results from the material fabrication process. Different approaches have been used to describe these phenomena [6–9].

In this work, we consider mechanical and electrical imperfect contact conditions, i.e., continuity conditions for the traction and the normal electrical displacement field are satisfied. However, continuity conditions for the mechanic displacement and the electric potential are not fulfilled. The mechanical imperfect contact is considered when the stresses are proportional to the jump of the mechanic displacement, and the electrical imperfect contact when the electric displacement field is proportional to the jump of the electric potential across the interface. This mechanical imperfection can be interpreted as the presence of springs between composite phases, so called "mechanical imperfect contact, spring type". The electrical imperfect contact is idealized as the existence of capacitors between constituents, i.e., electric potential difference between phases [10–12].

In the field of piezoelectric materials, due to their intrinsic coupled electro-mechanical behavior, there are induced electric charges, even for pure mechanical loading. Therefore, a key question is whether the existence of an interface electric barrier has any effects on the induced electro-mechanical fields or not.

The interface imperfection condition presented here is quite general, in the sense that any combination of mechanical and elec-

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trical imperfections can exist. The capacitor's capacity is a measure of the electrical imperfection. The complete electric barrier is effective when the capacity is equal to zero. When the capacity is large enough, the contact works as a perfect electrical interface. The degree of electrical imperfection is related to the degree of discontinuity in the electric potential function at the interface, i.e., the capacity determines the degree of the electrical imperfection.

The aim of this paper is to describe the role of mechanical and electrical imperfect interfaces on the elastic, piezoelectric and dielectric permittivity properties in piezoelectric materials based on the two scale asymptotic homogenization method (AHM) [12–14]. The complete set of elastic, piezoelectric and dielectric permittivity effective properties for two-phase composite with an electrical capacitor and a mechanical spring as an idealization of the imperfect conditions at the interface is the main contribution of the present work. Here, we expand the AHM formalism for any kind of imperfect contact piezoelectric composite. In addition, as an example, we develop the one dimension laminated composite. It may be used as a benchmark for more complex model validation.

2. Heterogeneous piezoelectric problems under electro-elastic imperfect contact

We consider a linear static heterogeneous piezoelectric boundary value problem, which is described by a coupled system of partial differential equations, where the elastic C_{ijkl}, piezoelectric e_{lij}, and dielectric permittivity d_{ij} coefficients are smooth, defined in the domain $\Omega \subset \mathbb{R}^3$,

$$\begin{aligned} &\left(C_{ijkl}(\mathbf{x})u_{k,l}(\mathbf{x}) + e_{lij}(\mathbf{x})\varphi_{,l}(\mathbf{x})\right)_{,j} + X_i = 0, \\ &\left(e_{ikl}(\mathbf{x})u_{k,l}(\mathbf{x}) - d_{ij}(\mathbf{x})\varphi_{,j}(\mathbf{x})\right)_{,i} = 0. \end{aligned} \quad (1)$$

This system, Eq. (1), is written in terms of the components of $\mathbf{u}$ displacement vector and φ electric potential in the domain Ω (see Fig. 1(a)), whose boundary is denoted by $\partial\Omega$, where $(\bullet)_{,j} = \partial(\bullet)/\partial x_j$. Also, Eq. (1) represents the static governing equations $\forall \mathbf{x} \in \Omega$ for the present problem, and its associated boundary conditions are given by

$$u_i(\mathbf{x})\big|_{\partial\Omega} = 0, \quad \sigma_{ij}(\mathbf{x})n_j\big|_{\partial\Omega} = S_i^0, \quad \varphi(x)\big|_{\partial\Omega} = \varphi_0, \quad D_i(\mathbf{x})n_i\big|_{\partial\Omega} = 0, \quad (2)$$

where S_i^0 and φ_0 are the prescribed mechanical load vector and electrical potential on $\partial\Omega$. Here, $i, j, k, l = 1, 2, 3$ and the summation convention over repeated Latin indices is considered. X_i and n_j are the components of the body force vector $\mathbf{X}$ and the outward unit normal vector $\mathbf{n}$.

The corresponding constitutive equations that allow the coupling of piezoelectric materials are

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + e_{lij}E_{,l}, \quad D_i = e_{ikj}\varepsilon_{kj} - d_{il}E_{,l} \quad (3)$$

and the strain–displacement equations and electrical field-potential equations for a quasi-static approach are

$$\varepsilon_{kl} = (u_{k,l} + u_{l,k})/2, \quad E_l = -\varphi_{,l}. \quad (4)$$

In Eqs. (3) and (4), σ_{ij} and ε_{kl} are the components of the stress and strain tensors $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$; E_1 and D_i are the components of the electrical field $\mathbf{E}$ and the electrical displacement vector $\mathbf{D}$.

