transversal
International Journal for the Historiography of Science

Transversal: International Journal for the Historiography of Science, 2024 (17): 1-15
ISSN 2526-2270
Belo Horizonte – MG / Brazil

Special Issue
History and Philosophy of Science in the Belle Époque

The Economy of Thought in Mach, Poincaré and Duhem

Roberto Estrada Olguín[1] [https://orcid.org/0000-0002-2378-3760]

Abstract:
In this work, we describe the concepts of complexes of sensations, economy of thought and causality, conventionalism in mathematics and natural science, distinction between metaphysics and science, on Ernst Mach, Henri Poincaré and Pierre Duhem. We intend to emphasize the characteristic common to the thought of these three thinkers: the economic function of thought. This ideas of Ernst Mach, Henri Poincaré and Pierre Duhem, give an image of science that does not distinguish between a context of discovery and a context of justification and, therefore, they do not see logic as a necessary and sufficient condition to analyze and justify scientific knowledge.

Keywords: Causality; Conventionalism; Metaphysics; Science; Philosophy

Received: April 23, 2024. Reviewed: October 10, 2024. Accepted: December 13, 2024.
DOI: http://dx.doi.org/10.24117/2526-2270.2024.i17.07

Introduction

Rudolf Haller has argued for the distinction between a *first* Vienna Circle and a *second* Vienna Circle, and that the commonly held image of the philosophy of science of the Vienna Circle – which Haller considers to be radically erroneous – is due to the position of the second Vienna Circle. According to that image, the members of that Circle held three basic assumptions: 1) a reductionist theory (according to which all statements must be reduced to statements about the immediate data); 2) the philosophy of the Circle was ahistorical, concerned only with logical-semantic analysis; and 3) the Vienna Circle promoted the idea of the cumulative progress of scientific theories and, therefore, an untenable conception of scientific change: "On various occasions I have already pointed out that this picture is wrong and requires radical correction. [...] The supposed dogmas of logical empiricism were criticized by the logical empiricists themselves, and some members of the Vienna Circle never held theses which would be contaminated by those just mentioned" (Haller 1991, 96).

According to Haller, this image of the philosophy of the Vienna Circle, besides being simplistic, is incorrect, because Haller thinks that the arguments put forward – even by some members of logical empiricism – to criticize the theses attributed to the logical empiricists,

[1] Roberto Estrada Olguin is a Professor in the Department of Humanities at the Autonomous University of Ciudad Juárez. Adress: Av. Universidad y Av. Heroico Colegio Militar S/N Zona Chamizal C.P. 32300. Ciudad Juárez, Mexico. E-mail: estradaa6@hotmail.com

were basically guided by the ideas of thinkers such as Ernst Mach and three French theorists, Pierre Duhem, Henri Poincaré and Abel Rey, ideas that influenced the members of the first Vienna Circle: "And indeed, it is these names which we meet again as guides for our orientation in our search for the main lines of thought of the first Vienna Circle" (Haller 1991, 97).

Haller has distinguished a first Circle, whose main members were Hans Hahn, Otto Neurath and Philipp Frank. And he states that this *first Circle* was of such paramount importance for the *second Circle* that it may well be justified to think that it was really Hans Hahn who was the real founder of the Vienna Circle.[2] Thus the conceptions of Mach and the three French theorists mentioned above had a profound influence on the three members of the first Circle.

For this reason, we will now present some of the ideas of three of the scientists who influenced the first Circle: Ernst Mach, Henri Poincaré and Pierre Duhem. From Ernst Mach we will deal with three ideas: the complexes of sensations; the economy of thought; and causality. From Henri Poincaré we will describe only his conventionalism in mathematics and natural science. And of Pierre Duhem we will discuss his distinction between metaphysics and science (as the study of phenomena); the economy of thought and science as convention. In doing so, we intend to emphasize the characteristic common to the thought of these three thinkers: the *economic* function of thought.

Both Ernst Mach and Henri Poincaré discuss the nature of knowledge through a critique of Kantian ideas concerning the nature of knowledge; mainly with the *a priori* origin of scientific knowledge and, therefore, with the existence of pure intuitions (as conditions of possibility of experience). And they relate these two problems to the Kantian idea of thing-in-itself or *things-in-themselves*. These ideas are discussed by these theorists always in the perspective of elucidating the nature of scientific knowledge. Of these three authors that are of our interest it seems that only Duhem does not – at least explicitly – have as the basis of his thought a dispute with Kantian ideas about the nature of science. We will now turn more specifically to the thinking of Mach, Poincaré and Duhem on the nature of science in order to show the line of thought that influenced the first Circle.[3]

The Economy of Thought in Ernst Mach

Ernst Mach has influenced philosophy, the history of science and science itself. Indeed, Ernst Mach, with his critique of Newtonian science, is considered a precursor of Einstein, thus contributing to science; in philosophy he is interpreted as a phenomenalist, contributing to the philosophy of knowledge; and, finally, in the history of science, his book *Die Mechanik* (1883) is considered essential for a better understanding of modern science.

According to Rudolf Haller, Mach's thought has not always been done justice when he is called a pure inductivist or a pure phenomenalist, and in the course of our exposition we will mention some of the exaggerations of this kind; for the moment we will mention the aspects of this philosopher's thought that influenced some of the members of the first Vienna Circle: 1) his rejection of the Kantian idea of *thing-in-itself*, for considering it as part of *metaphysics*; 2) science understood as *economy of thought*; and 3) the role of the idea of

[2] Karl Menger points out – supported by Philipp Frank – that Hans Hahn can really be regarded as the founder of the Vienna Circle. There are three reasons for thinking so: 1) that Hahn belonged to the trio of those who originally planned the Circle; 2) that it was Hahn who led the Circle into logic; and 3) that Hahn, until his death, was a prominent participating member. Let us add that it was at Hahn's insistence that Schlick took the chair of the philosophy of inductive sciences at the University of Vienna, recently vacated by the death of Adolf Stöhr (Hahn 1980, ix-xviii).

