Electr Eng
DOI 10.1007/s00202-017-0509-3

ORIGINAL PAPER

Process capability index for AC transformer under electrical harmonics

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Received: 3 September 2016 / Accepted: 23 January 2017

Abstract Nowadays, the quality of electronic devices is subjected to many variables such as electrical, thermal, mechanical and environmental factors. Voltage variation induced by electrical harmonics presented in the power lines is one of the main causes for electronic products to reduce their quality and performance in an operational environment. In this study, more significant variables are addressed, and a comparative study between voltage variations with and without electrical harmonics is presented. The case study presented in this paper is based on a transformer under voltage scenarios proposed in this study. The corresponding Cpk index in the transformer was estimated using a non-normal tool. The results of the case study showed that laptop computer under electrical harmonics has a lower Cpk rank and that Cpk value impacts directly the lifetime and performance of the device.

Keywords Process capability index · Electrical harmonics · Power quality issues · Inverse power law · Transformers · Reliability

1 Introduction

Today quality of any electronic device (ED) is subjected to many random factors such as manufacturing defects, power quality (PQ) and environmental effects. Some quality problems with those types of devices are produced by the alternate current (AC) power supply which does not provide a constant voltage in the majority of the time. This phenomenon affects directly to internal components, which are more vulnerable to voltage variations. One of the main causes of voltage variations in power lines is due to electrical harmonics (EH), which induces a corruption in the AC waveform and increases the voltage levels in the device. One of the devices which are more susceptible to EH are the transformers; that sensitivity may affect the reliability and the performance of the equipment, and can even modify the conditions of the process under which the equipment is manufactured.

Many authors have investigated the sources and effects of EH in the quality, reliability and performance of ED. Gosh et al. [4], De la Rosa [1], Wagner et al. [14] identified and classified harmonic sources in power lines and their main effects in ED. Effects of EH in transformers can be seen in [2,3,5,6,8,9,12–14].

In this paper, we use the process capability index (Cpk) in order to measure the ability of the ED to work under specification limits that was designed when an electrical variation is submitted. A case study is presented and compared the process with and without presence of EH in the process.

Finally, this paper is organized as follows: Section 2 presents a background of EH and its mathematical expression. Section 3 presents the statistical methodology followed in this paper. Section 4 presents the case study performed on laptop computers. The last section provides concluding remarks.

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2 Background of electrical harmonics

Today, one of the most accepted classifications that describe the types of electrical variations in the power line can be seen in [11]. Based on this classification [9] proposed an failure and mode effect analysis (FMEA) to determinate which electrical variation has more probability to damage an ED. The results showed that EH are more likely to damage the behavior and performance of ED under a real environment.

By definition EH are any nonlinear current or voltage patterns in an electrical distribution system. An EH should not be thought of as an acoustic or vibration harmonic, but simply as any electrical device that draws current unproportionally to voltage. EH are commonly produced by devices that rectifies AC voltage into a direct current (DC) voltage. Common harmonic-producing devices are lighting, computers and variable frequency drivers. An example of nonlinear vs linear loads can be seen in Fig. 1.

The EH can be expressed by the Fourier series; this is defined by:

$$y\left(t\right)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\left(n\omega_0 t\right)+b_n\sin\left(n\omega_0 t\right)\right] \tag{1}$$

By setting $\omega_0=2\pi f_0$, $\omega_n=n\omega_0$ and substituting in Eq. (1) the following is obtained:

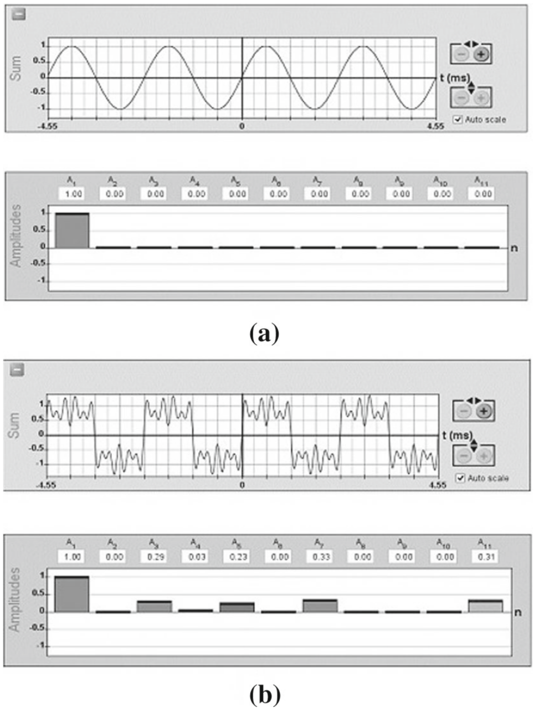

Fig. 1 Waveform produced in the electronic loads due to electrical harmonics. **a** Waveform and harmonics amplitude of linear loads. **b** Waveform and harmonics amplitude of nonlinear loads

$$y\left(t\right)=C_0+\sum_{n=1}^{\infty}C_n\mathrm{Cos}\left(w_n t-\theta_n\right) \tag{2}$$