In this work, the imperfect interface Γ is modeled via an idealization of a layer of zero thickness. Tractions components and normal electrical displacement are continuous across the interface. The jumps of the mechanical displacements and the electrical potential are proportional to tractions and normal electric displacement field components.

$$[\![\mathbf{T}]\!] = 0, \quad [\![\mathbf{D}\cdot\mathbf{n}]\!] = 0, \quad \text{on} \quad \Gamma \quad (5)$$

$$\mathbf{T}^{(\gamma)} = (-1)^{\gamma+1}\mathbf{K}[\![\mathbf{u}]\!], \quad \mathbf{D}^{(\gamma)}\mathbf{n}^{(\gamma)} = (-1)^{\gamma}M[\![\varphi]\!], \quad \text{on} \quad \Gamma, \quad (6)$$

where the upper script $\gamma = 1, 2$ is associated with each composite constituent. The double bracket represents the jump of the magnitudes at the interface $[\![\bullet]\!] = \bullet^{(2)} - \bullet^{(1)}$. Here, the traction vector is denoted by $\mathbf{T} = \boldsymbol{\sigma}\mathbf{n}$ with $\mathbf{T} = (\,T_n \quad T_t \quad T_s\,)^T$. The remaining magnitudes are the mechanical imperfect parameter tensor $\mathbf{K} = \begin{pmatrix} K_n & 0 & 0 \\ 0 & K_t & 0 \\ 0 & 0 & K_s \end{pmatrix}$, the mechanical displacement vector $\mathbf{u} = (\,u_n \quad u_t \quad u_s\,)^T$, and the electric field vector $\mathbf{D} = (\,D_n \quad D_t \quad D_s\,)^T$. The scalar M and φ represent the electrical imperfect parameter and the electrical potential, respectively. $\mathbf{n} = \left(\,n_n \quad n_t \quad n_s\,\right)^T$ is the outward unit normal vector. Here, the subscript n represents the normal component while t and s are the tangential components. The upper scripts $(\,)^T$ denote the transpose. $\mathbf{K}$ and M are the spring and capacitor constant type material parameters which have dimension of stress divided by length (N/m^3) and electrical displacement divided by electric potential (F/m^2), respectively.

3. Two scale asymptotic homogenization method

Let us assume that the material properties C_{ijkl}, e_{lij} and d_{il} are Y–periodic functions, and $Y = (0, Y_1) \times (0, Y_2) \times (0, Y_3)$ is the so-called periodic cell, see Fig. 1(b). Over Y, the material properties satisfy $C_{ijkl} = C_{ijkl}(\boldsymbol{\xi})$, $e_{lij} = e_{lij}(\boldsymbol{\xi})$ and $d_{il} = d_{il}(\boldsymbol{\xi})$, where $\xi = \left(\xi_1, \xi_2, \xi_3\right) \in Y$ is the so-called local or fast coordinate. Also, $\mathbf{x} = (x_1, x_2, x_3) \in \Omega$ is referred to as the global coordinate. Here, both scales are related by $\boldsymbol{\xi} = \mathbf{x}/\alpha$, where $\alpha = l/L(\alpha \ll 1)$ is a dimensionless small parameter which represents the ratio between the characteristic lengths of the periodic cell Y, (l), and the body as a whole (L).

Then, using the relation of scales, the system Eq. (1) can be transformed into a system of partial differential equations:

$$\begin{aligned} &\left(C_{ijkl}(\mathbf{x}/\alpha)u_{k,l}(\mathbf{x}/\alpha) + e_{lij}(\mathbf{x}/\alpha)\varphi_{,l}(\mathbf{x}/\alpha)\right)_{,j} + X_i = 0, \\ &\left(e_{ikl}(\mathbf{x}/\alpha)u_{k,l}(\mathbf{x}/\alpha) - d_{ij}(\mathbf{x}/\alpha)\varphi_{,l}(\mathbf{x}/\alpha)\right)_{,j} = 0, \end{aligned} \quad (7)$$

where C_{ijkl}, e_{lij} and d_{ij} are rapidly oscillating coefficients in the region Ω.

The solution of the system, Eq. (7), can be found by asymptotic expansions, as shown in [15],

$$u_i(\mathbf{x}) = \sum_{j=0}^{\infty} \alpha^j u_i^j(\mathbf{x}, \boldsymbol{\xi}), \quad \varphi(\mathbf{x}) = \sum_{j=0}^{\infty} \alpha^j \varphi_i^j(\mathbf{x}, \boldsymbol{\xi}), \quad (8)$$

where u_i^0 and φ^0 are independent functions of ξ. Also, u_i^j and φ^j represent corrections that mean variations of u_i^0 and φ^0 respectively.

Then, proceeding with the asymptotic formulation, due to the linearity of the problem, the first-order terms of the expansions are

$$\begin{aligned} &u_i^1(\mathbf{x}, \boldsymbol{\xi}) = {}_{pq}N_i^1(\boldsymbol{\xi})\partial u_p^0(\mathbf{x})/\partial x_q + {}_pP_i^1(\boldsymbol{\xi})\partial\varphi^0(\mathbf{x})/\partial x_p, \\ &\varphi^1(\mathbf{x}, \boldsymbol{\xi}) = {}_{pq}R^1(\boldsymbol{\xi})\partial u_p^0(\mathbf{x})/\partial x_q + {}_pQ^1(\boldsymbol{\xi})\partial\varphi^0(\mathbf{x})/\partial x_p, \end{aligned} \quad (9)$$

where the functions denoted as ${}_{pq}\mathbf{N} = {}_{pq}N_i^1(\boldsymbol{\xi})$, ${}_p\mathbf{P} = {}_pP_i^1(\boldsymbol{\xi})$, $\mathbf{R} = R^1(\boldsymbol{\xi})$ and $\mathbf{Q} = Q^1(\boldsymbol{\xi})$ are the periodic local functions, which are independent of $\mathbf{x}$.

