INTUITIONISTIC FUZZY DIMENSIONAL ANALYSIS FOR MULTI-CRITERIA DECISION MAKING

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Abstract. Dimensional analysis, for multi-criteria decision making, is a mathematical method that includes diverse heterogeneous criteria into a single dimensionless index. Dimensional Analysis, in its current definition, presents the drawback to manipulate fuzzy information commonly presented in a multi-criteria decision making problem. To overcome such limitation, we propose two dimensional analysis based techniques under intuitionistic fuzzy environments. By the arithmetic operations of intuitionistic fuzzy numbers, we describe the intuitionistic fuzzy dimensional analysis (IFDA) and the aggregated intuitionistic fuzzy dimensional (AIFDA) techniques. In the first technique, we consider only the handling of fuzzy information; and, in the second one we consider both quantitative (crisp) and qualitative (fuzzy) information typically presented together in a decision making problem. To illustrate our approaches, we present some numerical examples and perform some comparisons with other well-known techniques.

1. Introduction

Decision making is a common activity presented in the daily life; it raises to the process of choosing the best possible alternative A^*, from a set of alternatives $A = (A_1, \ldots, A_n)$, constructed on the provided decision information [59]. Additionally, since most decision-making problems contain multiple criteria/indicators, $x = (x_1, \ldots, x_m)$, used to describe the characteristics or performance of candidate alternatives, such problems are the so-called multi-criteria decision making (MCDM)[19, 29]. Therefore, during the last two decades, MCDM techniques have gain considerable attention due to its impact in real-world applications [64, 71], where numerous MCDM techniques have been developed that provide support in this complex task [1, 7, 10, 14, 19, 26, 30, 32, 34, 35, 44, 47, 48, 50, 52, 53, 63, 72]. Moreover, the most prominent that can be mentioned are AHP, TOPSIS, ANP, VIKOR, DEMATEL, and ELECTRE [43]. In other hand, Dimensional analysis (DA) is a mathematical technique that combines several heterogeneous criteria, expressed in different measurement scales, into a single dimensionless index [11, 12]. In 1993, Willis et al. proposed a modified version of DA to deal with MCDM problems [63]. Thereby, Braglia and Gabbrielli (2000) employed DA as a MCDM

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technique for the selection of industrial robots [13]. Similarly, DA has been applied in supplier appraisal problems [24]. Likewise, the main advantages from DA deal to integrate the opinions made by a cluster of decision makers (DMs) concerning diverse information, such as alternatives, criteria, and its importance. In the same time, due to the DA mathematical structure, the impact of the criteria on the alternatives are considered. Nevertheless, in its current definition DA only can manipulate crisp (non-fuzzy) numbers rather than any other kind of information, which limits the applications of DA to more extensive areas. Moreover, decision-making information provided by a group of DMs can be considered inaccurate and uncertain in some cases, due to the fact that DMs might take different backgrounds, expertise, and levels of knowledge of a subject under study [56, 68]. In general, it is often problematic for DMs to give "exact" measures about their preferences on the criteria and alternatives involved in a MCDM problem by using a crisp number [46, 51]. Thus, the data reflecting the DM's preferences might be manifested in a fuzzy nature [20, 38]. Fuzzy set theory, introduced by Zadeh in 1965 [72], tremendously accepted in the MCDM field as tool to solve [40], fuzziness in the judgements, preferences and perceptions of DMs. However, fuzzy sets are restricted to the modeling of uncertain environments [8, 15]. To address this limitation, in 1986, Atanassov introduced an extension of the classical fuzzy set, the so-called intuitionistic fuzzy set (IFS), whose elements have degrees of membership and non-membership, as a form of dealing with uncertain environments (in the classical fuzzy set the non-membership function fully complements the membership function to 1, which leads to not handling any uncertainty)[6]. Consequently, IFSs have been widely used in different MCDM problems successfully, e.g., medical diagnoses, personnel assessments, supplier selection, portfolio selection, among others [33, 36, 62]. Similarly, the treatment of MCDM problems have been studied by researchers over the last years (see e.g.,[64, 43, 39] and references therein); however, the literature still reveals the existence of in three main challenges in its evolution. First, the development of operators that might consider the interrelationship between criteria (information) which may affect directly the final decision [2, 25, 65, 16, 74]. Second, the fact that MCDM problems simultaneously may include both quantitative (tangible) and qualitative (intangible) data (most of the published works emphasizes only in one type of data)[27, 28, 41, 42, 45, 73]; and, third, the loss of information during the MCDM analysis by means of the transformation of the data (as many papers deal only with one type of information, they usually transform the data into the same type to perform the analysis)[21, 23, 49, 70, 37]. In fact, when crisp values are converted into another type of information, this last one might suffer from disproportionate loss, which may significantly decrease the accuracy of the MCDM technique and the consistency of final choice [64, 18]. Motivated by the advantages of the DA method and IFS, this paper proposes two MCDM methods by extending the DA to IFS environments. Additionally, arises dealing with the last two challenges mentioned in the paragraph above. In this sense, the originality and contribution of this paper can be summarized as follows. First, we propose DA under fuzzy environments to overcome the limitation of DA to deal with any other type of arguments rather than crisp data and extend its potential applications to more extensive areas. Second, our approach can simultaneously handle quantitative (tangible) and qualitative (intangible) information, commonly presented in a MCDM problem. The structure of this paper is organized as follows: In section 2, we will examine the backgrounds of DA and IFSs. In section 3, we define the two DA-based techniques for dealing with both types of information. In section 4, we present some numerical examples to illustrate our techniques and some analysis of the results. Finally, we provide the concluding remarks in section 5.

2. Background

This section discusses the preliminary concepts of DA and the IFSs.

2.1. Dimensional Analysis.

DA is a mathematical technique that systematically aggregates several heterogeneous criteria into a single dimensionless index [24] . DA as a MCDM technique assumes that there is an optimal solution (alternative) better than the rest, S^*. This optimal alternative is hypothetical, since it does not exist, and it is generated from the set of alternatives and criteria into evaluation in the MCDM problem. The best criteria are chosen to form what is called the ideal alternative, denoted by $S^* = (x_1^*, \ldots, x_j^*)$, where x_j^* is the best nominal value for criteria $j = 1, \ldots, m$. DA compares each alternative in evaluation with this ideal alternative to generate an index of similarity. The alternative showing the highest index of similarity is chosen as the best alternative to the MCDM problem. DA is defined as follows.

Definition 2.1. (**DA technique** [63]). Let $a_l^k(k = 1, \ldots, n)(l = 1, \ldots, m)$ and $S_l^* = a_j^*(l = 1, \ldots, m)$ represent a collection of crisp numbers. The DA technique is defined below

$$IS_i(a_1^k, \ldots, a_m^k) = \prod_{j=1}^{m} (\frac{a_l^k}{S_l^*})^{w_z} \tag{1}$$

where, IS_i is called the index of similarity for alternative i. Where a_1^k is the crisp evaluation of criterion l for alternative i, S_l^* is the crisp value of the ideal alternative for criterion l, and $w_j(z = 1, \ldots, m)$ is the corresponding crisp weight for criterion l satisfying $w_z > 0(z = 1, \ldots, m)$ and $\sum_{z=1}^{m} w_j = 1$

DA considers the impact of the criteria in its analysis. Criteria having a positive impact (benefit criteria), or which are desired to be maximized, have a positive sign in their corresponding weight $(+w_j)$. On the other hand, criteria having a negative impact (cost criteria), or which are desired to be minimized, have a negative sign in the corresponding weight $(-w_j)$.

2.2. Intuitionistic Fuzzy Set.

In 1986 Atanassov presented the concept of IFS [6, 5], whose mathematical description is explained as below.

Definition 2.2. (**I**FS) [5, 6]. Let a set Y be fixed. An IFS A in is determinate as below

$$A = \{\langle z, \mu_A(z), \nu_A(z)\rangle | z \in Z\} \tag{2}$$

where $\mu_A(z) \in [0, 1]$ and $\nu_A(z) \in [0, 1]$ satisfying $0 \leq \mu_A(z)+\nu_A(z) \leq 1$ for all $y \in Y$, and $\mu_A(z)$ and $\nu_A(z)$ are called the membership degree and non-membership degree of the element $z \in Z$, in A. In addition, $\boldsymbol{\pi}_A(z) = 1 - \mu_A(z) - \nu_A(z)$ is named the hesitancy degree of y to A, which represents the uncertainty number. Each pair of $(\mu_A(z), \nu_A(z))$ in A is named an IFN (intuitionistic fuzzy number), denoted by $\delta = (\mu_\delta, \nu_\delta)$. To rank two IFNs, [17] presented the score function $s(\delta) = \mu_\delta - \nu_\delta$ to obtain the score value of δ. Simultaneously, [31] determined the accuracy function $h(\delta) = \mu_\delta + \nu_\delta$ to appraise accuracy degree of α. In this sense, the score value and the accuracy value,[69] provides the conditions of relation between two IFNs, σ and δ, as follows

- If $s(\sigma) < s(\delta)$, *then* $\alpha < \beta$
- If $s(\sigma) = s(\delta)$, *then*
  - If $h(\sigma) = h(\delta)$, *then*, $\alpha = \beta$;
  - If $h(\sigma) < h(\delta)$, *then*, $\alpha < \beta$.