[3] Anastasios Brenner (1998-1999) and Thomas E. Uebel (1998-1999) have addressed the influence of Mach, Poncaré and Duhem on the thinking of the members of the Vienna Circle.

causality in science. Regarding the first point, listed in previous paragraphs, Ernst Mach tells us in *The Science of Mechanics*: "Properly speak the world is not composed of 'things' as its elements, but of colors, tones, pressures, spaces, times, in short what we ordinarily call individual sensations. The whole operation is a mere affair of economy" (Mach 1919, 483):

> To recognize that the world is composed of these individual sensations is the result of an analysis of that is immediately presented to us. According to Mach the colors and the sounds and the pressures, etc., present themselves combined in various ways; each of these combinations he calls a complex or composite tissue of sensations[4]. Now, certain sensations of these complexes, combined with spatial and temporal functions, appear as more stable, and these more stable sensations we designate with names by means of language; and they then appear as *bodies*. The attempt to analyze, i.e. to isolate, the constituent elements of complexes of senstantion may be in contradiction with the habit[5] of designating by names what is thought to be stable. The vague image which we have of a given permanent complex, being an image which does not perceptibly change when one or another of the component parts is taken away, gradually establishes itself as something which exists *by itself*. Inasmuch as it is possible to take away *singly* every constituent part without destroying the capacity of the image to *stand for* the totality and of being recognized again, it is imagined that it is possible to subtract *all* the parts and to have something still remaining. Thus arises the monstrous notion of a *thing in itself* unknowable and different from its *phenomenal* existence. (Mach 1897, 5-6)[6]

Mach conceives of all thought under *the principle of economy*, both in its function of naming *objects* and in the construction of scientific theories. Science is thus understood as a replacement of experiences, through the reproduction and anticipation of facts in thought. But this representation never reproduces experiences completely, but only that which is important for our practical interest. This is, in effect, a process of abstraction, which therefore also shows an economic tendency (Mach 1919, 479-481).

A law in the natural sciences, for example, is not something that exists in nature; exists in nature are instances of the law, which is the abstract form of these cases; the law has the function of sparing us the description of all the cases that the law comprises. A similar thing happens, and more clearly, in mathematics: multiplication, for example, is the abbreviated form of addition; algebra consists precisely of abstract formulas (by means of symbols) which are fulfilled – that is, the equality of the statement is fulfilled – for any numbers which are substituted.[7]

[4] Hume's influence on this idea is clear: "We have therefore no idea of substance, distinct from that of a collection of particular qualities, nor have we any other meaning when we either talk or reason concerning it" (Hume 1965, 16).

[5] *Habit* in the sense that Hume says that belief in the idea of causality is a product of habit and custom (Hume 1965, 155 ss). But it is important to note that, for Hume, ideas that are complex are derived and composed of simple and primitive ideas, whereas, for Mach, simple ideas are abstractions obtained from complexes of sensations.

[6] "Das dunkle Bild des Beständigen, welches sich nicht merklich ändert, wenn ein oder der andere Bestandteil ausfällt, scheint etwas für sich zu sein. Weil man jeden Bestandteil einzeln wegnehmen kann, ohne daß dies Bild aufhört, die Gesamtheit zu repräsentieren und wieder erkannt zu werden, meint man, man könnte alle wegnehmen und es bliebe noch etwas übrig. So entsteht in natürlicher Weise der anfangs imponierende, später aber als ungeheuerlich erkannte philosophische Gedanke eines (von seiner "Erscheinung" verschiedenen unerkennbaren) Dinges an sich" (Mach 1922, 5).

[7] Moreover it seems that the alphabet has the same function of abstraction, as other types of "writing" or languages, e.g. pictorial, ideographic, and syllabic, which are also ordered from lesser to greater degree of abstraction.

Now, in relation to the third aspect of Mach's critique of Kant's thought, we can say that also the idea of *causality*, or the relation of cause and effect, is a form through which thought *economizes*. But, according to Mach, the idea of *causality* does not represent experiences more adequately, so he has proposed its replacement by that of *function*, or the functional relation between the elements of an experience. That is of interest here, however, is Mach's perspective on causality as a form of the economy that characterizes thought. First, we might point out that causality, according to Mach, is the result of a process of abstraction.

The idea of cause and effect, more precisely, the relation between two facts in which one is cause and the other effect, arises from the habit of thought to emphasize only one part of a complex of circumstances. According to Mach, every phenomenon is the result of a mutual influence between the facts involved in the phenomenon in question; so that the mutual and reciprocal influence exerted by the elements of a phenomenon does not allow us to establish a one-sided relation; and we attribute to one part of the phenomenon a one-sided influence on the other part, thus giving rise to the relation of *causality*:

> Usually only two particularly striking components of a process are considered cause and effect. However, a more precise analysis of such a process almost always shows that the so-called cause is only a complement of a whole complex of circumstances which determines the so-called effect. Therefore, depending on whether one has noticed or overlooked this or that component of the complex, the complement in question is very different. (Mach 1906, 277, author's translation)[8]

Thus Mach asserts that there is no causal relation in nature, but a mutual influence of facts irreducible to a unilateral influence. The idea of causality is a provisional abstraction, which thought uses initially, but which with a deeper analysis of the complex of circumstances will be shown to be unnecessary. Only deep analysis cancels out the apparent simplicity of phenomena introduced by the superficiality of the idea of *causality*.

With this exposition of three aspects of Mach's thought, three ideas of great importance for the first Vienna Circle have also been shown: 1) the rejection of *metaphysics*, 2) the concept of *causality* is not a necessary principle of thought, since there is no such principle in nature (nor in human nature), but is an *arbitrary* instrument to simplify (*economize*) the facts in thought; and 3) science is understood – from the idea of the economy of thought – as an instrument that simplifies reality by *representing it*.