Substituting Eq. (8) into Eqs. (6) and (7), it is possible to obtain a set of recurrent problems that permit the general mathematical statement of the so-called local problems, the moduli of electro-elastic effective coefficients, and the equivalent homogenized problem, defined below.

The corresponding local problems under mechanical and electrical imperfect contact conditions, denoted as L and I $(p, q = 1, 2, 3)$, are now formulated.

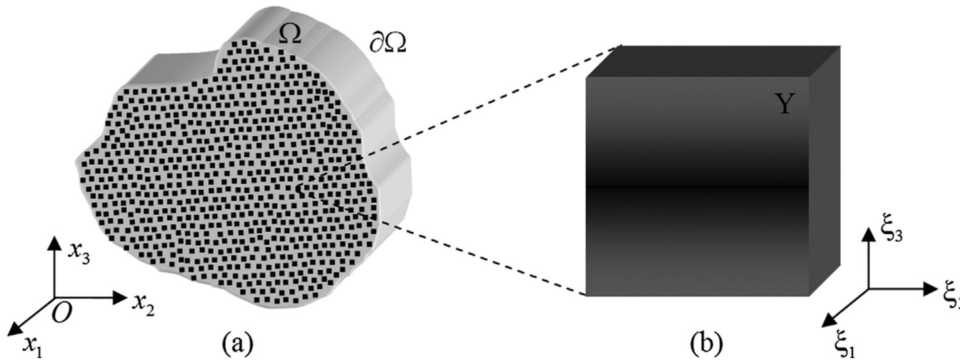

Fig. 1. (a) Heterogeneous piezoelectric material Ω. (b) A representative volume of analysis (periodic cell) Y.

The L problems seek to determine the periodic local functions ${}_{pq}N_k^1$ and R^1 in Y by mean of the following equations

$$ {}_{pq}\sigma_{ij,j} = 0, \quad {}_{pq}D_{i,i} = 0, \quad \text{in} \quad Y \tag{10}$$

$$ [\![{}_{pq}\mathbf{T}]\!] = 0, \quad [\![{}_{pq}\mathbf{D}\cdot\mathbf{n}]\!] = 0, \quad \text{on} \quad \Gamma \tag{11}$$

$$ {}_{pq}\mathbf{T}^{(\gamma)} = (-1)^{\gamma+1}\mathbf{K}[\![{}_{pq}\mathbf{N}]\!], \quad {}_{pq}\mathbf{D}^{(\gamma)}\mathbf{n}^{(\gamma)} = (-1)^{\gamma}M[\![{}_{pq}\mathbf{R}]\!] \quad \text{on} \quad \Gamma, \tag{12}$$

where each magnitude is written by components as

$$ {}_{pq}\mathbf{T} = \left({}_{pq}\sigma_{ij} + C_{ijpq}\right)_{|j}, \; {}_{pq}\mathbf{D} = \left({}_{pq}D_i + e_{ipq}\right)_{|j},$$

$$ {}_{pq}\sigma_{ij} = C_{ijkl} \;\; {}_{pq}N^1_{k|l} + e_{lij} \;\; {}_{pq}R^1_{|l}, \quad \text{and} \quad {}_{pq}D_i = e_{iklpq}N^1_{l|l} - d_{ilpq}R^1_{|l}.$$

Analogously, in the I problems, the local functions ${}_pP_k^1$ and Q^1 are sought in Y, which are results of the equations:

$$ {}_{p}\sigma_{ij,j} = 0, \quad {}_{p}D_{i,i} = 0, \quad \text{in} \quad Y \tag{13}$$

$$ [\![{}_{p}\mathbf{T}]\!] = 0, \quad [\![{}_{p}\mathbf{D}\cdot\mathbf{n}]\!] = 0, \quad \text{on} \quad \Gamma \tag{14}$$

$$ {}_{p}\mathbf{T}^{(\gamma)} = (-1)^{\gamma+1}\mathbf{K}[\![{}_{p}\mathbf{P}]\!], \quad {}_{p}\mathbf{D}^{(\gamma)}\mathbf{n}^{(\gamma)} = (-1)^{\gamma}M[\![{}_{p}\mathbf{Q}]\!], \quad \text{on} \quad \Gamma, \tag{15}$$

where each magnitude has the following components form

$$ {}_{p}\mathbf{T} = \left({}_{p}\sigma_{ij} + e_{ipq}\right)_{|j}, \quad {}_{p}\mathbf{D} = \left({}_{p}D_i + d_{ip}\right)_{|j},$$

$$ {}_{p}\sigma_{ij} = C_{ijklp}P^1_{k|l} + e_{lijp}Q^1_{|l}, \quad \text{and} \quad {}_{p}D_i = -e_{iklp}P^1_{k|l} + d_{ilp}Q^1_{|l}.$$

Additionally, the following conditions, over Y, are required to guarantee the unique solution of the local problems, i.e.,

$$ \langle {}_{pq}Nk1\rangle = 0, \quad \langle {}_{pq}R^1\rangle = 0, \quad \langle {}_{p}Pk1\rangle = 0 \quad \text{and} \quad \langle {}_{p}QI1\rangle = 0, \tag{16}$$

with $\langle\bullet\rangle = (1/|Y|)\int_Y(\bullet)\,dY$ denoting the volume average of ''•'' over Y, and |Y| is the volume of Y, $(\bullet)_{|j} = \partial(\bullet)/\partial\xi_j$.