Definition 2.3. (Arithmetic on IFNs proposed by [60, 69]). Let $\phi = (\mu_\phi, \nu_\phi)$, $\phi_1 = (\mu_{(\phi_1)}, \nu_{(\phi_1)})$, and $\phi_2 = (\mu_{(\phi_2)}, \nu_{(\phi_2)})$ be three IFNs and n a crisp number; then the following laws are valid

$$\phi_1 \oplus \phi_2 = (\mu_{\phi_1} + \mu_{\phi_2} - \mu_{\phi_1} \cdot \mu_{\phi_2}, \nu_{\phi_1} \cdot \nu_{\phi_2}). \tag{3}$$

$$\phi_1 \otimes \phi_2 = (\mu_{\phi_1} \cdot \mu_{\phi_2}, \nu_{\phi_1} + \nu_{\phi_2} - \nu_{\phi_1} \cdot \nu_{\phi_2}). \tag{4}$$

$$n\phi = (1 - (1 - \mu_\phi)^n, (\nu_\phi)^n), \quad n > 0. \tag{5}$$

$$\phi^n = ((\mu_\phi)^n, 1 - (1 - \nu_\phi)^n), \quad n > 0. \tag{6}$$

$$\frac{\phi_1}{\phi_2} = (x, \frac{\mu_{\phi_1}}{\mu_{\phi_2}}, \frac{\nu_{\phi_1} - \nu_{\phi_2}}{1 - \nu_{\phi_2}} | x \in X). \tag{7}$$

$\frac{\phi_1}{\phi_2}\, exists\, only\, if$

(1) $\phi_1 \leq \phi_2$
(2) $\phi_1 \leq \phi_2$
(3) $\phi_1 \leq \phi_2$
(4) $\phi_1 \leq \phi_2$

All results of the operations are also IFNs.

3. Intuitionistic Fuzzy Dimensional Analysis

In this section, we announce the intuitionistic fuzzy dimensional analysis (IFDA) and the aggregated intuitionistic fuzzy dimensional analysis (AIFDA) techniques for MCDM problems. The IFDA technique we are proposing can deal only with fuzzy information, represented by IFNs; alternatively, the AIFDA technique is capable of dealing with both types of information, crisp and fuzzy.

Based on equation (1), the description of IFDA is specified as follows.

Definition 3.1. (**Intuitionistic Fuzzy Dimensional Analysis technique**). Let $\alpha^t_{\hat{k}} = (\mu_{\alpha^t_{\hat{k}}}, \nu_{\alpha^t_{\hat{k}}})(t =, \ldots, n)(\hat{k} =, \ldots, m)$ represent a cluster of IFNs. The IFDA technique is well-defined as

$$IFIS_i(\alpha^t_1, \ldots, \alpha^t_m) = \bigotimes_{k=1}^{m} (\frac{\alpha^t_{\hat{k}}}{\psi^*_{\hat{k}}}) = \bigotimes_{k=1}^{m} (\beta^t_{\hat{k}})^{w_z} \tag{8}$$

where, $IFIS_i$ is named the intuitionistic fuzzy index of similarity for alternative i, where α_j^i is the intuitionistic fuzzy evaluation of criterion j for alternative i, ψ_j^* is the intuitionistic fuzzy value of the ideal alternative for criterion j, and $w_z(z=1,\ldots,m)$ is the corresponding crisp weight for criterion j satisfying $w_z > 0$ (z=1,…,m) and $\Sigma_{z=1}^{m} w_z = 1$ (the weight has to be a crisp number since it is not possible to power an IFN to an IFN).

Based on equations (3), (4), (5), (6), and (7) of IFNs described in Section 2, the next results are derived.

Theorem 3.2. *Let* $\beta_{\hat{k}}^t = (\mu_{\beta_{\hat{k}}^t}, \nu_{\beta_{\hat{k}}^t})(t=1,\ldots,n)(\hat{k}=1,\ldots,m)$ *represent a cluster of IFNs. Therefore, the combined value via IFIS is as well an IFN, and.*

$$IFIS_i(\alpha_1^t,\ldots,\alpha_m^t) = \bigotimes_{\hat{k}=1}^{m} (\beta_{\hat{k}}^t)^{w_z} = (\prod_{\hat{k}=1}^{m} (\mu_{\beta_{\hat{k}}^t})^{w_z}, 1 - \prod_{\hat{k}=1}^{m} (1-\nu_{\beta_{\hat{k}}^t})^{w_z}). \tag{9}$$

Proof. Based on equation (7) described in Section 2, equation (10) is derived as follows

$$\beta_{\hat{k}}^t = \frac{\alpha_{\hat{k}}^t}{\psi_{\hat{k}}^*} = (\frac{\mu_{\alpha_{\hat{k}}^t}}{\mu_{\alpha_{\hat{k}}^*}}, \frac{\nu_{\alpha_{\hat{k}}^t} - \nu_{\alpha_{\hat{k}}^*}}{1-\nu_{\alpha_{\hat{k}}^*}}) = (\mu_{\beta_{\hat{k}}^t}, \nu_{\beta_{\hat{k}}^t}) \tag{10}$$

then, by equation (6)

$$(\beta_{\hat{k}}^t)^{w_z} = ((\mu_{\beta_{\hat{k}}^t})^{w_z}, 1-(1-\nu_{\beta_{\hat{k}}^t})^{w_z}) \tag{11}$$

and according to equation (4) is proposed the equation (12)

$$\bigotimes_{\hat{k}=1}^{m} (\beta_{\hat{k}}^t)^{w_z} = ((\mu_{\beta_1^t})^{w_1}, \ldots, (\mu_{\beta_m^t})^{w_m},$$
$$(1-(1-(1-\nu_{\beta_1^t})^{w_1})), \ldots, (1-(1-(1-\nu_{\beta_1^t})^{w_1}))) \tag{12}$$

$$\bigotimes_{\hat{k}=1}^{m} (\beta_{\hat{k}}^t)^{w_z} = ((\mu_{\beta_1^t})^{w_1}, \ldots, (\mu_{\beta_m^t})^{w_m}, 1-((1-\nu_{\beta_1^t})^{w_1}), \ldots, (1-\nu_{\beta_1^t})^{w_1})) \tag{13}$$

$$\bigotimes_{\hat{k}=1}^{m} (\beta_{\hat{k}}^t)^{w_z} = (\prod_{\hat{k}=1}^{m} (\mu_{\beta_{\hat{k}}^t})^{w_z}, 1 - \prod_{\hat{k}=1}^{m} (1-\nu_{\beta_{\hat{k}}^t})^{w_z}). \tag{14}$$

If $\alpha_j^i \in BN$; then,

$$\beta_{\hat{k}}^t = \frac{\alpha_{\hat{k}}^t}{\psi_{\hat{k}}^*} = (\frac{\mu_{\alpha_{\hat{k}}^t}}{\mu_{\alpha_{\hat{k}}^*}}, \frac{\nu_{\alpha_{\hat{k}}^t} - \nu_{\alpha_{\hat{k}}^*}}{1-\nu_{\alpha_{\hat{k}}^*}}) = (\mu_{\beta_{\hat{k}}^t}, \nu_{\beta_{\hat{k}}^t}), \quad j \in BN \tag{15}$$

otherwise, if $\alpha_j^i \in C$; then,

$$\beta_{\hat{k}}'^t = \frac{\psi_{\hat{k}}^*}{\alpha_{\hat{k}}^t} = (\frac{\mu_{\alpha_{\hat{k}}^*}}{\mu_{\alpha_{\hat{k}}^t}}, \frac{\nu_{\alpha_{\hat{k}}^*} - \nu_{\alpha_{\hat{k}}^t}}{1-\nu_{\alpha_{\hat{k}}^t}}) = (\mu_{\beta_{\hat{k}}^t}, \nu_{\beta_{\hat{k}}^t}), \quad j \in C \tag{16}$$

following the next conditions,

$$\mu_{\alpha_{\hat{k}}^*} = ((max\, \mu_{\alpha_{\hat{k}}^t}) | \hat{k} \in BN), (min\, \mu_{\alpha_{\hat{k}}^t}) | \hat{k} \in C), \tag{17}$$

$$\nu_{\alpha_{\hat{k}}^*} = ((min\, \nu_{\alpha_{\hat{k}}^t}) | \hat{k} \in BN), (max\, \nu_{\alpha_{\hat{k}}^t}) | \hat{k} \in C), \tag{18}$$

Equations (17) and (18) represent the ideal alternative to criterion j under IFS nature, alike other MCDM techniques (e.g. TOPSIS).