Equation (2) is the compact form of the Fourier series, and it is widely employed to describe the AC waveform signal in magnitude and phase parameters. Since Eq. (2) represents an infinity summation of terms, the accuracy of this equation is defined by the number of harmonics added into the analysis. Nevertheless, in the real world measuring and computing a large number of EH can be expensive and unnecessary. Schneider electrics [10] established that higher-order harmonics (up to 50) are ineligible and are not necessary to be measured. However, if a good accuracy is required, harmonics of order up to 30 must be considered. Additionally, useful harmonics are of order 3, 5, 7, 11 and 13. Following the above recommendation Eq. (2) can be rewritten as:

$$\begin{aligned}y\left(t\right)_{\text{equivalent}}&=C_0+C_3\mathrm{Cos}\left(240\cdot t\right)\\&\quad+C_5\mathrm{Cos}\left(360\cdot t\right)+C_7\mathrm{Cos}\left(480\cdot t\right)\\&\quad+C_{11}\mathrm{Cos}\left(720\cdot t\right)+C_{13}\mathrm{Cos}\left(840\cdot t\right)\end{aligned} \tag{3}$$

In the following section, we use Eq. (3) in the statistical analysis in order to calculate the Cpk of the process when the EH are present in the component.

3 Statistical analysis

In this section, we provide all mathematical tools to measure the quality of the product under the voltage variation. In this case, we need to determine how to measure the quality of the product when it is subjected to EH. To measure the quality of the electronic product we use the operating days when the device is in use with the consumer. Based on information provided by Meeker and Escobar [7], the behavior of ED follows an Weibull distribution, and also based on the inverse power law (IPL) function, the behavior of a component under constant voltage is given by:

$$f\left(t,V\right)=\beta KV^n\left(KV^n t\right)^{\beta-1}e^{-\left(KV^n t\right)^{\beta}} \tag{4}$$

where $f(t,V)$ represents a quantifiable life measure, such as mean time to failure (MTTF). V represents the voltage level submitted in the component; finally, K and n are the parameters that describe the behavior of ED under constant voltage, and all parameters can be estimated via the maximum likelihood function. The percentile of the device under analysis can be calculated from the cumulative distribution function (CDF) of Eq. (4), which is expressed as:

$$t_{\text{percst}}=\frac{\mathrm{Ln}\left[1-F\left(t,V\right)\right]^{\frac{1}{\beta}}}{KV^n} \tag{5}$$

where $F(t, V)$ represents the cumulative distribution function (CDF) derived from Eq. (4).

In the electronic industry, non-normal lifetime of ED is very common due to the behavior of the semiconductors. Thus, the Cpk can be calculated as follows:

$$\mathrm{Cpk}_{\mathrm{cst}} = \frac{t_{\mathrm{percst}_{0.5}} - \mathrm{LSL}}{t_{\mathrm{percst}_{0.5}} - t_{\mathrm{percst}_{0.00135}}} \tag{6}$$

where LSL represents the lower specification limit. In this case only LSL is needed to calculate the C_{pk} because if the ED exceeds the minimal time, the quality of ED is better.

For the case when the device is under voltage variations in the power line, the model established in Eq. (4) needs to be transformed to capture the voltage variations based on the information provided by Méndez et al. [8,9], and the model presented in Eq. (4) can be modified as follows:

$$f\left(t, y\left(t\right)\right) = \left\{\beta\left[\frac{y\left(t\right)}{a}\right]^{n}\left[\int_{0}^{t}\left[\frac{y\left(u\right)}{a}\right]^{n}\mathrm{d}u\right]^{\beta-1}\right\}e^{-\left[\int_{0}^{t}\left[\frac{y(u)}{a}\right]^{n}\mathrm{d}u\right]^{\beta}} \tag{7}$$

in which α is a characteristic parameter of each device under analysis, and n measures the effect of the voltage in the device and $y\left(t\right) = y\left(u\right)$ represents a parametric function that describes the voltage profile in time-varying scenario.