For a heterogeneous and periodic medium Ω characterized by Y-periodic functions $C_{ijkl}(\boldsymbol{\xi})$, $e_{lij}(\boldsymbol{\xi})$, and $d_{il}(\boldsymbol{\xi})$ the original system Eq. (7) is transformed into a new equivalent system, defined on a homogeneous medium $\bar{\Omega}$, where the physical relations are now constant. Hence, the homogenized problem formulation associate is given as

$$\begin{cases} C^*_{ijklpq}u^0_{k,lj}(\mathbf{x}) + e^*_{lijpq}\varphi^0_{,lj}(\mathbf{x}) = 0, \\ e^*_{iklp}u^0_{k,li}(\mathbf{x}) - d^*_{ilp}\varphi^0_{,li}(\mathbf{x}) = 0, \end{cases} \quad \forall\mathbf{x} \in \bar{\Omega} \tag{17}$$

where C^*_{ijkl}, e^*_{lij} and d^*_{il} are called the elastic, piezoelectric, and dielectric effective coefficients. In Eq. (17), u^0_k and φ^0 are the homogenization solutions of the differential equations system.

The system Eq. (17) with the following boundary conditions

$$ {}_{pq}u^0_k(\mathbf{x})\big|_{\partial\Omega} = f_1(\mathbf{x}), \quad {}_{pq}\varphi^0(\mathbf{x})\big|_{\partial\Omega} = f_2(\mathbf{x}), \tag{18}$$

describes the fundamental equations of the theory of the linear piezoelectricity problem for a homogeneous medium $\bar{\Omega}$. The functions $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are infinitely differentiable on $\bar{\Omega}$.

The main problem to obtain such average properties and fields is to determine the Y-periodic solutions of the local problems L and I, that arise on the Y-periodic cell, in terms of the fast variable $\boldsymbol{\xi}$ [16]. Once they are solved, the effective coefficients C^*_{ijpq}, e^*_{kij}, and d^*_{ik} can be determined by using the following formulae:

Associated to the local problems L,

$$ C^*_{ijpq} = \left\langle C_{ijpq} + C_{ijklpq}N^1_{k|l} + e_{kijpq}R^1_{|k}\right\rangle, \tag{19}$$

$$ e^*_{ipq} = \left\langle e_{ipq} + e_{iklpq}N^1_{k|l} - d_{ikpq}R^1_{|k}\right\rangle. \tag{20}$$

Associated to the local problems I,

$$ e^*_{pij} = \left\langle e_{pij} + C_{ijklp}P^1_{k|l} + e_{kijp}Q^1_{|k}\right\rangle, \tag{21}$$

$$ d^*_{ip} = \left\langle d_{ip} - e_{iklp}P^1_{k|l} + d_{ikq}Q^1_{|k}\right\rangle. \tag{22}$$

The imperfect contact conditions, Eqs. (12) and (15), have been written $\forall\mathbf{x} \in \Gamma$ in the coordinate system (n, s, t) depending on the normal and tangential components on Γ.

Hence, the relations among the displacement **u**, and traction **T** vectors, and spring constant **K** are related to their Cartesian representations by the rotational matrices around the x_3-axis.

Thus, substituting Eq. (8) into Eqs. (5) and (6), and taking into account the θ rotation around x_3-axis, the imperfect contact conditions (Eqs. (12) and (15)) are rewritten as follows

- ${}_{pq}L$ local problems

In these problems, analogous expressions to Eqs. (10) and (12) are obtained. Here, the local functions ${}_{pq}\mathbf{N} = \left({}_{pq}N_n \quad {}_{pq}N_t \quad {}_{pq}N_s\right)^t$ and R are the pseudo-mechanic displacement vector and the pseudo-electric potential scalar. The homogenized components of the pseudo-traction tensor **T** and the pseudo-electric displacement vector **D** are

$$ {}_{pq}T_1^{(\gamma)} = \left({}_{pq}T_n^{(\gamma)} + C^{(\gamma)}_{ijpq}\right)n_j^{(\gamma)}, \quad {}_{pq}T_2^{(\gamma)} = \left({}_{pq}T_t^{(\gamma)} + C^{(\gamma)}_{ijpq}\right)n_j^{(\gamma)},$$

$$ {}_{pq}T_3^{(\gamma)} = \left({}_{pq}T_s^{(\gamma)} + C^{(\gamma)}_{ijpq}\right)n_j^{(\gamma)}, \quad {}_{pq}D_i^{(\gamma)} = \left(e^{(\gamma)}_{ipq} + C^{(\gamma)}_{ijkl}\,{}_{pq}N^{(\gamma)}_{k|l} + e^{(\gamma)}_{ipq}\,{}_{pq}P_l^{(\gamma)}\right)n_j^{(\gamma)}$$

- ${}_{p}I$ local problems

Similarly, equivalent expressions to Eqs. (13) and (15) are obtained. Here, the local functions ${}_p\mathbf{R} = \left({}_pR_n \quad {}_pR_t \quad {}_pR_s\right)^t$ and Q are the pseudo-mechanic displacement vector and the pseudo-electric potential scalar. The homogenized components of the pseudo-