Where

$$\psi_{\hat{k}}^* = (\mu_{\beta_{\hat{k}}^t}, \nu_{\beta_{\hat{k}}^t}) \tag{19}$$

This concludes the proof of **Theorem** 3.2. Hence, when a MCDM problem contains both benefit criteria (BN) and cost criteria (C) data, the following principles must be taken into account. □

Also, when a MCDM problem contains both crisp and fuzzy information (expressed by IFNs), the following definition is provided.

Definition 3.3. (Aggregated Intuitionistic Fuzzy Dimensional Analysis technique). Let $a_j^i(i = 1, \ldots, n)(j = 1, \ldots, l)$ and $S_j^* = a_j^*(\hat{k} = 1, \ldots, l)$ be a collection of crisp numbers, and $\alpha_j^i = (\mu_{\alpha_{\hat{k}}^t}, \nu_{\alpha_{\hat{k}}^t})(t = 1, \ldots, n)(\hat{k} = \check{l} + 1, \ldots, m)$ and $\psi_j^* = (\mu_{\alpha_{\hat{k}}^*}, \nu_{\alpha_{\hat{k}}^*})(\hat{k} = \check{l} + 1, \ldots, m)$ is a collection of IFNs. The AIFDA technique is defined as

$$AIFIS_i(a_1^t, \ldots, a_{\check{l}}^t, \alpha_{\check{l}+1}^t, \ldots, \alpha_m^t)$$
$$= (\prod_{j=1}^{\check{l}} (\frac{a_{\hat{k}}^t}{S_{\hat{k}}^*})^{w_z})(\bigotimes_{\hat{k}=\check{l}+1}^{m} (\frac{\alpha_{\hat{k}}^t}{\psi_{\hat{k}}^*})^{w_z}) = (\prod_{\hat{k}=1}^{\check{l}} (B_{\hat{k}}^t)^{w_z}) \otimes (\bigotimes_{\hat{k}=\check{l}+1}^{m} (\beta_{\hat{k}}^t)^{w_z}). \tag{20}$$

where, $AIFIS_i$ is named the aggregated intuitionistic fuzzy index of similarity for alternative i. Where a_j^i and $\alpha_j^i$are the crisp and the intuitionistic fuzzy evaluations of criterion $\hat{k}$ for alternative i, respectively, S_j^* and $\psi_{\hat{k}^*}$ are the crisp and the intuitionistic fuzzy values of the ideal alternative for criterion $\hat{k}$,respectively, and $w_z(z = 1, \ldots, m)$ is the corresponding crisp weight for criterion j satisfying $w_z > 0$ (z=1,...,m) and $\Sigma_{z=1}^m w_z = 1$.

Theorem 3.4. *Let $B_{\hat{k}}^t(t = l, \ldots, n)(\hat{k} = l, \ldots, l)$ be a cluster of crisp numbers, and $\alpha_j^i = (\mu_{\beta_{\hat{k}}^t}, \nu_{\beta_{\hat{k}}^t})(t = 1, \ldots, n)(\hat{k} = \check{l} + 1, \ldots, m)$ be a cluster of IFNs. Therefore, the combined value, by mean of AIFIS, is as well an IFN, and*

$$AIFIS_i(B_1^t, \ldots, B_{\check{l}}^t, \beta_{\check{l}+1}^t, \ldots, \beta_m^t) = (\prod_{\hat{k}=1}^{\check{l}} (B_{\hat{k}}^t)^{w_z}) \otimes (\bigotimes_{\hat{k}=\check{l}+1}^{m} (\beta_{\hat{k}}^t)^{w_z})$$
$$= (1 - (1 - \prod_{\hat{k}=\check{l}+1}^{m} (\mu_{\beta_{\hat{k}}^t})^{w_z})^{(\Pi_{\hat{k}=1}^{\check{l}} (B_{\hat{k}}^t)^{w_z})},$$
$$(1 - \prod_{\hat{k}=\check{l}+1}^{m} (1 - \nu_{\beta_{\hat{k}}^t})^{w_z})^{(\Pi_{\hat{k}=1}^{\check{l}} (B_{\hat{k}}^t)^{w_z})} \tag{21}$$

Proof. : Based on equation (9), it can be obtained the expression.

$$\bigotimes_{\hat{k}=\check{l}+1}^{m} (\beta_{\hat{k}}^t)^{w_z} = (\prod_{\hat{k}=\check{l}+1}^{m} (\mu_{\beta_{\hat{k}}^t})^{w_z}, 1 - \prod_{\hat{k}=\check{l}+1}^{m} (1 - \nu_{\beta_{\hat{k}}^t})^{w_z}), \tag{22}$$

thus, by equation (5)

$$n\dot{T} = (1 - (1 - \mu_{\dot{T}}(\dot{z}))^n, (\nu_{\dot{T}}(\dot{z})^n), \tag{23}$$

if we make

$$n = (\prod_{\hat{k}=1}^{\check{l}} (B_{\hat{k}}^t)^{w_z}), \tag{24}$$

and
$$\dot{T} = (\bigotimes_{\hat{k}=\check{l}+1}^{m} (\beta_{\hat{k}}^{t})^{w_z}) = (\prod_{\hat{k}=\check{l}+1}^{m} (\mu_{\beta_{\hat{k}}^{t}})^{w_z}, 1 - \prod_{\hat{k}=\check{l}+1}^{m} (1 - \nu_{\beta_{\hat{k}}^{t}})^{w_z})$$
$$= (\mu_A(x), \nu_A(x)), \tag{25}$$
then,
$$n\dot{T} = (\prod_{\hat{k}=1}^{\check{l}} (B_{\hat{k}}^{t})^{w_z}) \otimes (\bigotimes_{\hat{k}=\check{l}+1}^{m} (\beta_{\hat{k}}^{t})^{w_z})$$
$$= (1 - (1 - \prod_{\hat{k}=\check{l}+1}^{m} (\mu_{\beta_{\hat{k}}^{t}})^{w_z})^{(\Pi_{\hat{k}=1}^{\check{l}} (B_{\hat{k}}^{t})^{w_z})},$$
$$(1 - \prod_{\hat{k}=\check{l}+1}^{m} (1 - \nu_{\beta_{\hat{k}}^{t}})^{w_z})^{(\Pi_{\hat{k}=1}^{\check{l}} (B_{\hat{k}}^{t})^{w_z})} = n\dot{T} \tag{26}$$
which concludes the proof of **Theorem 3.4** □

When a MCDM problem contains both benefit criteria (BN) and cost criteria (C) data, the following principles must be considered. If $a_{\hat{k}}^{t} \in BN$ and $\alpha_{\hat{k}}^{i} \in BN$; then,
$$B_{\hat{k}}^{t} = \frac{\alpha_{\hat{k}}^{t}}{S_{\hat{k}}^{*}} \quad \hat{k} \in BN, \tag{27}$$
$$\beta_{\hat{k}}^{t} = \frac{\alpha_{\hat{k}}^{t}}{\psi_{\hat{k}}^{*}} = (\frac{\mu_{\alpha_{\hat{k}}^{t}}}{\mu_{\alpha_{\hat{k}}^{*}}}, \frac{\nu_{\alpha_{\hat{k}}^{t}} - \nu_{\alpha_{\hat{k}}^{*}}}{1 - \nu_{\alpha_{\hat{k}}^{*}}}) = (\mu_{\beta_{\hat{k}}^{t}}, \nu_{\beta_{\hat{k}}^{t}}) \quad \hat{k} \in BN, \tag{28}$$
Otherwise, if $a_{\hat{k}}^{t} \in C$ and $\alpha_{\hat{k}}^{i} \in C$; then,
$$B_{\hat{k}}^{\prime t} = \frac{S_{\hat{k}}^{*}}{\alpha_{\hat{k}}^{t}} \hat{k} \in C, \tag{29}$$
$$\beta_{\hat{k}}^{\prime t} = \frac{\psi_{\hat{k}}^{*}}{\alpha_{\hat{k}}^{t}} = (\frac{\mu_{\alpha_{\hat{k}}^{*}}}{\mu_{\alpha_{\hat{k}}^{t}}}, \frac{\nu_{\alpha_{\hat{k}}^{*}} - \nu_{\alpha_{\hat{k}}^{t}}}{1 - \nu_{\alpha_{\hat{k}}^{t}}}) = (\mu_{\beta_{\hat{k}}^{t}}, \nu_{\beta_{\hat{k}}^{t}}) \quad \hat{k} \in C. \tag{30}$$
Following the next conditions,
$$S_{j}^{*} = ((max\,(a_{\hat{k}}^{t}) | \hat{k} \in BN), (min\,(a_{\hat{k}}^{t}) | \hat{k} \in C)), \tag{31}$$
$$\mu_{\alpha_{\hat{k}}^{*}} = ((max\,\mu_{\alpha_{\hat{k}}^{t}}) | \hat{k} \in BN), (min\,\mu_{\alpha_{\hat{k}}^{t}}) | \hat{k} \in C)), \tag{32}$$
$$\nu_{\alpha_{\hat{k}}^{*}} = ((min\,\nu_{\alpha_{\hat{k}}^{t}}) | \hat{k} \in BN), (max\,\nu_{\alpha_{\hat{k}}^{t}}) | \hat{k} \in C)). \tag{33}$$
Likewise, equations (32) and (33) depict the ideal alternative to criterion $\hat{k}$ under IFS environment.