The percentile of Eq. (7) can be calculated as follows:

$$t_{\mathrm{pervar}} = \frac{\mathrm{Ln}\left[1 - F\left(t, y\left(t\right)\right)\right]}{\int_{0}^{t}\left(\frac{y(u)}{a}\right)^{n}} \tag{8}$$

where $F\left(t, y\left(t\right)\right)$ represents the CDF derived from Eq. (7).

Finally, the Cpk for voltage variations in the process can be calculated as follows:

$$\mathrm{Cpk}_{\mathrm{var}} = \frac{t_{\mathrm{pervar}_{0.5}} - \mathrm{LSL}}{t_{\mathrm{pervar}_{0.5}} - t_{\mathrm{pervar}_{0.00135}}} \tag{9}$$

4 Case study

In the case study, we made a comparison between voltage variations presented in the power line versus an assumption of constant voltage in the line. The ED selected for this case study are AC transformers which happen to be one of the most used ED in the present time (Fig. 2).

4.1 Preliminary

In the case of constant voltage scenario, an accelerated life testing (ALT) was performed in the AC transformer in order to obtain the failure time of the units under analysis. In this voltage scenario, we used a one-factor experiment design with 30 voltage levels (150 VAC, 185 VAC and 210 VAC) with 10 replicates for each level. The levels selected represent 1.22, 1.48 and 1.66 times the nominal voltage admitted by the AC transformer. The failure time data obtained in this experiment were recorded in days.

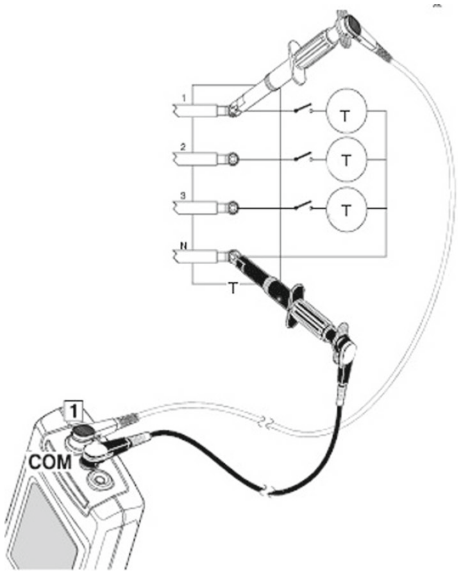

Fig. 2 Connection diagram to obtain the EH in the AC transformer

For the time-varying voltage scenario, an ALT was performed in the AC transformer; the time-varying voltage scenario was generated via Eq. (3). In order to obtain the amplitude of each harmonic presented in Eq. (3) and the RMS voltage value 30 measurements with the following replicates 150 VAC with 10 devices, 185 VAC with 10 devices and 210 VAC with 10 devices, all measurements were taken with FLUKE® 43b. The measurements were taken according to standard IEC 1000-3-2; in Fig. 3 is presented a connection diagram of FLUKE® 43b and the transformers. Additionally, in Table 1 are presented the results of the measurements of EH.

4.2 Cpk analysis for AC transformer under constant voltage

Based on the preliminaries of the case study, the AC transformers were subjected under an ALT and the failure times obtained for these procedures can be seen in Table 2.

To estimate the parameters β, K and n in the model established in Eq. 4, the maximum likelihood estimator (MLE) is used, and the results of the estimation were:

$$\begin{aligned} \beta &= 1.8116 \\ k &= 3.0187e^{-7} \\ n &= 3.3642 \end{aligned} \tag{10}$$

