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Estudio de la Teoría “Tau G improved”en aplicación de operaciones de proximidad y alcance con un cuerpo celeste

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Estudio de la Teoría “Tau G improved” en aplicación de operaciones de proximidad y alcance con un cuerpo celeste

Resumen del reporte técnico en español (máximo 250 palabras):
Antes de que una nave espacial alcance una posición estacionaria sobre un asteroide para realizar tareas orbitales, se deben realizar operaciones de proximidad. Dado que los movimientos de empuje de las naves espaciales dentro de la órbita de un objeto pequeño como un asteroide dependen únicamente de las fuerzas de aceleración proporcionadas por los propulsores, la correcta planificación de la ejecución de las trayectorias cercanas en la órbita del cuerpo celeste es de gran importancia. , por un lado, evitar que el vehículo alcance la velocidad de escape y, por otro, el mínimo consumo de combustible en la maniobra. En este artículo se muestra una simulación de planificación de trayectorias de encuentro de corto y largo alcance para alcanzar una posición estacionaria en un asteroide cercano a la Tierra (NEA). Para ello, se utiliza un método de planificación de trayectorias bio-inspirado basado en la teoría Tau-G, que explica cómo los movimientos de animales y humanos son dirigidos sensorialmente para acercarse a un objeto cerrando espacios, para alcanzar la posición final de la nave espacial cerca del cuerpo espacial.

Resumen del reporte técnico en inglés (máximo 250 palabras):
Before a spacecraft reaches a hovering position over an asteroid to perform orbital tasks close-proximity operations must be performed. Since the thrust movements of spacecraft within the orbit of a small object such as an asteroid depend only on the acceleration forces provided by the thrusters, the correct planning of the execution of the close trajectories in the orbit of the celestial body it is of great importance, firstly, to prevent the vehicle from reaching escape velocity and secondly the minimum use of fuel in the maneuver. In this paper a simulation of close-range and far-range rendezvous trajectories planning are shown to reach a hovering position on Near-Earth Asteroid (NEA). For this purpose, a Bio-Inspired trajectory planning method based on Tau-G theory, which explains how the movements of animals and humans are sensorially directed to approach an object by closing gaps, is used to reach the final position of the spacecraft close to the space body.

Palabras clave:
Bio-Inspired, hovering, proximity operations, close-range, far-range, Tau G Theory.

Usuarios potenciales (del proyecto de investigación).
Estudiantes de posgrado e investigadores en el área de dinámica espacial y mecánica orbital.