Where
$$\psi_{\hat{k}}^{*} = (\mu_{\beta_{\hat{k}}^{t}}, \nu_{\beta_{\hat{k}}^{t}}) \tag{34}$$

Therefore, the AIFDA technique successfully handles crisp values without any transformation, thereby avoiding reliability loss in results. In the following section, we present two numerical examples to provide a detailed illustration of both IFDA and AIFDA techniques.

4. **Numerical Illustrations**

In this section, we validate the application of the IFDA and AIFDA techniques to MCDM problems to demonstrate their applicability. Then, with the purpose of validating both techniques, we compared them with two well-known MCDM techniques, TOPSIS and AHP.

4.1. **IFDA for MCDM Problems.**

Most MCDM problems consider the subsequent steps to obtain a ranking of alternatives.

Step 1: Let $A = (A_1, \ldots, A_n)$ represent a group of n alternatives and $x = (\ddot{x}_1, \ldots, \ddot{x}_m)$ depict a group m criteria to be appraised, whose weight vector is $w = (w_1, \ldots, w_m)^T$, satisfying $w_z > 0$ (z=1,...,m) and $\Sigma_{z=1}^m w_z = 1$, where w_z denotes the preference grade of criterion x_j.Performance of A_i through criteria $x_{\hat{k}}$ is expressed by an IFN $\alpha_j^i = (\mu_{\alpha_j^i}, \nu_{\alpha_j^i})(t = 1, \ldots, n)(\hat{k} = 1, \ldots, m)$. All $\alpha_j^i = (\mu_{\alpha_j^i}, \nu_{\alpha_j^i})(t = 1, \ldots, n)(\hat{k} = 1, \ldots, m)$ are delimited in an intuitionistic decision matrix $IFDM = (\alpha_j^i)_{nxm}$ (See Table 1).

If all criteria X are of the same type (i.e. benefit criteria, BN), by means

	$\ddot{x}_1$	$\ldots$	$\ddot{x}_m$
A_1	$(\mu_{1,1}, \nu_{1,1})$	$\ldots$	$(\mu_{1,m}, \nu_{1,m})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_n	$(\mu_{n,1}, \nu_{n,1})$	$\ldots$	$(\mu_{1,m}, \nu_{1,m})$

Table 1. *IFDM*

of equations (15), (17), and (18) we obtain matrix $Y = (\beta_{\hat{k}}^t)_{nxm}$. On the other hand, if all criteria are of the type cost (C), by using equation (16) we obtain matrix $Z = (\beta_{\hat{k}}'^t)_{nxm}$. However, since a same MCDM problem generally involves both benefit and cost criteria, matrices Y and Z may be combined into the matrix $R = (\gamma_{\hat{k}}^t)_{nxm}$, where,

$$\gamma_{\hat{k}}^t = (\mu_{\gamma_{\hat{k}}^t}, \nu_{\gamma_{\hat{k}}^t}) = \begin{cases} \beta_{\hat{k}}^t, & \hat{k} \in BN; \\ \beta_{\hat{k}}'^t, & \hat{k} \in C. \end{cases} \tag{35}$$

Step 2: Apply the intuitionistic fuzzy dimensional analysis ($IFDA$) technique to aggregate all performance values $\gamma_k^t (t = 1, \ldots, n)(k = 1, \ldots, m)$ to obtain the intuitionistic fuzzy index of similarity, $IFIS_i$, corresponding to alternative $A_i (i = 1, \ldots, n)$.

$$IFIS_i(\gamma_1^t, \ldots, \gamma_m^t) = (\prod_{\hat{k}=1}^{m} (\mu_{\gamma_{\hat{k}}^t})^{w_z}, 1 - \prod_{\hat{k}=1}^{m} (1 - \nu_{\gamma_{\hat{k}}^t})^{w_z}). \tag{36}$$

Step 3: Compute the index of similarity. To rank any two IFNs, according to [17] the score function $S(IFIS_i) = (\mu_{IFIS_i} - \nu_{IFIS_i})$ to obtain the score value of an IFN. We used this score to obtain the overall index of similarity, IS_i, for alternative $A_i (i = 1, \ldots, n)$.

Step 4: Rank alternatives $A_i (i = 1, \ldots, n)$ in descending order according to the obtained IS_i values. Next, we provide a numerical case to thoroughly demonstrate the aforementioned technique.

Example 4.1. ***Application of IFDA Technique***. The problem consisted of selecting a CNC milling machine among three alternatives $A = (W, Y, Z)$ for an advanced manufacturing technology (AMT) training center. In general, AMT appraisal is observed as problematic task, due to the multiple criteria involved that is difficult to take into consideration in their totality. Current methods for AMT assessment and choice lack anthropometric factors and ergonomics (HFE) characteristics, since these are regularly ignored by DMs. For this reason, our example regarding the selection of a CNC milling machine is performed considering HFE criteria. In this sense, ergonomic appraisal of AMT may pose complications, since ergonomic conditions are several; moreover, it contains criteria that is intangible. Therefore, our objective is to employ a MCDM technique to evaluate and select the best CNC milling machine considering HFE criteria, which commonly are of qualitative (fuzzy) nature and are evaluated through individual assessments, represented by IFNs. Criteria analyzed are described as follows.

- **Skill level compatibility** ($\ddot{x}_1$): Equipment design criteria regarding equipment malleability to variances in practical skills of workers. *Benefit criterion.*
- **Training compatibility** ($\ddot{x}_2$): Equipment inventiveness criteria in terms of training required (excellence and time) and accessible, considering user requirements. *Benefit criterion.*
- **Access to machine and clearances**($\ddot{x}_3$): Equipment design criteria concerning degree of mobility and safety access to operators through its workspace and clearances. *Benefit criterion.*
- **Physical distribution of controls** ($\ddot{x}_4$): Equipment design criteria regarding layout of control-panel (buttons, knobs, levers, switches, etc.), providing safe and effective equipment operation. *Benefit criterion.*
- **Work station space**($\ddot{x}_5$): Criteria regarding magnitude and location of control command and the screen: size and kind of fonts used, flags, resolution and illumination, enabling human tasks. *Benefit criterion.*
- **Noise**($\ddot{x}_6$): Equipment design criteria associated to sound produced by the machine and its mechanisms and that may adversely impact to personnel of manufacturing area. *Cost criterion.*

Let $DM_k = (\mu_k, \nu_k, \pi_k)$ represent an IFN for assessment of k_{th} DM. In this manner, the importance of k_{th} DM is calculated as

$$\lambda_{\dot{k}} = \frac{\left(\mu_{\dot{k}} + \pi_{\dot{k}}\left(\frac{\mu_{\dot{k}}}{\mu_{\dot{k}}+\nu_{\dot{k}}}\right)\right)}{\sum_{\dot{k}=1}^{l}\left(\mu_k + \pi_{\dot{k}}\left(\frac{\mu_{\dot{k}}}{\mu_{\dot{k}}+\nu_{\dot{k}}}\right)\right)} \tag{37}$$

where $\lambda_{\dot{k}} > 0, \sum_{\dot{k}=1}^{z} \lambda_{\dot{k}} = 1$

To assess each criterion and rate its importance, three participating DMs used Table 2 shown below (Table 2 is a sample scale that can be used, but any other scale may be used considering IFNs).

Likewise, let $w_z^{(\dot{k})} = (\mu_z^{(\dot{k})}, \nu_z^{(\dot{k})}, \boldsymbol{\pi}_z^{(\dot{k})})$ represent an intuitionistic fuzzy value given

Linguistic expression	$IFN(\mu, \nu)$
Apprentice (Ap) / Very Insignificant (VI)	(0.10, 0.90)
Learner (Lr) / Insignificant (I)	(0.35, 0.60)
Capable (Ct) / Average (A)	(0.50, 0.45)
Skillful (S) / Imperative (Im)	(0.75, 0.20)
Dominant (D) / Very Significative (VS)	(0.90, 0.10)

Table 2. Linguistic Expression for Assessment Importance of DMs and Criteria

to criterion $\ddot{x}$ to k_th DM. Therefore, opinions from DMs about criteria may be integrated by IFWA operator proposed by [66]

$$w_z = IFWA(w_z^{(1)}, w_z^{(\dot{k})}, \ldots, w_z^{(l)}),$$

$$\lambda_1 w_z^{(1)} \otimes \lambda_1 w_z^{(\dot{k})} \otimes, \ldots, \otimes \lambda_1 w_z^{(l)} = [1 - \prod_{\dot{k}=1}^{m} (1 - \mu_j^{\dot{k}})^{\lambda_{\dot{k}}}, \prod_{\dot{k}=1}^{m} (\nu_j^{\dot{k}})^{\lambda_{\dot{k}}}]. \tag{38}$$

where $w_z = (\mu_z, \mu_z, \boldsymbol{\pi}_z)$ and $W = (w_1, \ldots, w_m)$. Hence, the preferences of each criterion are determinates as follows

$$w_z = \frac{\left(\mu_j + \boldsymbol{\pi}_j (\frac{\mu_j}{\mu_j + \nu_j})\right)}{\sum_{j=1}^{m} \left(\mu_j + \boldsymbol{\pi}_j (\frac{\mu_j}{\mu_j + \nu_j})\right)} \tag{39}$$

where $w_z > 0$ (z=1,...,m) and $\sum_{z=1}^{m} w_z = 1$

Coming back to our example, Table 3 shows the importance degree given to the DMs.