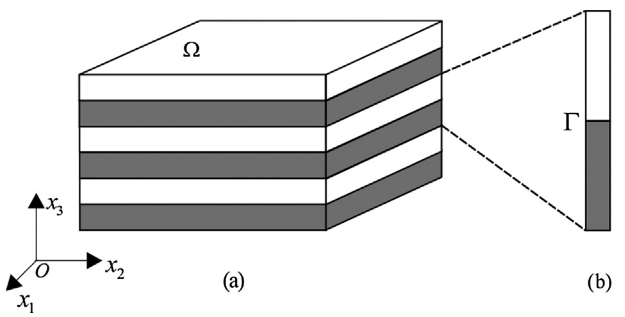

Fig. 2. (a) Heterogeneous laminate piezoelectric material. (b) One-dimensional piezoelectric composite.

traction tensor **T** and the pseudo-electric displacement vector **D** are

$$ {}_pT_1^{(\gamma)} = \left({}_pT_n^{(\gamma)} + e_{ijp}^{(\gamma)}\right) n_j^{(\gamma)}, \quad {}_pT_2^{(\gamma)} = \left({}_pT_t^{(\gamma)} + e_{ijp}^{(\gamma)}\right) n_j^{(\gamma)}, $$

$$ {}_pT_3^{(\gamma)} = \left({}_pT_s^{(\gamma)} + e_{ijp}^{(\gamma)}\right) n_j^{(\gamma)}, \quad {}_pD_i^{(\gamma)} = \left(e_{ijp}^{(\gamma)} + e_{ikl}^{(\gamma)}\,{}_pN_{k|l}^{(\gamma)} - d_{ip}^{(\gamma)}\,{}_pP_l^{(\gamma)}\right) n_j^{(\gamma)} $$

Thus, the local problem formulation, i.e., Eqs. (10) and (13), with the corresponding imperfect contact conditions on coordinate system (n, s, t) on Γ allows to find the local functions **N**, **P**, **R**, and **Q**. Once these local functions are obtained, the effective coefficients C_{ijkl}^*, e_{lij}^* and d_{il}^* can be computed by the expressions Eqs. (19)–(22).

4. An example of one-dimensional piezoelectric composite with imperfect contact conditions

As an illustrative example, in order to provide a numerical benchmark for the validation of theoretical models, we consider a laminate piezoelectric composite where the layers are distributed perpendicular to the x_3-axis (Fig. 2(a)). Particularly, a one-dimensional composite made by two components (Fig. 2(b)) is analyzed. Here, the regions occupied by the components in the unit periodic cell Y are denoted by Y_γ with $\gamma = 1, 2$; the local variable $\xi \in [0; 1] \subset \mathbb{R}$, with $\mathbb{R} = \{0\} \cup \{\mathrm{Y}_1\} \cup \{\mathrm{Y}_2\} \cup \{1\}$, $\mathrm{Y}_1 = (0; \mathrm{V}_1)$ and $\mathrm{Y}_2 = (\mathrm{V}_1; 1)$. E_γ, e_γ, and d_γ denote Young's modulus, piezoelectric and dielectric coefficients for each component, respectively. From now on, the local variable $\boldsymbol{\xi}$ will not be written.

For this problem, the 1-periodic local functions are declared as N, P, R, and Q. Following the previous procedure, these local functions are determined by means of the one-dimensional local problems, denoted as L and I, as result of the formulation given in Eqs. (10) and (13), in Y_γ.

Hence, the one-dimensional local problems are formulated as follows,

$$ L: \begin{cases} d/d\xi\left[E_\gamma + d/d\xi\left(E_\gamma N^{(\gamma)} + e_\gamma R^{(\gamma)}\right)\right] = 0, \\ d/d\xi\left[e_\gamma + d/d\xi\left(e_\gamma N^{(\gamma)} - d_\gamma R^{(\gamma)}\right)\right] = 0, \end{cases} \tag{23} $$

$$ I: \begin{cases} d/d\xi\left[e_\gamma + d/d\xi\left(E_\gamma P^{(\gamma)} + e_\gamma Q^{(\gamma)}\right)\right] = 0, \\ d/d\xi\left[d_\gamma - d/d\xi\left(e_\gamma P^{(\gamma)} - d_\gamma Q^{(\gamma)}\right)\right] = 0, \end{cases} \tag{24} $$

under the uniqueness conditions and periodicity of the local functions

$$ \begin{matrix} N^{(\gamma)}(0) = 0, R^{(\gamma)}(0) = 0, \\ N^{(1)}(0) = N^{(2)}(1), R^{(1)}(0) = R^{(2)}(1) \end{matrix} \quad \text{for } L \text{ local problem} \tag{25} $$

$$ \begin{matrix} P^{(\gamma)}(0) = 0, Q^{(\gamma)}(0) = 0, \\ P^{(1)}(0) = P^{(2)}(1), Q^{(1)}(0) = Q^{(2)}(1) \end{matrix} \quad \text{for } I \text{ local problem} \tag{26} $$

Additionally, the imperfect contact conditions, defined in Eqs. (11), (12), (14) and (15), can be written on $\xi = \mathrm{V}_1$ as follows