1. Introducción

Las misiones de encuentro han sido un pilar de la exploración espacial desde el programa Apolo hasta las operaciones actuales con la Estación Espacial Internacional. Hoy en día, existe un interés creciente en varios tipos de misiones que no sólo dependen del encuentro y el acoplamiento en satélites artificiales dentro de la órbita terrestre baja, sino que también dependen del encuentro y el aterrizaje en un asteroide cercano a la Tierra. Por ejemplo, la exploración espacial ha permitido el descubrimiento de nuevos cuerpos celestes que se identifican como potencialmente explorables. De los numerosos cuerpos pequeños que se encuentran en el espacio exterior, los asteroides cercanos a la Tierra (NEA) son de interés debido a su accesibilidad y al riesgo de colisión con la Tierra. Además, los NEA podrían representar recursos materiales que pueden ayudar a mantener sostenible la exploración espacial. Los NEA también se conocen como Objetos Cercanos a la Tierra (NEO) y se caracterizan por sus órbitas heliocéntricas. Una propiedad fundamental de un NEA es su masa total ya que controla la atracción gravitacional que se ejerce sobre una nave espacial. Además, las perturbaciones no gravitacionales también están asociadas con el Sol y la órbita heliocéntrica debido a la transferencia de impulsos de los fotones solares que chocan y se alejan del cuerpo en órbita. Tanto las fuerzas gravitacionales como las no gravitacionales ejercen una fuerza externa sobre la nave espacial en las proximidades del cuerpo celeste que puede afectar las maniobras de encuentro y aterrizaje. En este trabajo se considera el estudio de la dinámica del asteroide con respecto a las perturbaciones gravitacionales y solares y se incluyen en la formulación matemática como fuerzas externas adicionales que pueden cambiar el curso de la trayectoria planificada. En las siguientes secciones se incluyen estrategias de orientación relacionadas con operaciones de encuentro y de proximidad entre una nave espacial y un pequeño cuerpo celeste. Además, en este artículo se propone un estudio del diseño de trayectorias de maniobras de encuentro de largo y corto alcance hacia la exploración del asteroide 99942 Apophis (Figura 1). El objetivo principal de esta investigación es analizar el uso de un método bioinspirado basado en la teoría Tau (Lee, 1976; Lee, 1993, Lee, 2003, Lee, 2009). A diferencia de otros métodos de orientación como el presentado por Yuya Mimasu, et al. (2020), el método de orientación propuesto describe cómo la teoría de los movimientos naturales que realizan los seres vivos para acercarse a un objeto cerrando espacios, se puede utilizar para la planificación de trayectorias de operaciones de proximidad. Dado que todos los espacios se pueden cerrar al mismo tiempo, se pueden lograr las condiciones finales de la trayectoria, incluida una velocidad relativa cero o pequeña, si se desea. Para validar el enfoque se crea un estudio de simulación en el espacio para recrear las maniobras orbitales en las proximidades del pequeño cuerpo celeste, los resultados de la simulación revelan las ventajas de los métodos utilizados. Se consideran parámetros orbitales como perturbaciones (radiación gravitacional potencial y presión solar) y parámetros de nave espacial y parámetros de control. La dinámica de la trayectoria orbital se simula en un control de trayectoria de seguimiento basado en el controlador PD-PWPF (Valenzuela Najera R., et. al., 2020).

2. Planteamiento.

Las misiones de encuentro y aterrizaje han sido un pilar de la exploración espacial desde el programa Apolo hasta las operaciones actuales con la Estación Espacial Internacional. Actualmente sigue habiendo un interés creciente en varios tipos de misiones que no solo se basan en el encuentro y el acoplamiento, sino que también se basan en el encuentro y el aterrizaje en un asteroide. Por ejemplo, la exploración del espacio ha permitido el descubrimiento de nuevos cuerpos celestes que se identifican como potencialmente explorables. De los numerosos cuerpos pequeños en el espacio exterior, los asteroides cercanos a la Tierra (NEA) son de interés debido a su accesibilidad y amenaza de colisión con la Tierra. Además, NEA podría plantear recursos materiales que pueden ayudar a mantener la exploración espacial sostenible. Los NEA, también conocidos como objetos cercanos a la Tierra (NEO), se caracterizan por sus órbitas heliocéntricas. Una propiedad fundamental de un NEA es su masa total ya que controla la atracción gravitatoria que se ejerce sobre una nave espacial. Además, las perturbaciones no gravitacionales también están asociadas con el sol y la órbita heliocéntrica debido a la transferencia de impulsos de los fotones solares que golpean y se alejan del cuerpo en órbita. Tanto las fuerzas gravitatorias como las no gravitatorias ejercen una fuerza externa sobre la nave espacial en las proximidades del cuerpo celeste que puede afectar las maniobras de encuentro y aterrizaje. Las maniobras espaciales son una tarea desafiante principalmente debido al entorno sin aire y de gravedad reducida, lo que puede causar rebotes descontrolados si la trayectoria no se planifica adecuadamente.

2.1 Antecedentes

El trabajo realizado en (Valenzuela Najera R., et. al., 2020) se realizó un estudio de aplicación de la teoría Tau (Coupling-Tau & Intrinsic Tau) para demostrar que es posible realizar planeación de trayectorias de aterrizaje de una nave espacial sobre un cuerpo celeste. Los resultados obtenidos muestran como se puede iniciar el movimiento desde una posición flotante con cero o cierta cantidad de velocidad inercial hasta una posición final de velocidad y aceleración cero. El planteamiento de las ecuaciones servirá de punto de partida crear nuestro propio algoritmo en este proyecto.