Decision Maker	1	2	3
Linguistic expression	S	S	S
IFN	(0.75,0.2)	(0.75,0.2)	(0.75,0.2)
Weight	1/3	1/3	1/3

Table 3. Importance Degree Given of DMs

As mentioned earlier, the three DMs rated each criterion based on their importance in the selection of the best CNC milling machine considering HFE criteria.

All opinions are integrated by means of equation (38).

$$W\{\ddot{x_1}, \ddot{x_2}, \ddot{x_3}, \ddot{x_4}, \ddot{x_5}, \ddot{x_6}\} = \begin{bmatrix} (0.527, 0.416) \\ (0.718, 0.262) \\ (0.527, 0.416) \\ (0.718, 0.262) \\ (0.747, 0.229) \\ (0.567, 0.378) \end{bmatrix}^T$$

The Table 4 displays the ratings given by the DMs to the criteria.

Finally, the importance degree of each criterion is computed by equation (39)

$$(\ w_{\ddot{x}_1} = 0.142, w_{\ddot{x}_2} = 0.185, w_{\ddot{x}_3} = 0.142, w_{\ddot{x}_4} = 0.185, w_{\ddot{x}_5} = 0.194, w_{\ddot{x}_6} = 0.152\ )$$

$$\sum_{z=1}^{6} w_z = 0.142 + 0.185 + 0.142 + 0.185 + 0.194 + 0.152 = 1.$$

Decision Maker	Criteria					
	$\ddot{x}_1$	$\ddot{x}_2$	$\ddot{x}_3$	$\ddot{x}_4$	$\ddot{x}_5$	$\ddot{x}_6$
DM_1	Im	Ap	Im	VI	VS	Lr
DM_2	Lr	D	Lr	VS	I	Im
DM_2	Lr	Im	Lr	Im	Im	A

Table 4. Importance Degree of Criteria for Selecting a CNC Milling Machine

Linguistic expression	$IFN(\mu, \nu)$
Extremely Poor (EP) / Extremely Little (EL)	(0.10, 0.90)
Very Poor (VP) / Very Little (VL)	(0.10, 0.75)
Poor (P) / Little (L)	(0.25, 0.60)
Medium Poor (MP) / Medium Little (ML)	(0.40, 0.50)
Fair-minded (F) / Average (A)	(0.50, 0.40)
Middle Good (MG) / Middle High (MH)	(0.60, 0.30)
Respectable (R) / Tall (T)	(0.70, 0.20)
Great (G) / Excessive (E)	(0.80, 0.10)
Exceptional (EX) / Enormously High (EH)	(1.00, 0.00)

Table 5. Linguistic Expression for Evaluation of the Alternatives

In order to evaluate the three alternatives (CNC milling machines), we used Table 5. Thus, DMs can make their evaluations in common terms such as linguistic expressions (Table 5 is a sample scale just a scale that can be used, but any other scale may be used considering IFNs).
All preferences of DMs had to be involved into a final intuitionistic decision matrix (FIFDM). In this sense, we used $IFWA$ operator to integrate all different evaluations into a single assessment, thus obtaining $R = (\alpha_{\hat{k}}^{t})_{nxm}$

$$\begin{aligned} \alpha_{\hat{k}}^{t} &= (\alpha_{\hat{k}}^{t^{(1)}}, \alpha_{\hat{k}}^{t^{(k)}}, \alpha_{\hat{k}}^{t^{(l)}}) \\ &= \lambda_1 \alpha_{\hat{k}}^{t^{(1)}} \otimes \lambda_1 \alpha_{\hat{k}}^{t^{(k)}} \otimes, \ldots, \otimes \lambda_1 \alpha_{\hat{k}}^{t^{(l)}} \\ &= [1 - \prod_{\hat{k}=1}^{l} (1 - \mu_{\hat{k}}^{t})^{\lambda_k}, \prod_{\hat{k}=1}^{l} (\nu_{\hat{k}}^{t})^{\lambda_k}]. \end{aligned} \tag{40}$$

where $\alpha_{\hat{k}}^{t} = (\mu_{A_t}(\ddot{x}_{\hat{k}}), \nu_{A_t}(\ddot{x}_{\hat{k}}))(t = 1, \ldots, n; \hat{k} = 1, \ldots, l)$
Then, the FIFDM is defined as

$$FIFDM = \begin{pmatrix} \alpha_1^1 & \ldots & \alpha_m^1 \\ \vdots & \ddots & \vdots \\ \alpha_1^1 & \ldots & \alpha_m^n \end{pmatrix} \tag{41}$$

Specifically,

$$FIFDM = \begin{pmatrix} (\mu_{A_1}(\ddot{x}_1), \nu_{A_1}(\ddot{x}_1)) & \ldots & (\mu_{A_1}(\ddot{x}_m), \nu_{A_1}(\ddot{x}_m)) \\ \vdots & \ddots & \vdots \\ (\mu_{A_n}(\ddot{x}_1), \nu_{A_n}(\ddot{x}_1)) & \ldots & (\mu_{A_n}(\ddot{x}_m), \nu_{A_n}(\ddot{x}_m)) \end{pmatrix} \tag{42}$$

Decision Maker	Machine	$\ddot{x}_1$	$\ddot{x}_2$	$\ddot{x}_3$	$\ddot{x}_4$	$\ddot{x}_5$	$\ddot{x}_6$
DM1	W	P	P	MG	MG	R	ML
	Y	F	T	T	T	MG	A
	Z	T	T	MG	MG	T	VL
DM2	W	B	B	MG	F	G	ML
	Y	T	T	G	G	T	ML
	Z	G	G	R	MG	R	VL
DM3	W	MB	P	MG	MP	F	L
	Y	T	MG	VG	MP	MP	L
	Z	G	T	MG	F	T	ML

Table 6. Ratings of Criteria for Selecting a CNC Milling Machine

For our case study, Table 6 shows ratings or evaluations from DMs for each of the three alternatives (CNC milling machines) based on the criteria previously stated.

Then, the final intuitionistic decision matrix, FIFDM, is

$$FIFDM = \begin{bmatrix} (0.304, 0.565) & (0.250, 0.600) & (0.600, 0.300) & (0.507, 0.391) & (0.689, 0.200) & (0.354, 0.531) \\ (0.644, 0.252) & (0.670, 0.229) & (0.771, 0.126) & (0.670, 0.215) & (0.584, 0.311) & (0.392, 0.493) \\ (0.771, 0.126) & (0.738, 0.159) & (0.637, 0.262) & (0.569, 0.330) & (0.700, 0.200) & (0.214, 0.655) \end{bmatrix}$$

Once we collected and integrated the data from DMs, we applied the IFDA approach, following the aforementioned steps.

Step 1: Let $A = (W, Y, Z)$ represent the three alternatives and $x_{\hat{k}} = (x_1, \ldots, x_6)$ represent the six criteria to be appraised, whose weight vector is $w = (0.142, 0.185, 0.142, 0.185, 0.194, 0.152)^T$, satisfying $w_z > 0$ $(z = 1, \ldots, 6)$ and $\sum_{z=1}^{6} w_z = 1$, where w_z denotes the importance degree of criterion $\ddot{x}_{\hat{k}}$.

Performance of A_i with respect to criteria $\ddot{x}_{\hat{k}}$ is expressed by an IFN $\alpha_{\hat{k}}^t = (\mu_{\alpha_{\hat{k}}^t}, \nu_{\alpha_{\hat{k}}^t})(t = 1, 2, 3)(\hat{k} = 1, \ldots, 6)$.