The Economy of Thought in Henri Poincaré

The last two ideas – the idea of causality and the idea of science as an instrument of simplification – are the ones that Henri Poincaré will develop in his conception of science.[9] These two ideas carry in germ the affirmation that Kant's synthetic *a priori* judgments do not

---

[8] "Gewöhnlich werden nur zwei besonders auffallende Bestandteile eines Vorganges als Ursache und Wirkung aufgefaßt. Die genauere Analyse eines solchen Vorganges zeigt aber dann fast immer, daß die sogenannte Ursache nur ein Komplement eines ganzes Komplexes von Umständen ist, welcher die sogenannte Wirkung bestimmt. Deshalb ist auch, je nachdem man diesen oder jenen Bestandteil des Komplexes beachtet oder übersehen hat, das fragliche Komplement sehr verschieden".

[9] Brenner has quoted Poincaré himself to point out his agreement with Mach on the economic function of thought. "Poincaré évoque, dans *Science et Méthode,* en 1908, la notion d'économie intellectuelle proposée par Mach: 'Le célèbre philosophe viennois a dit que le rôle de la science est de produire l'économie de pensée de même que la machine produit l'économie d'effort. Et cela est très juste' […]. Cette notion scande les deux premiers chapitres de l'ouvrage; Poincaré donne ainsi une nouvelle expression à des pensées qu'il a déjà émises. Le savant doit sélectionner les faits; sa tâche consiste à condenser les résultats" (Brenner 1998-1999, 33).

exist in natural science and geometry and, therefore, the affirmation these sciences are based on definitions that simplify reality, and not on the aforementioned judgments. Let us see what Poincaré thinks on this point. Henri Poincaré turned his attention to the nature of the *synthetic a priori judgments*, which according to Kant, are the foundation of scientific knowledge – and especially of the axioms of geometry. Already in Mach's perspective on causality and sensation we can suspect that just as for Mach the notion of *thing-in-itself* is a metaphysical invention, so too for Poincaré the axiomatic principles of geometry and the principles of natural science are a conventional element.[10]

Poincaré is often said to have asserted that the judgments of science are neither synthetic a priori nor empirical judgments, but *conventional definitions*, which are proposed for use as scientific instruments. However, the word convention must be taken with care in the context of Poincaré's thought. For this philosopher the principles of the sciences are always based on experience, supported by induction; but not logically deduced from it, but produced by the spirit, through its power of generalization, through the scientist's heuristic capacity[11]. In the idea that the laws or hypotheses of science are neither synthetic a priori judgments nor empirical judgments, is implicitly Mach's idea that science is a "representation" of facts (or complex of sensations). Poincaré claims that scientific laws are *conventional definitions*. This perspective of Poincaré about scientific laws is the Kantian question of what is the provenance of those laws.

Poincaré first questions the nature of mathematical reasoning as accepted by Kant. In order to elucidate the nature of mathematical reasoning, Poincaré begins by asserting that there is a contradiction in the very possibility of this reasoning: if this reasoning is not deductive, what provides the correctness, which no one can deny, to the inferences of mathematical reasoning? On the contrary, if all the propositions of mathematics can be derived from one another by the rules of logic, why is not all mathematics reduced to a huge tautology? For example, in *La science et l'Hypothèse* he states:

> No doubt we may refer back to axioms which are at the source of all these reasonings. If it is felt that they cannot be reduced to the principle of contradiction, if we decline to see in them any more than experimental facts which have no part or lot in mathematical necessity, there is still one resource left to us: we may class them among

[10] However, according to Uebel, Poincaré does not completely reject the Kantian a priori: "For Poincaré, geometry was a special case among the mathematical sciences the foremost of which, arithmetic, was viewed as synthetic a priori. Geometry was treated differently from arithmetic – namely, conventionally – because in its case the choice between three alternative axiomatizations was left undetermined, as Friedman explains [...]. (By contrast, the neopositivists, following Schlick, ascribed to Poincaré a general argument from underdetermination and discounted not only his reliance on the synthetic a priori, but also his presuppositions concerning the hierarchy of the sciences that were made untenable by relativity theory)" (Uebel 1998-1999, 77).

[11] This is why Poincaré says that the accumulation of data does not constitute one theory, induction alone does not produce any hypothesis. A hypothesis is a generalization and in order to generalize an *act* of creation is necessary, which is not the product of induction: "Cannot we be content with experiment alone? No, that is impossible; that would be a complete misunderstanding of the true character of science. The man of science must work with method. Science is built up of facts, as a house is built of stones; but an accumulation of facts is no more a science than a heap of stones is a house" (Poincaré 1905 [1902], 141). "Ne pouvons nous nous contenter de l'expérience toute nue? Non, cela est impossible; ce serait méconnaître complètement le véritable caractère de la science. Le savant doit ordonner; on fait la science avec des faits comme une maison avec des pierres; mais une accumulation de faits n'est pas plus une science qu'un tas de pierres n'est une maison" (Poincaré 1917 [1902], 168).

> a priori synthetic views. But this is no solution of the difficulty – it is merely giving it a name. (Poincaré 1905 [1902], 2)[12]

The paradox of mathematics, according to the French philosopher, is not resolved by accepting that the principles of science are synthetic a priori judgments, because the procedure of derivation of axioms would remain analytical. Moreover, he adds that all mathematicians declare the intention of generalizing a proposition that is already known. The question arises here: how is it that mathematics proceeds from the particular to the general? What then a deductive science is? "Even if these consequences are challenged, it must be granted that mathematical reasoning has of itself a kind of creative virtue, and is therefore to be distinguished from the syllogism" (Poincaré 1905 [1902], 3).[13]

That creative virtue is the power of generalizing an operation which is true for particular cases, which is valid for all cases; it is a creative virtue because it is not obtained by experience but from the spirit itself. Poincaré is thinking of definitions of the following type: $x + 1$, which can be defined as the addition of the unit to any number; and assuming the definition of the number 1, the numbers 2, 3 and 4 can be derived:

> A) 1+1=2, to the number one we add the unit;
> B) 2+1=3, to the number two we add the unit;
> C) 3+1=4, to the number tree we add the unit, and so on.[14]

The whole definition is fully generalized when it is defined as $x + a$: the addition of any number x to any other number a. The association theorem, the commutation theorem and the distribution theorem can then be derived. If one wants to verify any of these theorems for a specific number, one only has to construct the formula by recurrence, that is, by repetition of the initial definition, up to the desired number. Thus, the general formula of recurrence is: if the theorem is valid for n–1, then it is valid for n (if valid for 2–1, then it is valid for 2).