2.2 Marco teórico

Las ecuaciones y conceptos teóricos han quedado descritos en el documento de exposición del presente proyecto.

3. Objetivos (general y específicos)

Demostrar mediante el análisis matemático la aplicación de la *Teoría Tau G Improved* en aplicaciones de movimientos de alcance en operaciones espaciales.

- Analizar la dinámica del cuerpo celeste objeto de estudio
- Analizar el método de definición de trayectoria basado en la teoría TAU
- Formular matemáticamente las operaciones de proximidad con el cuerpo celeste
- Determinar la dinámica orbital correspondiente al encuentro de una nave espacial
- al cuerpo celeste.
- Realizar simulaciones de la evolución de la trayectoria en SIMULINK®

4. Metodología.

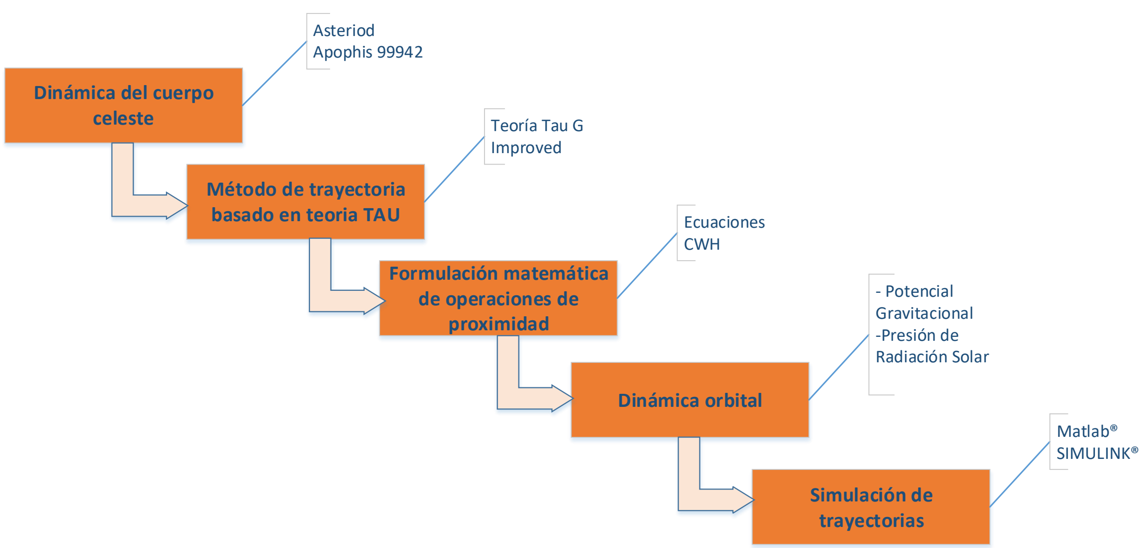

Figura.4.1: Metodología simulador de trayectoria en operaciones de proximidad en maniobras espaciales.

5. **Instituciones, organismos o empresas de los sectores social, público o productivos participantes (Si aplica).**
   Universidad Autónoma de Cd. Juárez / Instituto de Ingeniería y Tecnología / Departamento de Ingeniería Industrial y Manufactura / Programa de Ingeniería en Sistemas Automotrices.

6. **Resultados.**
   Se obtienen como resultados un estudio de simulación donde se puede apreciar la evolución de la trayectoria dadas las condiciones iniciales para 2 maniobras espaciales, la sección de anexos muestra el trabajo realizado para el proyecto.

7. **Productos generados**
   Se ha generado un artículo el cual se encuentra en proceso de revisión interna antes del envío a solicitud de publicación a un *journal*, por motivos de confidencialidad no se muestra el resultado ya que no ha sido publicado aún.

8. **Conclusiones.**
   Mediante el estudio realizado como parte de las actividades del proyecto, se ha demostrado la factibilidad del uso de la *Teoría Tau G improved* en maniobras espaciales de acercamiento y operaciones de proximidad, por lo tanto es importante señalar que sería importante en continuar aplicando el concepto de la teoría en posibles misiones espaciales futuras en satélites artificiales y/o pequeños cuerpos celestes dentro o fuera de la órbita terrestre.