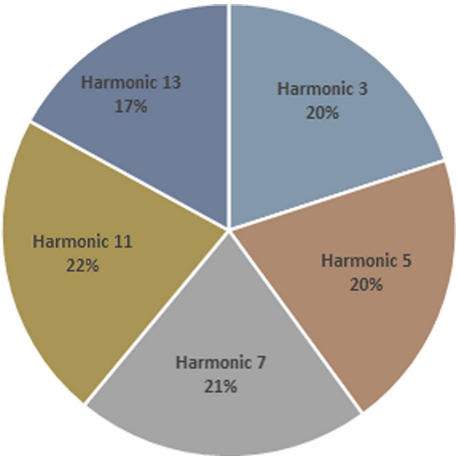

Fig. 3 Connection diagram to obtain the EH in the AC transformer

Table 1 Harmonics measured for each transformer under electrical harmonics

Unit	VAC	Harmonics					RMS value
		3	5	7	11	13	
1	150	14	23	12	19	13	154.593
2	150	3	19	3	8	4	151.522
3	150	11	5	2	11	3	150.930
4	150	2	17	7	6	12	151.730
5	150	21	21	20	9	11	154.868
6	150	14	17	3	20	8	153.160
7	150	18	4	18	17	20	154.444
8	150	20	13	6	24	17	154.822
9	150	10	21	12	23	3	154.023
10	150	18	2	6	15	15	152.689
11	185	19	18	5	19	15	188.470
12	185	23	23	1	4	13	188.332
13	185	5	5	14	20	9	186.955
14	185	14	2	6	8	9	186.027
15	185	12	9	18	17	13	187.702
16	185	14	18	2	14	15	187.537
17	185	18	15	15	23	15	189.085
18	185	1	9	2	12	5	185.688
19	185	17	1	7	3	2	185.949
20	185	19	6	19	16	2	187.731
21	210	13	14	14	24	3	213.162
22	210	3	6	23	7	1	211.858
23	210	10	10	13	1	9	210.855
24	210	24	6	0	15	22	212.002
25	210	19	17	19	12	9	213.427
26	210	5	17	23	18	14	214.007
27	210	2	24	20	8	15	213.049
28	210	2	23	17	1	18	212.713
29	210	8	11	24	24	6	213.244
30	210	20	8	24	19	10	213.544

Table 2 Failure times of transformer under ALT and constant voltage

Unit	Stress level (VAC)	Failure times (days)
1	150	154.247
2	150	561.192
3	150	616.395
4	150	952.158
5	150	1478.151
6	150	1556.866
7	150	1607.536
8	150	1821.316
9	150	1828.624
10	150	1985.781
11	185	26.064
12	185	180.382
13	185	262.421
14	185	607.317
15	185	642.631
16	185	718.582
17	185	860.597
18	185	996.653
19	185	1143.219
20	185	1379.666
21	210	125.962
22	210	229.750
23	210	234.110
24	210	262.541
25	210	290.168
26	210	366.946
27	210	376.379
28	210	597.124
29	210	710.827
30	210	713.012

To obtain the Cpk of transformer under constant voltage, it is necessary to transform the accelerated data of Table 1 to the real failure time. To do that, the accelerator factor needs to be calculated, and this factor is calculated as follows:

$$A_F = \left(\frac{V_A}{V_u}\right)^n \tag{11}$$

where A_F is the relation between the real life and the accelerated life. V_A = is the accelerated voltage level; V_U = is the use voltage level. n is obtained from Eq. (4).

Based on the data in Table 2 and the results obtained in (10), the real lifetime of the device is calculated as follows $A_F \cdot Failure\ time$; the results are shown in Table 3; consider a $V_U = 110V$.

Based on the results presented in Table 3, the results obtained in (10) and by setting the LSL = 365 days which

Table 3 Failure times of transformer at real condition time under constant voltage

Unit	Failure time (days)
1	437.894
2	1593.177
3	1749.896
4	2703.098
5	4196.349
6	4419.816
7	4563.664
8	5170.567
9	5191.315
10	5637.471
11	149.831
12	1036.954
13	1508.564
14	3491.254
15	3694.261
16	4130.874
17	4947.272
18	5729.408
19	6571.966
20	7931.218
21	1109.164
22	2023.079
23	2061.469
24	2311.821
25	2555.094
26	3231.167
27	3314.229
28	5258.007
29	6259.228
30	6278.466

Table 4 Failure times of AC transformer under EH

Unit	Stress level	Failure time (days)
1	154.593	108.228
2	151.522	234.922
3	150.93	574.648
4	151.73	673.065
5	154.868	796.372
6	153.16	1121.873
7	154.444	1165.625
8	154.822	1209.269
9	154.023	1526.648
10	152.689	1970.570
11	188.47	159.012
12	188.332	167.674
13	186.955	258.928
14	186.027	260.785
15	187.702	507.996
16	187.537	670.646
17	189.085	747.649
18	185.688	761.932
19	185.949	1252.033
20	187.731	2228.103
21	213.162	54.470
22	211.858	200.474
23	210.855	294.344
24	212.002	381.254
25	213.427	420.362
26	214.007	509.460
27	213.049	518.741
28	212.713	891.143
29	213.244	937.927
30	213.544	1031.821