	Alternatives		
Criteria	W	Y	Z
$\ddot{x_1}$	(0.304,0.565)	(0.644,0.252)	(0.771,0.126)
$\ddot{x_2}$	(0.250,0.600)	(0.670,0.229)	(0.738,0.159)
$\ddot{x_3}$	(0.600,0.300)	(0.771,0.126)	(0.637,0.262)
$\ddot{x_4}$	(0.507,0.391)	(0.670,0.215)	(0.569,0.330)
$\ddot{x_5}$	(0.689,0.200)	(0.584,0.311)	(0.700,0.200)
$\ddot{x_6}$	(0.354,0.531)	(0.392,0.493)	(0.214,0.655)

Table 7. Intuitionistic Decision Matrix IFDM

All $\alpha_{\hat{k}}^t = (\mu_{\alpha_{\hat{k}}^t}, \nu_{\alpha_{\hat{k}}^t})(t = 1, 2, 3)(\hat{k} = 1, \ldots, 6)$ are presented via intuitionistic decision matrix IFDM=$(\alpha_{\hat{k}}^t)_{3x6}$ (see Table 7). Since criteria $\ddot{x_1}, \ddot{x_2}, \ddot{x_3}, \ddot{x_4}$ and $\ddot{x_5}$ are benefit criteria, whereas criteria $\ddot{x_6}$ is a cost criterion, we combined both types of criteria into the matrix $R = (\gamma_{\hat{k}}^t)_{3x6}$, where

$$\psi = ((0.771, 0.126), (0.738, 0.159), (0.771, 0.126), (0.670, 0.215), (0.700, 0.200), (0.214, 0.655)). \tag{43}$$

	Alternatives		
Criteria	W	Y	Z
$\ddot{x_1}$	(0.394,0.502)	(0.836,0.144)	(1.000 0.000)
$\ddot{x_2}$	(0.339,0.525)	(0.908,0.083)	(1.000 0.000)
$\ddot{x_3}$	(0.778,0.199)	(1.000,0.000)	(0.826,0.156)
$\ddot{x_4}$	(0.757,0.224)	(1.000,0.000)	(0.850,0.146)
$\ddot{x_5}$	(0.689,0.200)	(0.985,0.000)	(1.000,0.000)
$\ddot{x_6}$	(0.604,0.264)	(0.546,0.320)	(1.000,0.000)

Table 8. Intuitionistic Fuzzy Combined Matrix R

And $R = (\gamma^t_{\hat{k}})_{3x6}$ (see Table 8).

Step 2: We applied the IFDA technique to aggregate all performance values $\gamma^t_{\hat{k}}(t = 1, 2, 3)(\hat{k} = 1, \ldots, 6)$ with the aim of obtaining an intuitionistic fuzzy index of similarity, $IFIS_i$, corresponding to alternative $A_i(i = 1, 2, 3)$. As a result, $IFIS$ is calculated by equation (36)

$$IFIS(\gamma^t_1, \gamma^t_2, \gamma^t_3, \gamma^t_4, \gamma^t_5, \gamma^t_6) = (\prod_{\hat{k}=1}^{6} (\mu_{\gamma^t_{\hat{k}}})^{w_z}, 1 - \prod_{\hat{k}=1}^{6} (1 - \nu_{\gamma^t_{\hat{k}}})^{w_z}). \quad (44)$$

Therefore, $IFIS_1 = (0.607, 0.304), IFIS_2 = (0.893, 0.118), IFIS_3 = (0.944, 0.052)$.

Step 3: Then, we computed the index of similarity, IS_i, for A_i.

$$IS_1 = 0.303, \quad IS_2 = 0.725, \quad IS_3 = 0.892.$$

Step 4: Finally, we rank the alternatives A_i in descending order according to obtained the values of IS_i. Since results showed ZYW, CNC milling machine **Z** is selected as the best alternative to our problem, considering HFE criteria.

4.2. **AIFDA for MCDM Problems.** Typically, MCDM problems involve quantitative (tangible) and qualitative (intangible) criteria implicit simultaneously integrated into the evaluation process [22]. Thus, in this section we present the use of the second technique, the aggregated intuitionistic fuzzy dimensional analysis (AIFDA), which has the potential to work with both types of information. In AIFDA, some criteria may be treated with crisp values and other with linguistic expressions represented by IFNs. In this sub-section, we present this technique with the same four steps typically encountered in MCDM problems.

Step 1: Let $A = (A_1, \ldots, A_n)$ represent a group of n alternatives and $X = (\ddot{x}_1, \ldots, \ddot{x}_m)$ depict a group of m criteria to be appraised, whose weight vector is $w = (w_1, \ldots, w_m)^T$, satisfying $w_z > 0(z = 1, \ldots, m)$ and $\sum_(z = 1)^m w_z = 1$, where w_j denotes the importance degree of criterion x_j. The appraisal of A_i through criteria x_j in crisp numbers $\alpha^t_{\hat{k}}(t = 1, \ldots, n)(\hat{k} = 1, \ldots, \check{l})$ and IFNs $\alpha^t_{\hat{k}} = \mu_(\alpha^t_{\hat{k}}), \nu_(\alpha^t_{\hat{k}}))(t = 1, \ldots, n)(\hat{k} = \check{l} + l, \ldots, m)$.

All $\alpha^t_{\hat{k}}(t = 1, \ldots, n)(\hat{k} = 1, \ldots, \check{l})$ and IFNs $\alpha^t_{\hat{k}} = (\mu_{\alpha^t_{\hat{k}}}, \nu_{\alpha^t_{\hat{k}}})(t = 1, \ldots, n)(\hat{k} = 1 + l, \ldots, m)$ are contained via aggregated intuitionistic decision matrix $AIFDM = (a\alpha^t_{\hat{k}})_{nxm}$ (see Table 9).

If all criteria, X, are of the same type (i.e. benefit criteria, BN), according to equation (27) and (31), we obtain the matrix $Y_1 = (B^t_{\hat{k}})_{nxl}$, and through equations (28), (32), and (33) we obtain the matrix $Y_2 = (\beta^i_j)_{nx(m-l)}$.

	$\ddot{x}_1$	$\dots$	$\ddot{x}_l$	$\ddot{x}_1$	$\dots$	$\ddot{x}_m$
A_1	$a_{1,1}$	$\dots$	$a_{1,\check{l}}$	$(\mu_{1,\check{l}+1}, \nu_{1,\check{l}+1})$	$\dots$	$(\mu_{1,m}, \nu_{1,m})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_n	$a_{n,1}$	$\dots$	$a_{n,\check{l}}$	$(\mu_{n,\check{l}+1}, \nu_{n,\check{l}+1})$	$\dots$	$(\mu_{1,m}, \nu_{1,m})$

Table 9. *AIFDM*

On the other hand, if all criteria are of type cost (C), using equation (29) we obtain the matrix $Z_1 = (B_{\hat{k}}^{\prime t})_{nxl}$, and based on equation (30) we obtain the matrix $Z_2 = (\beta_{\hat{k}}^{\prime t})_{nx(m-l)}$. However, MCDM problems usually deal with both benefit and cost criteria at the same time. Thus, matrices Y_1 and Z_1 can be combined into matrix $P = (\rho_{\hat{k}}^t)_{nxl}$, where

$$\rho_{\hat{k}}^t = \begin{cases} B_{\hat{k}}^t, & \hat{k} \in BN; \\ B_{\hat{k}}^{\prime t}, & \hat{k} \in C. \end{cases} \tag{45}$$

Note that matrix ρ_j^i is normalized and matrices Y_2 and Z_2 can be combined into matrix $R = (\gamma_{\hat{k}}^i)_{nx(m-l)}$, where

$$\gamma_{\hat{k}}^t = \begin{cases} \beta_{\hat{k}}^t, & \hat{k} \in BN; \\ \beta_{\hat{k}}^{\prime t}, & \hat{k} \in C. \end{cases} \tag{46}$$

Step 2: Apply the AIFDA technique to aggregate all performance values $\rho_{\hat{k}}^t(t = 1, \dots, n)(\hat{k} = 1, \dots, \check{l})$, and $\gamma_{\hat{k}}^t(t = 1, \dots, n)(k = \check{l} + 1, \dots, m)$ to obtain aggregated intuitionistic fuzzy index of similarity, $AIFIS_i$, corresponding to alternative $A_i(i = 1, \dots, n)$.

$$\begin{aligned} &AIFIS_i(\rho_1^t, \dots, \rho_{\check{l}}^t, \gamma_{\check{l}+1}^t, \dots, \gamma_m^t) \\ &= (1 - (1 - \prod_{\hat{k}=\check{l}+1}^{m} (\mu_{\gamma_{\hat{k}}^t})^{w_z})^{(\prod_{\hat{k}=1}^{\check{l}} (\rho_{\hat{k}}^t)^{w_z})}, \\ &(1 - \prod_{\hat{k}=\check{l}+1}^{m} (1 - \nu_{\gamma_{\hat{k}}^t})^{w_z})^{(\prod_{\hat{k}=1}^{\check{l}} (\rho_{\hat{k}}^t)^{w_z})}). \end{aligned} \tag{47}$$

Step 3: Compute the index of similarity. To rank any two IFNs, such as stated by [17] the score function $S(AIFIS_i) = \mu_(AIFIS_i) - \nu_(AIFIS_i)$ to obtain the score value of an IFN. We used this score to obtain the overall index of similarity, IS_i, for alternative $A_i(i = 1, \dots, n)$.

Step 4: Rank alternatives $A_i(i = 1, \dots, n)$ in descending order according to the values of IS_i. Once again, we provide a numerical example to illustrate the aforementioned procedure.