- Associated to the local problem L

$$ \begin{cases} E_1 + d/d\xi(E_1N^{(1)} + e_1R^{(1)}) - E_2 - d/d\xi(E_2N^{(2)} + e_2R^{(2)}) = 0, \\ e_1 + d/d\xi(e_1N^{(1)} - d_1R^{(1)}) - e_2 - d/d\xi(e_2N^{(2)} - d_2R^{(2)}) = 0, \end{cases} $$
$$ \begin{cases} E_\gamma + d/d\xi(E_\gamma N^{(\gamma)} + e_\gamma R^{(\gamma)}) = (-1)^{\gamma+1}K[\![N]\!], \\ e_\gamma + d/d\xi(e_\gamma N^{(\gamma)} - d_\gamma R^{(\gamma)}) = (-1)^{\gamma+1}M[\![R]\!], \end{cases} \quad \text{on } \xi = \tau. \tag{27} $$

- Associated to the local problem I

$$ \begin{cases} e_1 + d/d\xi(E_1P^{(1)} + e_1Q^{(1)}) - e_2 - d/d\xi(E_2P^{(2)} + e_2Q^{(2)}) = 0, \\ d_1 - d/d\xi(e_1P^{(1)} - d_1Q^{(1)}) - d_2 + d/d\xi(e_2P^{(2)} - d_2Q^{(2)}) = 0, \end{cases} $$
$$ \begin{cases} e_\gamma + d/d\xi(E_\gamma P^{(\gamma)} + e_\gamma Q^{(\gamma)}) = (-1)^{\gamma+1}K[\![P]\!], \\ d_\gamma - d/d\xi(e_\gamma P^{(\gamma)} - d_\gamma Q^{(\gamma)}) = (-1)^{\gamma}M[\![Q]\!], \end{cases} \quad \text{or } \xi = \tau. \tag{28} $$

Then, from Eqs. (23) and (24), a homogeneous system, Eqs. (27) and (28), is derived and the solutions are obtained. Here, the coefficients E_γ, e_γ and d_γ are constants over Y. Also, the expressions for each local functions as a linear function of $\boldsymbol{\xi}$ can be written as $N^{(\gamma)} = A_1^{(\gamma)}\xi + A_2^{(\gamma)}$, $R^{(\gamma)} = A_3^{(\gamma)}\xi + A_4^{(\gamma)}$, $P^{(\gamma)} = B_1^{(\gamma)}\xi + B_2^{(\gamma)}$ and $Q^{(\gamma)} = B_3^{(\gamma)}\xi + B_4^{(\gamma)}$. Using (25) and (26), the coefficients $A_\alpha^{(\gamma)}$ and $B_\alpha^{(\gamma)} (\alpha = 1, 2, 3, 4)$ follow the linear relations:

$$ A_1^{(2)} = -A_2^{(2)}, \quad A_3^{(2)} = -A_4^{(2)} \tag{29} $$

$$ B_1^{(2)} = -B_2^{(2)}, \quad B_3^{(2)} = -B_4^{(2)} \tag{30} $$

The contact condition over the interface given in Eq. (27) and the relations (29) yield the system of linear equations

$$ \begin{bmatrix} -(KV_1 + E_1) & -KV_2 & -e_1 & 0 \\ -KV_1 & -(KV_2 + E_2) & 0 & -e_2 \\ -e_1 & 0 & MV_1 + d_1 & MV_2 \\ 0 & -e_2 & MV_1 & MV_2 + d_2 \end{bmatrix} \begin{bmatrix} A_1^{(1)} \\ A_1^{(2)} \\ A_3^{(1)} \\ A_3^{(2)} \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ e_1 \\ e_2 \end{bmatrix} \tag{31} $$

In the same way, dealing with the Eqs. (28) and (30), we obtain the following corresponding linear system of equation

$$ \begin{bmatrix} -(KV_1 + E_1) & -KV_2 & -e_1 & 0 \\ -KV_1 & -(KV_2 + E_2) & 0 & -e_2 \\ -e_1 & 0 & MV_1 + d_1 & MV_2 \\ 0 & -e_2 & MV_1 & MV_2 + d_2 \end{bmatrix} \begin{bmatrix} B_1^{(1)} \\ B_1^{(2)} \\ B_3^{(1)} \\ B_3^{(2)} \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ -d_1 \\ -d_2 \end{bmatrix} \tag{32} $$

The volume fractions of each component are V_1, and $V_2 = 1 - V_1$.

The effective coefficients for one-dimensional piezoelectric composites are derived from Eqs. (19) and (22), as

$$ \begin{aligned} L&: \begin{cases} E^* = \langle E\rangle + (E_1V_1A_1^{(1)} + E_2V_2A_1^{(2)}) + (e_1V_1A_3^{(1)} + e_2V_2A_3^{(2)}), \\ e^* = \langle e\rangle + (e_1V_1A_1^{(1)} + e_2V_2A_1^{(2)}) - (d_1V_1A_3^{(1)} + d_2V_2A_3^{(2)}), \end{cases} \\ I&: \begin{cases} e^* = \langle e\rangle + (E_1V_1B_1^{(1)} + E_2V_2B_1^{(2)}) + (e_1V_1B_3^{(1)} + e_2V_2B_3^{(2)}), \\ d^* = \langle d\rangle - (e_1V_1B_1^{(1)} + e_2V_2B_1^{(2)}) + (d_1V_1B_3^{(1)} + d_2V_2B_3^{(2)}). \end{cases} \end{aligned} \tag{33} $$

Finally, using the solutions of the systems Eqs. (31), (32), the effective coefficients can be written as

$$ E^* = \frac{\langle E/\tau\rangle + 1/\mathrm{M}}{\Delta}, \tag{34} $$

$$ e^* = \frac{\langle e/\tau\rangle}{\Delta}, \tag{35} $$

$$ d^* = \frac{\langle d/\tau\rangle + 1/K}{\Delta}, \tag{36} $$

where

$\Delta = (\langle E/\tau\rangle + 1/M)(\langle d/\tau\rangle + 1/M) + \langle e/\tau\rangle^2$, $\tau_\gamma = E_\gamma d_\gamma + e_\gamma^2$ and $\gamma = 1, 2$.