is equal to 1 year of warranty of the device, the C_pk of the transformer under constant voltage is:

$$\mathrm{Cpk}_{\mathrm{cst}} = \frac{3592 - 365}{3592 - 149} = 0.937 \tag{12}$$

Constant voltage scenario offers an approximation of the devices lifetime when it is under a real environment scenario, and is a widely accepted technique when a reliability and quality analysis is performed on an electrical or an ED. However, constant voltage conditions are difficult to achieve due to the large amount of interference that exist in the power lines. As a consequence, in the following subsection we performed an ALT of the same product and we introduced the EH to understand the devices performance and Cpk under electrical variation.

4.3 Cpk analysis for AC transformer under EH

Based on preliminaries of the experiment, the laptop computer was under a time-varying voltage ALT and under the EH obtained in Table 1. The failure times obtained for these procedures are shown in Table 4.

Following the same procedure as established in Sect. 4.2, the estimation of the parameters β, a and n based on the data presented in Table 4 is:

$$\begin{aligned} \beta &= 1.4609 \\ a &= 1.1456e+5 \\ n &= 1.02117 \end{aligned} \tag{13}$$

Following the same procedure as Sect. 4.2, we need to calculate the real failure time of the transformer under EH.

Table 5 Failure times of transformer at real condition time under EH

Unit	Stress level (VAC)	Failure time (days)
1	110	153.231
2	110	325.855
3	110	793.901
4	110	934.903
5	110	1129.559
6	110	1573.316
7	110	1648.676
8	110	1714.684
9	110	2153.298
10	110	2754.846
11	110	275.648
12	110	290.446
13	110	445.166
14	110	446.085
15	110	876.945
16	110	1156.685
17	110	1300.371
18	110	1300.894
19	110	2140.742
20	110	3846.945
21	110	107.081
22	110	391.641
23	110	572.240
24	110	745.324
25	110	827.421
26	110	1005.581
27	110	1019.218
28	110	1748.089
29	110	1844.554
30	110	2032.127

However, based on Eq. 11, the data presented in Table 4 and the results presented in 13, the results are shown in Table 5.

Based on the results presented in Table 5, the results obtained in (13) and by setting the LSL= 365 days which is equal to 1 year of warranty of the device, the C_pk of the transformer under EH voltage is:

$$\mathrm{Cpk}_{\mathrm{var}} = \frac{1180 - 365}{1180 - 93} = 0.75 \qquad (14)$$

In this part of the analysis, a lower value of the Cpk index is due to the presence of the electrical variations submitted into the device. Low Cpk shows that the transformer is susceptible to power variations, especially when these variations are caused by EH. Figure 3 shows the effect of each harmonic submitted into the laptop computer. The consequences of this susceptibility are reflected directly into the performance and the lifetime of the transformer.

5 Discussion

To verify both Cpk scenarios, we can compare the parameter value calculated for the reliability model into the constant and time-varying voltage scenario. The results of the estimation show a significant difference. In both scenarios the parameter n represents the effect (how much harm is accumulated throughout the piece when a voltage stress is applied).

For constant voltage scenario, parameter n has a big value in comparison with EH scenario and that means a high effect of the voltage into the device causes a low performance and a considerable reduction of its lifetime. But this affirmation is refuted in the EH scenario because the effects of parameter n are mitigated by the other parameters β and a.

Parameters a and k have the same meaning and represent a characteristic of each product, which can be resistance, power consumption, etc. In practice for AC transformers, parameters a and k are related to internal components such as cooling tubes, winding resistance and the oil's viscosity. By comparing the values of a and k, we can conclude that the internal components of AC transformer exposed to EH suffer greater degradation of their properties, which consequently leads to a low performance of the product as a whole.

The differences in the values of the parameters β, K, a and n affect directly the estimation of the lifetime of the AC transformer. As a way to show this difference, the MTTF value is calculated in both voltage scenarios. The MTTF of the laptop computer under constant voltage is calculated as $\mathrm{MTTF} = \frac{1}{KV^n} \cdot \Gamma\left(\frac{1}{\beta} + 1\right)$. As a consequence, by taking the values obtained in 10 and setting the operational voltage of the device as $120VAC$ the MTTF = 2695.19 days. The MTTF of the AC transformer under time-varying voltage is calculated as:

$$\begin{aligned} &\mathrm{MTTF}\,(x\,(t)) \\ &\quad = t\left[\left\{\beta\left[\frac{y\,(t)}{a}\right]^n\left[\int_0^t\left[\frac{y\,(u)}{a}\right]^n \mathrm{d}u\right]^{\beta-1}\right\}\right. \\ &\qquad \left.\times\, e^{-\left[\int_0^t\left[\frac{y(u)}{a}\right]^n \mathrm{d}u\right]^{\beta}}\right]\mathrm{d}t \end{aligned}$$