According to Poincaré, the simple repetition of a formula in particular cases in order to verify it is an analytical procedure; but the enunciation of the general formula cannot be obtained by the process of verification, because the general formula incorporates the *mathematical infinite* and the process of verification is by essence finite. The creative virtue of the spirit is this incorporation of the *infinite*.

Mathematical reasoning, like that of the natural sciences, is a procedure from the particular to the general; from particular cases a general formula is obtained, how is it then that mathematics possesses has universal character?, according to Poincaré, this universal character is the difference between induction by recurrence (mathematical induction) and induction in the natural sciences; the answer to this question is given by Poincaré in the following terms:

[12] "Sans doute, on peut remonter aux axiomes qui sont à la source de tous les raisonnements. Si on juge qu'on ne peut les réduire au principe de contradiction, si on ne veut pas non plus y voir des faits expérimentaux qui ne pourraient participer à la nécessité mathématique, on a encore la ressource de les classer parmi les jugements synthétiques a priori. Ce n'est pas résoudre la difficulté, c'est seulement la baptizer" (Poincaré 1917 [1902], 10).

[13] "Si l'on se refuse à admettre ces conséquences, il faut bien concéder que le raisonnement mathématique a par lui-même une sorte de vertu créatrice et par conséquent qu'il se distingue du syllogisme" (Poincaré 1917 [1902], 11).

[14] With these definitions it can be shown that 2 + 2 = 4. We can define the operation $x + 2$ as follows: $x + 2 = (x + 1) + 1$; substituting the value of $x = 2$ we obtain $2 + 2 = (2 + 1) + 1$; by definition B) we obtain $(2 + 1) + 1 = 3 + 1$ and by definition C) we obtain that $3 + 1 = 4$ or $2 + 2 = 4$.

> Why then is this view imposed upon us with such an irresistible weight of evidence? It is because it is only the affirmation of the power of the mind which knows it can conceive of the indefinite repetition of the same act, when the act is once possible. The mind has a direct intuition of this power, and experiment can only be for it an opportunity of using it, and thereby of becoming conscious of it. (Poincaré 1905 [1902], 13)[15]

As for the science of nature, like mathematical reasoning, it proceeds by means of definitions. The law of inertia, for example, says Poincaré, is not a priori since it was unknown to the ancients, but neither is it a law but neither is it a law inferred from experience since in experience no body is found to be free from all resistance. The principle of inertia is the generalization of particular cases to all cases; all the principles of science are generalizing definitions.[16]

> The principles of mechanics are therefore presented to us under two different aspects. On the one hand, there are truths founded on experiment, and verified approximately as far as almost isolated systems are concerned; on the other hand, there are postulates applicable to the whole of the universe and regarded as rigorously true. If these postulates possess a generality and a certainty which falsify the experimental truths from which they were deduced, it is because they reduce in final analysis to a simple convention that we have a right to make, because we are certain beforehand that no experiment can contradict it.
>
> This convention, however, is not absolutely arbitrary; it is not the child of our caprice. We admit it because certain experiments have shown us that it will be convenient, [...]. (Poincaré 1905 [1902], 135-136)[17]

Thus we see how it is that the procedure is the same as the one we have shown in mathematical reasoning, this is a process of induction and the ability to generalize. A

---

[15] "Pourquoi donc ce jugement s'impose-t-il à nous avec une irrésistible évidence? C'est qu'il n'est que l'affirmation de la puissance de l'esprit qui se sait capable de concevoir la répétition indéfinie d'un même acte dès que cet acte est une fois possible. L'esprit a de cette puissance une intuition directe et l'expérience ne peut être pour lui qu'une occasion de s'en servir et par là d'en prendre conscience" (Poincaré 1917 [1902], 23-24).

[16] Every generalization is a hypothesis, which is why Poincaré says: "Thus, by generalization, every fact observed enables us to predict a large number of others; only, we ought not to forget that the first alone is certain, and that all the others are merely probable. However solidly founded a prediction may appear to us, we are never absolutely sure that experiment will not prove it to be baseless if we set to work to verify it" (Poincaré 1905 [1902], 143-144). "Ainsi, grâce à la généralisation, chaque fait observé nous en fait prévoir un grand nombre; seulement nous ne devons pas oublier que le premier seul est certain, que tous les autres ne sont que probables. Si solidement assise que puisse nous paraître une prévision, nous ne sommes jaimas sûr absolument que l'expérience ne la démentira pas, si nous entreprenons de la vérifier" (Poincaré 1917 [1902], 171).

[17] "Les principes de la mécanique se présentent donc à nous sous deux aspects différents. D'une part, ce sont des vérités fondées sur l'expérience et verifies d'une façon très approchée en ce qui concerne des systèmes presque isolés. D'autre part, ce sont des postulats applicables à l'ensemble de l'univers et regardés comme rigoureusement vrais. Si ces postulats possèdent une généralité et une certitude qui faisaient défaut aux vérités expérimentales d'où ils sont tirés, c'est qu'ils se réduisent en dernière analyse à une simple convention que nous avons le droit de faire, parce que nous sommes certains d'avance qu'aucune expérience ne viendra la contredire.