Table 1
Effective properties for composite C-91/P-82 using the expression from [2] and the present model, K = 10^7 (GPa/m) and M = 10^7 (10^{-10} F/m^2).

	Mechanical and electrical perfect contact conditions					
	E^* (GPa)		e^* (C/m^2)		(d^* 10^{-10}F/m)	
V_1	Ref. [17]	Present model	Ref. [17]	Present model	Ref. [17]	Present Model
0.0	118.300	118.299	26.400	26.399	110.900	110.899
0.4	116.953	116.952	24.806	24.805	148.056	148.054
0.8	115.401	115.400	22.634	22.633	200.835	200.832
1.0	114.500	114.499	21.200	21.199	236.600	236.595

Table 2
Influence of mechanical imperfection parameter in the effective elastic coefficient considering electrical perfect contact.

	Young modulus E^* for M = 10^7 (10^{-10} F/m^2)					
	Mechanical imperfection K (GPa/m) variation					
V_1	10^5	10^4	10^3	10^2	10^1	10^0
0.0	118.16	116.92	105.79	54.192	9.221	0.992
0.4	116.82	115.60	104.71	53.907	9.212	0.992
0.8	115.27	114.09	103.46	53.575	9.203	0.991
1.0	114.37	113.20	102.74	53.380	9.197	0.991

Table 3
Influence of electrical imperfection parameter in the effective coefficient considering mechanical perfect contact.

	Young modulus E^* for K = 10^7 (GPa/m)					
	Electrical imperfection M (10^{-10} F/m^2) variation					
V_1	10^5	10^4	10^3	10^2	10^1	10^0
0.0	118.37	118.99	124.57	151.34	175.94	180.58
0.4	117.01	117.56	122.31	141.76	155.88	158.23
0.8	115.45	115.90	119.67	132.43	139.70	140.78
1.0	114.54	114.94	118.13	127.85	132.72	133.41

5. Validation and numerical results

The expression (Eqs. (12) and (15)) of the L and I local problems defined on Γ are rewritten as

$$(-1)^{\gamma+1}{}_{pq}\mathbf{T}^{(\gamma)}[\![{}_{pq}\mathbf{N}]\!]^{-1} = \mathbf{K}, \quad (-1)^{\gamma}{}_{pq}\mathbf{D}^{(\gamma)}\mathbf{n}^{(\gamma)}[\![{}_{pq}\mathbf{R}]\!]^{-1} = M, \tag{37}$$

$$(-1)^{\gamma+1}{}_{p}\mathbf{T}^{(\gamma)}[\![{}_{p}\mathbf{P}]\!]^{-1} = \mathbf{K}, \quad (-1)^{\gamma}{}_{p}\mathbf{D}^{(\gamma)}\mathbf{n}^{(\gamma)}[\![{}_{p}\mathbf{Q}]\!]^{-1} = M. \tag{38}$$

From Eqs. (37) and (38), we notice that the continuity condition of the local functions is satisfied when the mechanical or electrical imperfection parameters approach to infinity, i.e., the perfect contact condition between the constituents of the composite is established. Hence, the effective coefficients, Eqs. (34)–(36), are analytically reduced to those reported in Ref. [17] with large enough values for the imperfect parameters K and M. Eqs. (34)–(36) show the significant role of mechanical K and electrical M imperfection parameters for the expressions in the effective coefficients of the composite.

For the numerical analysis, we considered a composite (Fig. 2) whose constituents are C-91 and P-82 [18]. The data used for computations, for C-91 (P-82), are E = 114.5(118.3) GPa, e = 21.2(26.4) C/m^2 and d = 236.6(110.9) × 10^{-10} C/Vm. Table 1 shows a comparison of the numerical output of the present model for K = 10^7 GPa/m and M = 10^{-3} GPa/m with the values calculated using the expressions reported in Ref. [17]. Here, mechanical and electrical perfect contacts are considered. Also, the effective properties for the Young modulus, piezoelectric and dielectric permittivity for different A (C-91) volume fractions V_f are shown. We observe a good match between the two models.

Table 4
Influence of mechanical imperfection parameter in the effective piezoelectric coefficient considering electrical perfect contact.

	Effective piezoelectric e^* for M = 10^7 (10^{-10}F/m^2)					
	Mechanical imperfection K (GPa/m) variation					
V_1	10^5	10^4	10^3	10^2	10^1	10^0
0.0	26.369	26.091	23.607	12.093	2.058	0.221
0.4	24.777	24.519	22.208	11.434	1.954	0.210
0.8	22.607	22.375	20.292	10.507	1.805	0.194
1.0	21.175	20.960	19.022	9.883	1.703	0.184

Table 5
Influence of electrical imperfection parameter in the effective piezoelectric coefficient considering mechanical perfect contact.