Thus by taking the values obtained in (13) and setting the voltage submitted in the device due to the EV presented in Eq. 3, the $\mathrm{MTTF}_{\mathrm{EH}} = 2373.81$ days. The differences obtained in the MTTF of both voltage scenarios showed that the AC transformers under EH reduce their lifetime by 592 days. That difference impacts directly in the estimation of the Cpk when any electrical variation is considered in the process. In

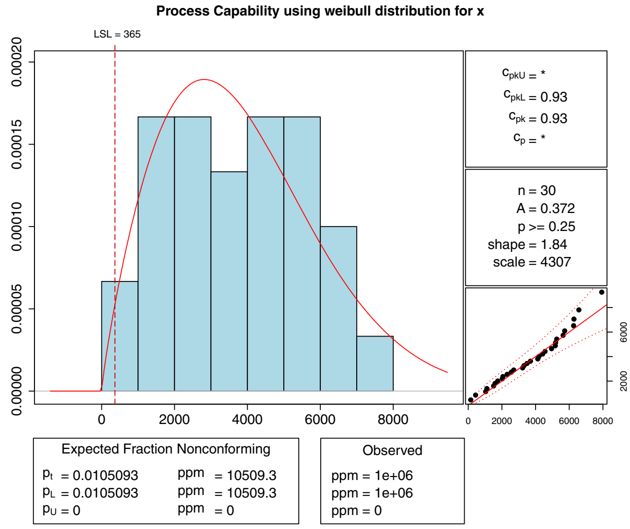

Fig. 4 Cpk for AC transformers under constant voltage

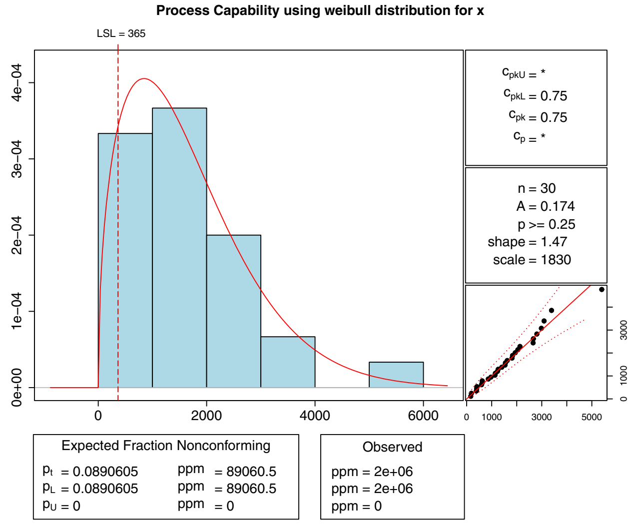

Fig. 5 Cpk for AC transformers under EH

this case, the EH can reduce the estimation of the lifetime of the device when it is in a real environment. Figure 4 shows the Cpk graph for constant voltage scenario. In Fig. 5 shows the Cpk graph for EH scenario.

6 Concluding remarks

The presented paper showed a quality comparative study for constant voltage versus time-varying voltage using the indices of Cpk for non-normal data. Based on the information provided by Seymour [11], the variable with greater effect (distortion into AC waveform) in the performance of electronic equipment is the EH. Since the EH affect the life of the device, its analysis involved quality and reliability tools. Results showed that ED are not designed for time-varying voltage scenarios; consequently, these affect the performance and lifetimes of the devices. This conclusion is supported by the results obtained in Eq. 12 and 14, and it is reflected in the MTTF calculated in discussion section for both voltage scenarios proposed in this study. This study differs with traditional analysis because the failure rate of the device under a real environment has a time-varying behavior.

The proposed analysis offers a way to understand the behavior of ED under real voltage environments. In addition, the model in this study provides a guide for future research. For example, to analyze the effects of other EV into the ED, it is possible to add more EV in $y(t)$, see references [8,9]. In addition, a new reliability model can be proposed via cumulative damage model (CDM) with random failures, and with this methodology it would be possible to measure the effects of EV, such as the Cpk.

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