Cette convention n'est pourtant pas absolument arbitraire; elle ne sort pas de notre caprice; nous l'adoptons parce que certaines expériences nous ont montré qu'elle serait commode" (Poincaré 1917 [1902], 162-163).

combination of contingent experience and universal creative virtue; this combination is expressed in the above quotation by the two aspects under which the principles of the sciences are shown to us: *truths founded on experiment* and *postulates applicable to the whole of the universe and regarded as rigorously true.*

On the other hand we can also see that the idea of *generalization* is similar to Mach's idea of the *economy of thought*, since generalization turns out to be more *comfortable* than the extremely large series of particular cases; moreover, science proceeds by *conventional definitions* and it is clearly shown that logical deduction is not sufficient to produce science. In short, according to Mach and Poincaré, science is constituted by the facts of experience and by *generalization*, which functions under the principle of *economy of thought* by means of the *abstraction* of which Mach speaks, which makes it possible to handle an indeterminate number of particular cases; science includes an element which functions as an *instrument* which makes it possible to simplify the complexity of phenomena.[18] These generalizations are *conventional definitions* which we rightfully accept, because they allow us to know a complex reality in a simplified way; and they are not *a priori* synthetic judgments because they can be modified and changed as has happened with the principles of Euclidean geometry.

The Economy of Thought in Pierre Duhem

Pierre Duhem thinks, as Poincaré and Mach, that science is a *representation*. Although for the former, what the hypotheses of science represent are no longer – as for Mach – strictly *facts* but the *empirical laws* of science.[19] Besides it is important to highlight that for both Poincaré and Duhem, *facts* are not complexes of elements. However, before going into the details of Duhem's thought, let us enumerate the aspects in which this thought stands out for the matter at hand, in order to make our exposition as clear as possible:

a) Like Mach, Duhem rejects the idea that science aims at *metaphysical* explanation. However, we note that the meaning of *metaphysics* is not exactly the same for both scientists.

b) Like Mach, Duhem also attributes an *economic* role to science; here, it seems following the same idea as Mach.

c) On the other hand, like Poincaré, Duhem thinks that scientific theories include some arbitrary aspect, especially in the *definitions* and hypotheses of science.

d) Unlike to Poincaré and Mach, Duhem thinks that science is not only a vocabulary into which facts expressed in vulgar language are translated in order to make the expression of those facts more comfortable. Duhem thinks that science adds theoretical content that is not derived from the empirical facts.

And e) Contrary to Poincaré, Duhem states that, although it is possible to construct several theories for the explanation of the same type of phenomena, these theories are not equivalent. There are according to him, three criteria for choosing between these theories (which we will point out later).

---

[18] According to Uebel, Poincaré distinguishes three types of conventions: "For Poincaré, like for Mach, science is a system of classifying data and their regularities so as to facilitate predictions. In *Science and Hypothesis* Poincaré specified three types of conventions, over and above those of naming things and specifying units of measurement: the axioms of geometry; the 'principles' of mechanics, for instance, Newton's three laws; and the methodological maxim of simplicity" (Uebel 1998-1999, 76-77).

[19] According to Brenner Duhem points out that such a representation is also a classification: "Quand Duhem définit la théorie physique, non pas comme une explication, mais comme une représentation abstraite, il est proche de Mach. Aussi ne sommes-nous pas étonné de le voir invoquer la notion d'économie de pensée. Mais Duhem ajoute tout de suite une précision: cette représentation est aussi une classification, qui tend de plus en plus vers une classification naturelle" (Brenner 1998-1999, 34).

With regard to the distinction between *metaphysics* and *science*, Duhem thinks that this distinction is based on another more fundamental distinction: the difference between appearance and reality. A distinction in which reality is hidden in appearance; according to Duhem, science does not deal with *reality* at all, but only with appearances or phenomena. The problem of whether there are particles or waves behind luminous phenomena is a metaphysical problem, not a scientific one. The theory of the ether is also a matter of the metaphysics; the science has only phenomena as its object and only the *representation* of the laws of these phenomena as its aim.

Note that for Mach the idea of *thing-in-itself* is an illusion created through a process of abstraction of all sensations and the assumption that after having abstracted all sensible qualities something would still remain, this *something* that supposedly remains was designated as *thing-in-itself*. Moreover, the reality hidden behind the phenomena is not exactly the Kantian *noumenon*, a *noumenon* which would correspond to the *thing-in-itself*. Thus *metaphysics* begins to have a multitude of meanings; in this time for Mach it is constituted by notions or ideas created by the imagination; and for Duhem it is a supposed *reality* that is hidden behind the phenomena, but for neither of them it is a condition of possibility of experience.

Since according to Duhem the scientific theories can say nothing about ultimate reality – things in themselves – but has only phenomena as its object, then, Duhem asks, what is the usefulness of a scientific theory? Duhem's answer is that the usefulness of scientific theories lies in the fact that they carry an *economic function*", i.e., they "represent" in a smaller number of hypotheses the manifold experimental laws, which carry an "economic function also. Thus, for example, Duhem says in *La Théorie Physique. Son Objet et sa Structure* (1906):

> First of all, instead of a great number of laws offering themselves as independent of one another, each having to be learnt and remembered on its own account, physical theory substitutes a very small number of propositions, viz., fundamental hypotheses. [...] Such condensing of a multitude of laws into a small number of principles affords enormous relief to the human mind, which might not be able without such an artifice to store up the new wealth it acquires daily.
>
> The reduction of physical laws to theories thus contributes to that *intellectual economy* in which Ernst Mach sees the goal and directing principle of science (Duhem 1991 [1906], 21).[20]

Duhem finds the conventional elements of theories in the successive operations by which a theory is constituted, in this succession in which *intellectual economy* is applied. In order to better understand Duhem's thinking in this respect, we will set out what these operations consist of and show how they are successive. And we will point out the relevance of the remaining aspects of the list we have given.

The first operation is the *definition* – like Poincaré Duhem thinks that science starts from definitions – and the measurement of physical quantities. To define a physical *notion*, for example the notion of heat, a certain magnitude must correspond to it and this magnitude must be capable of *addition*, i.e., it must be capable of being added. For example,

[20] "Tout d'abord, à un très grand nombre de lois qui s'offrent à nous comme indépendantes les unes des autres, dont chacune doit être apprise et retenue pour son propre compte, la théorie substitue un tout petit nombre de propositions, les hypothèses fondamentales. [...] Une telle condensation d'une foule de lois en un petit nombre de principes est un immense soulagement pour la raison humaine qui ne pourrait, sans un pareil artifice, emmagasiner les richesses nouvelles qu'elle conquiert chaque jour. La réduction des lois physiques en théories contribue ainsi à cette *économie intellectuelle* en laquelle M. E. Mach voit le but, le principe directeur de la Science" (Duhem 1906, 29).

the notion of heat has the following characteristics: a body can be hotter, less hot or equally hot than another body.