	Effective piezoelectric e^* for K = 10^7 (GPa/m)					
	Electrical imperfection M (10^{-10} F/m^2) variation					
V_1	10^5	10^4	10^3	10^2	10^1	10^0
0.0	26.370	26.110	23.764	12.518	2.184	0.236
0.4	24.769	24.444	21.607	10.000	1.569	0.166
0.8	22.588	22.188	18.848	7.524	1.074	0.112
1.0	21.150	20.710	17.144	6.298	0.860	0.089

Table 6
Influence of mechanical imperfection parameter in the effective dielectric permittivity coefficient considering electrical perfect contact.

	Effective dielectric permittivity d^* for M = 10^7 (10^{-10} F/m^2)					
	Mechanical imperfection K (GPa/m) variation					
V_1	10^5	10^4	10^3	10^2	10^1	10^0
0.0	110.97	111.59	117.13	142.82	165.22	169.32
0.4	148.12	148.66	153.56	176.42	196.53	200.22
0.8	200.88	201.34	205.42	224.61	241.68	244.84
1.0	236.64	237.04	240.63	257.55	272.69	275.50

Table 7
Influence of imperfection electrical parameter in the effective dielectric permittivity coefficient under mechanical perfect contact.

	Effective dielectric d^* for K = 10^7 (GPa/m)					
	Electrical imperfection M (10^{-10} F/m^2) variation					
V_1	10^5	10^4	10^3	10^2	10^1	10^0
0.0	110.78	109.68	99.830	52.584	9.173	0.991
0.4	147.84	145.90	128.96	59.687	9.367	0.993
0.8	200.43	196.88	167.25	66.759	9.525	0.995
1.0	236.04	231.13	191.33	70.291	9.595	0.996

Tables 2–7 show the effect of the mechanical and electrical imperfections on the effective coefficients of the composite for different contact conditions and for A (C-91) volume fraction V_f running for 0.0, 0.4, 0.8 and 1.0. In all tables, it can be seen that the influence of the imperfection is marked, regardless of the constituent volume fraction.

Table 8
Influence of simultaneous mechanical and electrical imperfection parameters in the effective piezoelectric coefficients.

Effective piezoelectric moduli for different combinations of mechanical K (GPa/m) and electrical M (10^{-10} F/m^2) imperfections					
K	M	V_1	E^*	e^*	d^*
10^2	10^7	0.4	53.907	11.434	176.415
	10^5		53.920	11.414	176.108
	10^3		55.018	9.719	149.962
	10^1		60.920	0.613	9.463
10^7	10^2	0.4	141.757	10.000	59.687
10^5			141.559	9.986	59.697
10^3			124.159	8.759	60.562
10^1			9.341	0.659	66.276

As reported in Table 2, the effective Young modulus E^* changes for different values of mechanical imperfection, as measured by the parameter K under electrical perfect contact. As K decreases, the Young modulus decreases. This is an expected result, because the mechanical imperfection is considered as a spring at the interface, and the softer (lower value of K) this spring is, the weaker the transmission of the mechanical impulse between the phases turns out to be. It implies a lower composite Young modulus. For any fixed K value, the V_f tendency of the effective Young modulus is the same as the one observed for the perfect contact case.

Table 3 shows the effect of the electrical imperfection M on the Young modulus E^* for a mechanical perfect contact. It can be observed that the Young modulus is considerably affected by the electrical imperfection. Nonetheless, the effect of the mechanical imperfection in Table 2 is 4 orders of magnitude greater, which is expected.

In Tables 4 and 5, the influence of the mechanical and electrical imperfect contact conditions on effective piezoelectric coefficients is shown. The same influence is reported in Tables 6 and 7 for the effective dielectric constant. We observe that the dielectric permittivity is more sensitive to changes in the electrical imperfection than to changes in the mechanical imperfection. This is consistent with the fact that the effective Young's modulus is more sensitive to K and it is also an expected result. Table 8 reports the combined effect of mechanical and electrical imperfections on the effective properties for the volume fraction equal to 0.4. As with Tables 1–5 where one the mechanical and electrical fields can be considered continuous, we see that that the variation in one of the imperfection constants causes a decrease in orders of magnitude of one of the effective coefficients while causing a slight increase in the other effective property. In both cases, decreasing M or decreasing K, we see a decrease of the piezoelectric property.

6. Conclusions

The formulation of imperfect spring and capacitor types is studied for piezoelectric composites. Local problems under mechanical and electrical imperfect contact conditions are derived using the two-scale homogenization method. The general analytic expressions for the elastic, piezoelectric and dielectric permittivity effective coefficients are derived, and a particular example for one-dimensional piezoelectric composite under mechanical and electrical imperfect contacts conditions is reported. The effective coefficients depend on the properties of the constituents, their volume fractions, the mechanical and electrical imperfect parameters. The formulae of the effective coefficients are able to reduce to those expressions reported for perfect constituents contact when the imperfect parameters tend to infinity. The mechanical and electrical imperfect parameters affect significantly the behavior of the effective coefficients E^*, e^* and d^* in the composite. This effect can be described as a weakening of the effective properties due to constituent debonding.

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