9. **Mecanismos de transferencia. (Si aplica).**
   No aplica.

10. **Contribución e impacto del proyecto.**
    En función de los objetivos y la metodología establecidos, la principal contribución definida por el presente estudio es el de ofrecer una contribución a la comunidad científica (investigadores y estudiantes) un método alternativo de generación de trayectoria de vehículos no tripulados aéreos o terrestres. Los objetivos de recorrido a corta distancia previamente bien definido, donde las condiciones iniciales marcan el recorrido hacia un punto en el espacio tridimensional hasta una condición final de velocidad diferente de cero son alcanzables mediante la aplicación de la ***Teoría Tau G Improved,*** donde las condiciones de la evolución de la trayectoria, perfil de velocidades y aceleración, pueden ser cambiados solo por el simple hecho de modificar la constante tau.

11. Impacto económico, social y/o ambiental en la región.

La aplicación del método puede ser ampliamente utilizado en aplicaciones robóticas, vehículos terrestres, aéreos, (tripulados y no tripulados), así como el principal objeto de estudio: manipulación de sondas espaciales en maniobras dentro y fuera de la órbita terrestre. La aplicación de la teoría Tau para la planeación de trayectorias en estos tipos de vehículos, es un concepto relativamente nuevo que es relacionado con los sistemas bio-inspirados, los cuales tienen un gran impacto en aplicaciones de desarrollos tecnológicos en el área de robótica. Nuevas líneas de investigación pueden generarse del estudio en este proyecto en aplicaciones de vehículos autónomos y no tripulados.

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13. Anexos.

Anexo A. Articulo generado en revisión interna

Bio-Inspired Method for Performing Proximity Operations to Reach a Hovering Position.

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Abstract— **Maneuvers in space between two space objects, one called a chaser, which is a spacecraft, and the other called a target, which is usually an artificial satellite or celestial body, are frequently performed in what we know as Low Earth Orbit. (LEO). Most of the artificial satellites of our planet are there. Such space movements for different purposes, are rendezvous and docking maneuvers, which should be guided according to The Clohessy–Wiltshire equations, which describes a simplified model of orbital relative motion, in which the target is in a circular orbit, and the chaser spacecraft is in an elliptical or circular orbit. Similarly, space maneuvers in the vicinity of small celestial bodies are essential movements to advance the status of exploration, collection of geographic data, measurement of physical properties among other orbit tasks. Before a spacecraft reaches a hovering position over an asteroid to perform orbital tasks close-proximity operations must be performed. Since the thrust movements of spacecraft within the orbit of a small object such as an asteroid depend only on the acceleration forces provided by the thrusters, the correct planning of the execution of the close trajectories in the orbit of the celestial body it is of great importance, firstly, to prevent the vehicle from reaching escape velocity and secondly, the minimum use of fuel in the maneuver. In this paper a simulation of close-range and far-range rendezvous trajectories planning are shown to reach a hovering position on Near-Earth Asteroid (NEA). For this purpose, a Bio-Inspired trajectory planning method based on Tau-G theory, which explains how the movements of animals and humans are sensorially directed to approach an object by closing gaps, is used to reach the final position of the spacecraft close to the space body. The solution of Clohessy-Wiltshire equations is used to find the required state variables at each time instant. In addition, the required space maneuvers affected by disturbances and orbital uncertainties are described in the work and are analyzed in Simulink by means of a PD-PWPF trajectory tracking control. The results of the simulation describe the performance of the spatial movements, highlighting a final velocity different than zero at each stage, which sometimes is necessary to maintain the spaceship with residual velocity while it is in hovering position. Additionally, the switching control signal required by the thrusters is calculated by the PWPF. The advantages of the method with respect to a commonly used approach are that different kinematic behaviors can be obtained by modifying only the single variable named the Tau constant.**

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1. INTRODUCTION.