And, in addition, the quantity heat must be added, i.e., we must be able to know that a body A is as hot as a body B, or that it is as hot as body B plus body C. Thus, the notion *heat* is made to *correspond* to a magnitude: temperature. By virtue of this *correspondence*, the one becomes the symbol (the temperature) of the other (the heat). But there is no natural relation between them (the heat is pleasant or unpleasant, it comforts or burns us, and the temperature can be added to another temperature, be multiplied or divided by a number, the temperature is a *represented* value by a symbol and, as such, is not pleasant or unpleasant).

According to Duhem the operation just described has a high degree of arbitrariness, since there is a certain freedom in deciding which specific magnitude we have to make correspond to the physical *notion*. Thus in our example, the temperature can be measured in degrees Celsius or Fahrenheit or some other. Such a decision is "conventional", which Poincaré will put it a year later with respect to the *definitions* of geometry.

Once the physical notions have been defined and measured, the second operation is to *choose the hypotheses*. The hypotheses are, on the one hand, the expression – usually mathematical – of the relations between the various physical notions, for example, the relation between the temperature and the pressure (a directly proportional relation, the higher the temperature the higher the pressure) and, on the other hand, the hypotheses constitute the foundations on which the theory will be built. In other words, the *hypotheses* are similar to the definitions of geometry as Poincaré would claim.

In the third operation the *hypotheses are combined* under the rules of mathematics, inferring the consequences from the theory (as, for example, the theorems of geometry or the predictions of the natural sciences) and finally, in the fourth operation, the consequences (the theorems) of the theory can be translated into judgments or propositions about the physical properties of bodies, to be compared with the experimental laws, which the theory is intended to represent. Duhem gives the example of universal attraction. The law of universal gravitation is based on Kepler's laws, but it is not just a symbolic translation of them, it is something more; for example, on "Quelques réflexions au sujet des théories physiques", says Duhem:

> What are the experimental laws on which it [the theory of gravity] is based? Kepler's laws. What is the exact translation of these laws into the symbolic language that creates the definitions of rational mechanics?
>
> "The sun exerts on each planet a force of attraction in inverse ratio of the square of the distance from the sun to the planet. The forces exerted by the sun on the various planets are to each other like the masses of those planets. The planets exert no force on the sun".[21] (Duhem 1987, 9, author's translation)

This proposition speaks of a particular case, it is the formulation of an *experimental law* (Duhem speaks of experimental laws as the regularities observed in a specific case), what this experimental law affirms it does for the sun and the planets, on the contrary a theoretical law is not limited to this formulation of a particular case, but refers in a general way to every possible case; let us see with Duhem what the theoretician has to do to transform this experimental law into a theoretical law:

---

[21] "Quelles sont les lois expérimentales sur lesquelles elle repose? Les lois de Képler. Quelle est la traduction exacte de ces lois dans la langage symbolique que créent les définitions de la mécanique rationnelle? Le soleil exerce sur chaque planéte une force attractive en raison inverse du carré de la distance du soleil á la planéte. Les forces exercées par le soleil sur diverses planètes sont entre elles comme les masses de ces planétes. Les planètes n'exercent aucune force sur le soleil".

> Newton corrects, as we said, the preceding proposition: instead of saying that, according to Kepler's laws, the planets exert no action on the sun, Newton states that every planet exerts on the sun an action equal and directly opposite to that which it receives from it.
>
> Is Newton content with this correction? No, he adds a proposition which experience has not given him: namely, that if the sun were replaced by some other body, the actions exerted on the various planets would be multiples of the relation of the mass of that new body to the mass of the sun.
>
> Is that all? Not yet; Newton generalizes the result obtained, and it is only by this generalization that he can state the fundamental principle of his theory:
>
> Two material bodies whose dimensions are negligible in relation to their distance are subject to a mutual attraction proportional to the product of the masses of the two bodies and in inverse ratio to the square of the distance separating them.[22] (Duhem 1987, 9-10, author's translation)

This new proposition, product of the correction of the formulation of the law attributing an action of the planet on the sun, of the addition of a new proposition which is not verifiable by experience and of the generalization of the experimental law, no longer speaks of a particular case, it is a universal theoretical law, it does not limit itself to affirming what it enunciates only for the sun and the planets, but for all possible bodies. Let us see how Duhem concludes what results from this process of hypothesis generation:

> What did Newton do? Did he take for a hypothesis the symbolic translation of one or more experimental laws? No, he has taken by hypothesis a [universal] proposition whose experimental laws placed at the beginning of his theory are only the exact or merely approximate particular consequences [of that universal law].
>
> This is the general procedure employed by all theorists. In order to formulate their hypotheses they choose some of the experimental laws whose whole is to be covered by their theory; then, by way of correction, of generalization, of analogy, they compose a proposition of which its laws are exact or merely approximate consequences, and it is this proposition which they take for their hypothesis.[23] (Duhem 1987, 10, author's translation)

---

[22] "[...] Cette proposition que nous venons d'enoncer, Newton la corrige; puis il lui adjoint une nouvelle proposition non vérifiable par l'expérience; puis il généralise le résultat obtenu. Newton corrige, avons-nous dit, la proposition précedente: au lieu que, d'aprés les lois de Képler, les planétes n'exercent aucune action sur le soleil, Newton énonce que toute planéte exerce sur le soleil une action égale et directement opposée á celle qu'elle en recoit. Newton se contente-t-il de cette correction? Non, il ajoute une proposition que l'expérience en lui fournit pas: á savoir, que si le soleil était remplacé par un autre corps, les actions exercées sur les diverses planètes seraient multipliées par la rapport de la masse de ce nouveau corps á la masse du soleil. Est-ce tout? Non encore; Newton généralise le résultat obtenu, et ce seulement par cette généralisation qu'il peut énoncer le principe fondamental de sa théorie: Deux corps matériels, dont les dimensions sont négligeables par rapport á leur distance, sont soumis á une attraction mutuelle proportionnelle au produit des masses des deux corps et en raison inverse du carré de la distance qui le sépare".