Example 4.2. Application of AIFDA Technique. For this case, we apply the AIFDA technique in the evaluation and selection process of an agricultural tractor. Selection and evaluation criteria characterizing farm tractors are selected by revising existing literature and questioning farmers and machinery salespersons, but also by assuming some characteristics of criteria for AMT selection and evaluation. To conduct the AIFDA analysis, we researched in governmental agencies such as SAGARPA (Secretariat of Agriculture, Livestock, Rural Development, Fisheries, and Food) and SEDER (Secretariat of Rural Development). As for advanced technology in agriculture (ATA) vendors, we visited the ATA user at their business locations. The problem consisted of selecting a tractor among six alternatives. Evaluation and selection criteria had both qualitative and quantitative information to be integrated into the same evaluation and selection process. Analyzed criteria are defined as follows.

- **Original cost of tractor** ($\ddot{x}_1$): depicting the amount of money, stated in Mexican pesos, that the rural association must pay for the tractor in a onetime payment. The lowest possible price is desired (cost criterion) and would be represented by a crisp number.
- **Rated power** ($\ddot{x}_2$): Belong to engine power. This criterion is stated in HP (horsepower); the maximum value is desired (benefit criterion), and it would be given by a crisp value.
- **Cylinders** ($\ddot{x}_3$): are the quantity of cylinders in the machine. It would be measured with a crisp value, and lowest values are desired (cost criterion), since number of cylinders are associated with diesel consumption.
- **Displacement** ($\ddot{x}_4$): volume swept by all pistons inside the cylinders of an ignition engine in a single movement, from top dead center (TDC) to bottom dead center (BDC). Depicting in crisp number, and the lowest values are desired (cost criterion).
- **Operators safety** ($\ddot{x}_5$): Belong a subjective value (fuzzy) that indicated DM assessments regarding operators safety. Maximum values are desired (benefit criterion).
- **Service** ($\ddot{x}_6$): After-sale customer service from suppliers. Subjective value (fuzzy) representing DMs assessments regarding services to be obtained from the purchase. Maximum values are desired (benefit criterion).

The first four attributes are quantitative and can be measured directly through units (tangible); however, the last two criteria are qualitative (intangible).

Also, in this example the decision group was integrated by five DMs, whose importance degree is shown in Table 10. Note that Table 2 introduced in ***Example 4.1*** was used for rating of k_{th}DM. As in the first example, all DMs provided their opinions (assessments) regarding the importance degree of each criterion for selecting an agricultural tractor.

Decision maker	Linguistic Term	IFN	Weight
1	I	(0.35,0.60)	0.12
2	Ct	(0.50,0.45)	0.17
3	Im	(0.75,0.20)	0.25
4	A	(0.90,0.05)	0.29
5	Ct	(0.50,0.45)	0.17

TABLE 10. Importance Degree of Decision Makers

$$W\{\ddot{x}_1, \ddot{x}_2, \ddot{x}_3, \ddot{x}_4, \ddot{x}_5, \ddot{x}_6\} = \begin{bmatrix} (0.801, 0.141) \\ (0.425, 0.525) \\ (0.490, 0.452) \\ (0.553, 0.392) \\ (0.755, 0.180) \\ (0.666, 0.528) \end{bmatrix}^T$$

All opinions are integrated by means of equation (38). Besides, Table 11 shows assessments from DMs.

Decision Maker	Criteria					
	$\ddot{x}_1$	$\ddot{x}_2$	$\ddot{x}_3$	$\ddot{x}_4$	$\ddot{x}_5$	$\ddot{x}_6$
DM_1	Im	I	VI	I	Im	Im
DM_2	Im	A	I	A	Im	A
DM_3	VS	I	I	I	D	D
DM_4	Im	A	Im	Im	A	I
DM_5	Im	I	I	A	Im	A

Table 11. Importance Degree of Criteria for Selecting a CNC Milling Machine

Then, importance degree of each criterion is computed by equation (39)

$$w_z = \left[w_{\ddot{x}_1} = 0.215, w_{\ddot{x}_2=}0.116, w_{\ddot{x}_3} = 0.136, w_{\ddot{x}_4} = 0.152, w_{\ddot{x}_5} = 0.203, w_{\ddot{x}_6} = 0.178\right].$$

$$\sum_{z=1}^{6} w_z = 0.215 + 0.116 + 0.136 + 0.152 + 0.203 + 0.178 = 1.$$

Successively, since $\ddot{x}_5$ and $\ddot{x}_6$ are two qualitative criteria, we employed Table 5. Thus, DMs can provide their assessments on criteria in common terms, such as linguistic expressions.

	Decision Maker									
	DM_1		DM_2		DM_3		DM_4		DM_5	
Alternatives	$\ddot{x}_5$	$\ddot{x}_6$	$\ddot{x}_5$	$\ddot{x}_6$	$\ddot{x}_5$	$\ddot{x}_6$	$\ddot{x}_5$	$\ddot{x}_6$	$\ddot{x}_5$	$\ddot{x}_6$
A_1	G	G	R	G	VL	EP	VL	E	VP	ML
A_2	R	T	VL	T	MG	VL	L	VL	MG	L
A_3	A	MH	T	A	EP	L	L	R	T	L
A_4	R	T	MG	MP	A	ML	E	L	F	MG
A_5	R	A	L	MG	EX	ML	EP	A	MP	T
A_6	MG	VL	MG	L	EP	EH	F	T	A	T

Table 12. Ratings of Alternatives

Table 12 displays the assessments of each DM expressed in linguistic terms.Once we collected and integrated information provided by DMs, we applied the AIFDA technique, (***see Example 4.1***) to map the assessments provided by DMs, following the aforementioned steps.

Thus, the final intuitionistic decision matrix, FIFDM, in the last two criteria$(\ddot{x}_5, \ddot{x}_6)$ can be expressed as

$$FIFDM = \begin{pmatrix} (0.371, 0.476) & (0.650, 0.226) \\ (0.465, 0.410) & (0.360, 0.497) \\ (0.448, 0.439) & (0.505, 0.327) \\ (0.487, 0.397) & (0.686, 0.191) \\ (0.686, 0.191) & (0.537, 0.359) \\ (0.456, 0.452) & (0.641, 0.236) \end{pmatrix}$$

Step 1: Let $A = (A_1, \ldots, A_6)$ represent the six alternatives and $X = (\ddot{x}_1, \ldots, \ddot{x}_m)$ depict a group of m criteria to be appraised, whose weight vector is $w = (0.216, 0.114, 0.132, 0.149, 0.205, 0.183)^T$, satisfying $w_z > 0(z = 1, \ldots, 6)$ and $\sum_{z=1}^{6} w_z = 1$, where w_z denotes the importance degree of criterion $x_{\hat{k}}$. The appraisal of A_i through criteria x_j in crisp numbers $a_{\hat{k}}^t(t = 1, \ldots, 6)(\hat{k} = 1, \ldots, 4)$ and IFNs $\alpha_{\hat{k}}^t = \mu_(\alpha_{\hat{k}}^t, \nu_(\alpha_{\hat{k}}^t(t = 1, \ldots, 6)(\hat{k} = \check{l} + l, \ldots, m)$.

All $a_{\hat{k}}^t(t = 1, \ldots, 6)(\hat{k} = 1, \ldots, 4)$ and IFNs $\alpha_{\hat{k}}^t = \mu_(\alpha_{\hat{k}}^t), \nu_(\alpha_{\hat{k}}^t))(t = 1, \ldots, n)(\hat{k} = \check{l}+l, \ldots, m)$. are contained via aggregated intuitionistic decision matrix $AIFDM = ((a\alpha)_{\hat{k}}^t)_{nxm}$ (see Table 13).

	$\ddot{x}_1$	$\ddot{x}_2$
A_1	(0.371,0.476)	(0.650,0.226)
A_2	(0.465,0.410)	(0.360,0.497)
A_3	(0.448,0.439)	(0.505,0.372)
A_4	(0.487,0.397)	(0.447,0.436)
A_5	(0.686,0.191)	(0.537,0.359)
A_6	(0.456,0.452)	(0.641,0.419)

Table 13. AIFDM

And P combined with R is shown in Table 14.