[23] "Qu'a donc fait Newton? A-t-il pris pour hypothèse la traduction symbolique d'une ou de plusieurs lois expérimentales? Nullement. Il a pris pour hypothèse une proposition dont les lois expérimentales placées au début de sa théorie sont seulement approchées. C'est lá le procédé général employé par tous les théoriciens. Pour formuler leurs hypothèses, ils font choix de quelques-unes des lois expérimentales dont l'ensemble doit être embrassé par leur théorie; puis, par voie de correction, de généralisation, d'analogie, ils composent une proposition dont ces lois soient des conséquences exactes ou simplement approchées, et c'est cette proposition qu'ils prennent pour hypothèse".

This correction and this analogy and this generalization is that makes Duhem think, unlike to Poincaré, that science is not only the vocabulary with which to express more easily the phenomena of reality, but that science is theory, that is, the scientist brings non-empirical ideas into the formulation of his principles. This idea is not only important because it is different from Poincaré's view; it also means that the "addition" that the scientist provides is an interpretation of the facts; an interpretation which, of course, need not be the only one. The important thing, then, is that, on the one hand, this correction is an addition and that, on the other hand, both this correction and this generalization are an essential part of science and, therefore, the ideas which are not provided by experience, but that the scientist adds are an essential part of science.

To a certain extent this aspect of science, i.e. the generalization of ideas that are not derived from experience, was set aside by the intellectual heritage that had a strong influence on the formation of what Rudolf Haller has called the second Vienna Circle; proof of this is that the philosophy of science that followed the Vienna Circle recovered this aspect of science in its critique of the philosophy of logical positivism. Moreover, the latter called "metaphysics" any idea that was not founded on empirical data, which shows that metaphysics underwent a further change in meaning with logical empiricism. In addition, Duhem formulated what has subsequently been called the *underdetermination* of theories, i.e., the possibility of constructing different and logically incompatible theories for the same set of phenomena, all these theories logically equivalent. This idea shows that Duhem was aware of the limits of logic for the problem of theoretical choice:

> However, of the same class of phenomena there may be several theories, all founded on clearly stated hypotheses, all logically constructed, all in sufficient agreement with the facts they purport to represent: Optics offers us a striking example. Logically all these theories are acceptable; are they all equivalent? No logical criterion decides between them; is it the case that we can have no reasonable ground for preferring one to the other?
>
> Three characteristics can help us to choose between these different theories; these are:
>
> The extent of the theory
> The number of hypotheses
> And the nature of the hypotheses.[24] (Duhem 1987, 31, author's translation)

Already points one and two show Duhem's idea according to which thought proceeds economically: the extension of the theory and the number of hypotheses is aimed at pointing out that a theory that covers the most with the least number of hypotheses is better. Clearly, then, we can see from this quotation that for Duhem, although logic cannot offer us an ultimate and definitive criterion for choice, this does not mean that we have to regard the different theories as equivalent. But it should not be understood by this that logic plays no

[24] "Mais, d'une même classe de phénomènes, il peut exister plusieurs théories, toutes fondées sur des hypothèses clairement énoncées, toutes logiquement construites, toutes en accord satisfaisant avec les faits qu'elles prétendent représenter. [...] Logiquement, toutes ces théories sont acceptables; en résulte-t-il qu'elles soient toutes équivalentes? Aucun criterium logique ne décide entre elles, en résulte-t-il que nous ne poissions avoir aucun motif raisonnable de préférer l'une à l'autre?

Trois caractères peuvent nous servir à choisir entre ces différentes théories; ce sont:
L'étendue de la théorie;
Le nombre des hypothèses;
La nature des hypothèses."
It must be said that Duhem never again enumerated a list of criteria like this in his work.

role in theoretical choice; on the contrary, according to Duhem, logic also demands three requirements for theories:

> 1) that its fundamental assumptions are not self-contradictory propositions,
> 2) that the hypotheses are not contradictory to the rest of the hypotheses of the theory,
> 3) finally, that the hypotheses are chosen in such a way as to derive mathematically from their totality the consequences which represent the set of experimental laws with sufficient approximation. (Duhem 1991 [1906], 219-220)

Duhem has also expounded what later in the field of philosophy of science was called *holism*, i.e. the idea that scientific hypotheses are not tested in isolation but that in scientific experiments it is a whole theoretical system that is subjected to testing and, therefore, that the prediction of a phenomenon is made by taking into account the whole theoretical system. Therefore if the experiment results in the non-occurrence of the predicted phenomenon, then the *error* which is the cause of the non-occurrence of the phenomenon cannot be located in the theoretical system. Duhem expresses it in the following way:

> A physicist decides to demonstrate the inaccuracy of a proposition; in order to deduce from this proposition the prediction of a phenomenon and institute the experiment which is to show whether this phenomenon is or is not produced, in order to interpret the results of this experiment and establish that the predicted phenomenon is. not produced, he does not confine himself to making use of the proposition in question; he makes use also of a whole group of theories accepted by him as beyond dispute. The prediction of the phenomenon, whose nonproduction is to cut off debate, does not derive from the proposition challenged if taken by itself, but from the proposition at issue joined to that whole group of theories; if the predicted phenomenon is not produced, not only is the proposition questioned at fault, but so is the whole theoretical scaffolding used by the physicist. The only thing the experiment teaches us is that among the propositions used to predict the phenomenon and to establish whether it would be produced, there is at least one error; but where this error lies is just what it does not tell us. (Duhem 1991 [1906], 185)[25]

In Duhem holism is the preliminary step for his critique of the idea of crucial experiment, and serves as a further argument for Duhem's claim that logic is a necessary condition for the evaluation of theories, but not a sufficient condition; logical requirements must be complemented by what he calls *le bon sens*, which is assimilated to *l'esprit de finesse* that Pascal opposes to *l'esprit géométrique*. Good sense plays the role of weighing the good

---

[25] "Un physicien se propose de démontrer l'inexactitude d'une proposition; pour déduire de cette proposition la prévision d'un phénomène, pour instituer l'expérience qui doit montrer si ce phénomène se produit ou ne se produit pas, pour interpréter les résultats de cette expérience et constater que le phénomène prévu ne s'est pas produit, il ne se borne pas à faire usage de la proposition en litige; il emploie encore tout un ensemble de théories, admises par lui sans conteste; la prévision du phénomène dont la non-production doit trancher le débat ne découle pas de la proposition litigieuse prise isolément, mais de la proposition litigieuse jointe à tout cet ensemble de théories; si le phénomène prévu ne se produit pas, ce n'est pas la proposition litigieuse seule qui est mise en défaut, c'est tout l'échafaudage théorique dont le physicien a fait usage; la seule chose que nous apprenne l'expérience, c'est que, parmi toutes les propositions qui ont servi à prévoir ce phénomène et à constater qu'il ne se produisait pas, il y a au moins une erreur; mais où gît cette erreur, c'est ce qu'elle ne nous dit pas" (Duhem 1906, 303).

reasons that scientists may have for their theoretical choices (Duhem 1906, 356-359; 1991 [1906], 216-218).

It is important to note that the philosophy of science (faced with the problem of rationality) is currently questioning the traditional notion of rationality, and a notion of rationality has emerged onto the battlefield that comes from Aristotle and his concept of *phronesis*, which in modern terms we can call practical rationality; Duhem's concept of *bon sens* can well be understood as that practical rationality whose main characteristic is that it is based on deliberation as a process of choosing between various possible alternatives.

Conclusion

In summary we can say that the ideas of Ernst Mach, Henri Poincaré and Pierre Duhem, give an image of science that does not distinguish between a *context of discovery* and a *context of justification* and, therefore, they do not see logic as the necessary and sufficient method to analyze and justify scientific knowledge. The ideas of these authors are grounded in the history of science, i.e. the picture of science that these ideas suggest to us is obtained through knowledge of the history of science, which in this context is understood to be part of the *philosophical* analysis of science. The inclusion of historical knowledge of science in the *philosophical* study of science has as some of its consequences the following points:

1) that a theory, a physical theory, is not constructed by a purely inductive process, but that for such construction the activity of thought is necessary, an activity whose function is, according to the three thinkers we have just analyzed, to *economize* the effort of the spirit. This *economizing* function is carried out through the generalization and analogy of ideas which are not inferred from observations, what logical positivism would call metaphysics.

2) that logic is a necessary, but not sufficient condition for:

a) the choice of hypotheses

b) the choice of theories.

3) that truth has nothing to do with *reality* but with the consequences of the theories and the laws enunciated by the experimenters (the regularities observed in particular cases).

This brief exposition of some of the ideas of Mach, Poincaré and Duhem, ideas that influenced the representative members of the first Vienna circle, shows that much of the notion of science of these thinkers was left out by the representative members of the second Vienna Circle. Philipp Frank already pointed out that one of his differences with Mach was the little or no importance attributed by the latter to logic. And Pierre Duhem has pointed out the insufficiency of logic to provide a satisfactory explanation of science, particularly to account for scientific change or choice of theories.

However this perspective was bypassed by an upsurge of logic as the method of philosophy in the analysis of science. Of course, the intellectual heritage of the second Circle was a very different one from that of the first Circle. It was a heritage, basically, of new research in logic and mathematical logic. But it was neither Hans Hahn, nor Otto Neurath, nor Philipp Frank – the members of the first Circle – nor Moritz Schlick, who developed logical analysis for the explanation of science, but Rudolph Carnap who explicitly left the history of science – as well as psychology and sociology of the science – outside the scope of the philosophy of science. That is, the whole *context of discovery*, Carnap claimed, is not the province of philosophy, but only and exclusively *the context of justification*.

References

Brenner, Anastasios. 1998-1999. "Les voies du positivisme en France et en Autriche: Poincaré, Duhem et Mach". *Philosophia Scientiae* 3 (2): 31-42.

Duhem, Pierre. 1906. *La Théorie Physique. Son Objet et sa Structure*. Paris: Chevalier et Rivière Éditeurs.

Duhem, Pierre. 1987. *Premices Philosophiques*, edited by Stanley Jaki. Leiden: E. J. Brill.

Duhem, Pierre. 1991 [1906]. *The Aim and Structure of Physical Theory*. Princeton: New Jersey Princeton University Press.

Hahn, Hans. 1980. *Empiricism, Logic, and Mathematics*. London: Brian McGuinnes Editor, Riedel Publishing Company.

Haller, Rudolph. 1991. "The first Vienna Circle". In *Rediscovering the Forgotten Vienna Circle*: 95-108, edited by Thomas E. Ubel. Netherlands: Klumer Academic Publishers.

Hume, David. 1965. *A Treatise of Human Nature*. Oxford: Oxford University Press.

Mach, Ernst. 1897. *Contributions to the Analysis of the Sensations*. Chicago: The Open Court Publishing Company.

Mach, Ernst. 1906. *Erkenntnis und Irrtum. Skizzen zur Psychologie der Forschung*. Leipzig: Verlag von Johann Ambrosius Barth.

Mach, Ernst. 1919. *The Science of Mechanics. A Critical and Historical Account of its Development*. Chicago: The Open Court Publishing Company.

Mach, Ernst. 1922. *Die Analyse der Empfindungen und das Verhältnis des Physischen zum Psychischen*. Jena: Verlag von Gustav Fischer.

Poincaré, Henri. 1905 [1902]. *Science and Hypothesis*. New York: The Walter Scott Publishing Co.

Poincaré, Henri. 1917 [1902]. *La Science et l'Hypotèse*. Paris: Ernest Flammarion.

Uebel, Thomas E. 1998-1999. "Fact, hypothesis and convention in Poincaré and Duhem". *Philosophia Scientiae* 3 (2): 75-94.