	$\ddot{x}_1$	$\ddot{x}_2$	$\ddot{x}_3$	$\ddot{x}_4$	$\ddot{x}_5$	$\ddot{x}_6$
A_1	(748,223.0)	(80)	(4)	(4530)	(0.371,0.476)	(0.650,0.226)
A_2	(520,730.0)	(75)	(4)	(4500	(0.465,0.410)	(0.360,0.497)
A_3	(425,232.5)	(80)	(4)	(4070)	(0.448,0.439)	(0.505,0.372)
A_4	(649,477.5)	(100)	(6)	(6000)	(0.487,0.397)	(0.447,0.436)
A_5	(585,305.0)	(95)	(4)	(4000)	(0.686,0.191)	(0.537,0.359)
A_6	(702,590.0)	(110)	(6)	(6000)	(0.456,0.452)	(0.641,0.419)

Table 14. Aggregated Intuitionistic Fuzzy Combined $PandR$

Criteria $\ddot{x}_1, \ddot{x}_3$, and $\ddot{x}_4$ are cost criteria, whereas $\ddot{x}_2, \ddot{x}_5$, and $\ddot{x}_6$ are benefit criteria. Thus, we combined them into matrix $P = (\rho_{\hat{k}}^t)_{6x4}$ and $R = (\gamma_{\hat{k}}^t)_{6x2}$, where

$$\psi^* = ((425, 232.5), (110), (4), (4000), (0.686, 0.191), (0.650, 0.226)), \quad (48)$$

Step 2: We used AIFDA technique to aggregate all performance values $\rho_{\hat{k}}^t(t = 1, \ldots, 6)(\hat{k} = 1, \ldots, 4)$ and $\gamma_{\hat{k}}^t(t = 1, \ldots, 6)(\hat{k} = 5, 6)$ to obtain the aggregated intuitionistic fuzzy index of similarity, $AIFIS_i$, corresponding to alternative $A_i(i = 1, \ldots, 6)$, and it is calculated by equation (47)

$$AIFIS_i(\rho_1^t, \rho_2^t, \rho_3^t, \rho_4^t, \gamma_1^t, \gamma_2^t) = (1 - (1 - \prod_{\hat{k}=5}^{6} (\mu_{\gamma_{\hat{k}}^t})^{w_z})^{(\prod_{\hat{k}=1}^{4} (\rho_{\hat{k}}^4)^{w_z})}, (1 - \prod_{\hat{k}=5}^{6} (1 - \nu_{\gamma_{\hat{k}}^t})^{w_z})^{(\prod_{\hat{k}=1}^{4} (\rho_{\hat{k}}^4)^{w_z})}) \quad (49)$$

Therefore,

$$AIFIS_1 = (0.907, 0.064), AIFIS_2 = (0.837, 0.127), AIFIS_3 = (0.868, 0.114),$$

$$AIFIS_4 = (0.919, 0.068), AIFIS_5 = (0.972, 0.028), AIFIS_6 = (0.957, 0.041).$$

Step 3: Then, we computed the index of similarity, IS_i, for alternative $A_i(i = 1, , 6)$.

$$[IS_1 = 0.843, IS_2 = 0.710, IS_3 = 0.754, IS_4 = 0.851, IS_5 = 0.944, IS_6 = 0.916] .$$

Step 4: Finally, we sorted alternatives $A_i(i = 1, \ldots, 6)$ in descending order according to obtained IS_i values. Since we obtained $A_5 > A_6 > A_4 > A_1 > A_3 > A_2$, the $\mathbf{A_5}$ was selected as the best alternative (best tractor) to solve our problem. As a result, AIFDA is proficient in handling crisp values without transforming them into another type of data or terms, which allows to preserving reliability of results.

4.3. **Discussion of Results.** In this section, we offer a comparative analysis of both IFDA and AIFDA with similar MCDM approaches, specifically TOPSIS and AHP to verify the reliability of our proposed techniques. In order to confirm the effectiveness of both IFDA and AIFDA techniques, we conducted a comparative analysis with respect to the intuitionistic fuzzy TOPSIS (IF-TOPSIS) [9] and the intuitionistic fuzzy AHP (IF-AHP)[67], by using the same evaluation data for all cases and doing the necessary adjustments for the analysis. The results obtained from the first example provided in this research (IFDA approach) are presented in Table 15.

Alternatives	IFDA	IF-TOPSIS	IF-AHP
W	3	3	3
Y	2	2	2
Z	1	1	1

Table 15. Comparison Analysis- Ranking-A

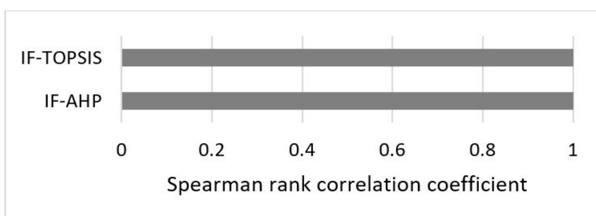

Figure 1. Correlation Between IFDAś Final Ranking and Other MCDM Techniques

As it can be observed, the ranking is the same for all methods, and alternative **Z** (CNC milling machine) is selected as the best one. Also, to illustrate correspondence between IFDA rankings and the other MCDM approaches, we used the Spearmans rank correlation coefficient [55] as scale for evaluating the relationship between two ranks.

The Spearmans coefficient is a real number between -1 and 1, where 1 indicates a one-to-one correspondence of compared rankings, whereas -1 shows opposition of the ranks. Figure 1 shows the Spearmans rank correlation coefficients obtained for our first example. The results show that rankings of the other two MCDM techniques TOPSIS and AHP are identical to the rankings produced by our technique.

Alternatives	IFDA	IF-TOPSIS	IF-AHP
A_1	5	3	4
A_2	4	5	6
A_3	6	4	5
A_4	2	6	3
A_5	1	1	1
A_6	3	2	2

Table 16. Comparison Analysis- Ranking-B

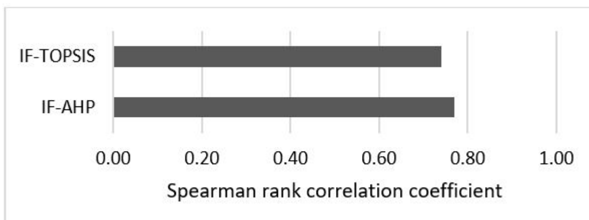

Figure 2. Correlation Between AIFDAś Final Ranking and Other MCDM Techniques

For our second example using AIFDA, we conducted the same analysis. Table 16 shows the rankings produced by the compared approaches. Once again, we employed the Spearmans rank correlation coefficient to evaluate the relationship between two rankings, and such results are shown in Figure 2. As shown in Table 16, IF-AHP, IF-TOPSIS, and AIFDA approaches suggested the same best alternative (A_5), although there is a slightly difference in the final ranking of the other alternatives.

Based on the Spearmans rank results is demonstrated that there is a significant correspondence between our approach and the two MCDM approaches most commonly reported in the literature as AHP and TOPSIS. However, we think our proposal has some advantages in relation to the other two. First, in comparison with AHP, our technique does not need pairwise comparisons which facilities its computation. Besides, there is still some issues in the fuzzy versions of AHP related to the reciprocity and transitivity conditions. Hence, the DMs opinion will most likely be inconsistent [54, 61]. Moreover, for the case if the consistency ratio (CR) is more than 10 percent, the method requires to repeat the pairwise comparisons to meet this validation (Saaty, 1980). Additionally, in relation with TOPSIS, this last one is criticized by the use of the Euclidean distance [4, 57, 58]; if other type of distance (e.g. Mahalanobis) is used then the final ranking might be different. As a result, TOPSIS is very dependent on the type of distance employed. Besides, we argue that our approaches are simpler to apply since the computation involved are not complex at all (basic math operations), which may extend their potential applications to more extensive areas. From the results, we can state our techniques can provide similar results to the most reported techniques in the literature. Thus, DA can be taken as a convenient MCDM technique since its computation results quite simple. It is important to mention that although IFDA and AIFDA raise their level of complexity when considering both types of information, the other techniques also do so, however, the nature of DA is maintained.

5. Conclusions

The multi-criteria decision making problem is considered as a complex subject including typically both quantitative and qualitative criteria. In this paper, we have proposed two techniques that have the potential to manipulate both qualitative and quantitative criteria by integrating the participation of numerous decision makers, whose knowledge is often expressed by personal opinions or preferences. On one hand, IFDA might provide the guidelines and directions to support decisions when available information belongs to the fuzzy environment.

On the other hand, AIFDA might offer support in cases when MDCM problems include both crisp and fuzzy information at the same time. Similarly, AIFDA avoids loss and modification of the original decision information, and thus guarantees reliability of the MCDM technique and consistency of the final decision results. Also, we provided an example for both, IFDA and AIFDA to illustrate their feasibility and applicability. Finally, results from the comparative analysis with respect to AHP and TOPSIS showed that the two proposed methods are effective and suitable for MCDM problems. Though the results are promising, we will seek to extend our proposals to handle other types of information with additional generalizations of the fuzzy sets. In addition, the consideration of the comparisons must be strengthened. Future work will be centered on evaluation the effects of these approaches on medium-large MCDM problems.

Nomenclature

- IFDA: intuitionistic fuzzy dimensional analysis
- AIFDA: aggregated intuitionistic fuzzy dimensional analysis
- $IFIS_i$: intuitionistic fuzzy index of similarity for alternative i
- $AIFIS_i$: aggregated intuitionistic fuzzy index of similarity for alternative i
- IFDM: intuitionistic decision matrix
- FIFDM: final intuitionistic decision matrix
- $a^t_{\hat{k}}$: crisp evaluation of criterion j for alternative i
- $S^*_{\hat{k}}$: crisp value of the ideal alternative for criterion j
- $\alpha^t_{\hat{k}}$: intuitionistic fuzzy evaluation of criterion j for alternative i
- $\psi^t_{\hat{k}}$: intuitionistic fuzzy value of the ideal alternative for criterion j
- $\rho^t_{\hat{k}}$: vector of crisp benefit and cost criteria for alternative i
- $\gamma^t_{\hat{k}}$: vector of intuitionistic fuzzy benefit and cost criteria for alternative i

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