Rendezvous missions have been a mainstay of space exploration from the Apollo program through present day operations with the International Space Station. Nowadays, there is a growing interest in several mission types that not only rely on rendezvous and docking on artificial satellites inside low Earth-orbit, but also rely on rendezvous and landing on Near-Earth Asteroid. For example, the exploration space exploration has allowed the discovery of new celestial bodies which are identified as potentially explorable. From the numerous small bodies in outer space, Near-Earth Asteroids (NEA) are of interest due to their accessibility and Earth collision threat. Besides, NEAs might pose material resources that can help to maintain space exploration sustainable. NEAs are also known as Near-Earth Objects (NEO) and are characterized by their heliocentric orbits. A fundamental property of an NEA is its total mass since it controls the gravitational attraction that is exerted on a spaceship. As well non-gravitational disturbances are also associated with the sun and the heliocentric orbit due to the impulse transfer of the solar photons that strike and recede from the orbiting body. Both gravitational and non-gravitational forces exert an external force on the spacecraft in the vicinity of the celestial body that can affect the rendezvous and landing maneuvers. A study of the dynamics of the asteroid with respect to gravitational and solar disturbances is considered in this work and included in the mathematical formulation as additional external forces that can change the course of the planned trajectory. In the following sections guidance strategies related to rendezvous and close-proximity operations between a spacecraft and a small celestial body are included. Additionally, in this paper a study of far-range and close-range rendezvous maneuvers trajectory design towards asteroid 99942 Apophis exploration are proposed (Figure 1). First space explorations of the asteroid 99942 Apophis placed it in a place of potential threat of collision against planet Earth, for this reason, some researchers drew on the task of designing specific experimental missions at low cost (Scheeres 2006; Howard & Gillet, 2007; Wagner & Wie 2009).

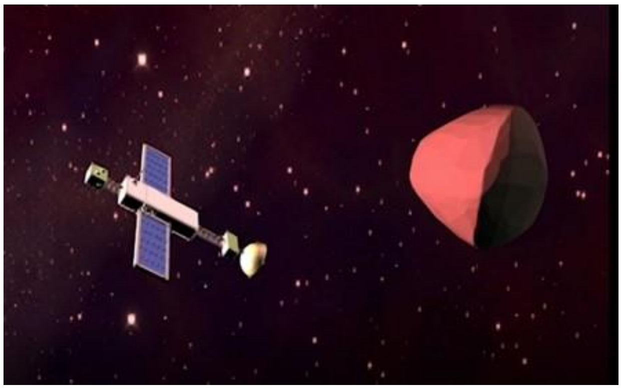

Figure 1. Close-proximity operations on Apophis Asteroid: Illustration. (Credits: Angel E. Garibay Valenzuela).

Whatever will be the mission, operations of rendezvous which are defined as: Transfer from phasing orbit to first aim point in close vicinity of target, are required. Some technologies to achieve orbital transfer to celestial bodies such as solar sail rendezvous (Mengali & Quarta, 2009; Zeng, Gong & Li 2014) and optimal low thrust (Zhang, JungFeng & ShenngPing, 2014; Rassoto, Armellin & DiLizzia, 2016) have been studied. Then when the phasing is completed, far-range and close-range rendezvous operations defined as: Reduction of relative distance to target acquisition of final approach line, are required. The final line approach is typically referred to in the literature as the status conditions ready for the capture of the target (Goodman, 2001; Wertz & Bell, 2003; Franz, 2007; Sherryl 2003; Yazhong, Jin & Guojin, 2014; Spencer 2015; Jewinson 2017; Kumar 2018; Starek, 2016) generally described as a second spacecraft, the first spacecraft that performs the maneuvers is known as chaser.

The main purpose of this research is to analyze the use of a bio-inspired method based on Tau-theory (Lee, 1976; Lee, 1993, Lee, 2003, Lee, 2009). Unlike other guidance methods such as the one presented by Yuya Mimasu, et al. (2020), the proposed guidance method describes how the theory of natural movements performed by living beings to approach an object by closing gaps, it can be used for close-proximity operations trajectory planning. Since the gaps can all be closed at the same time, the final conditions of the trajectory can be achieved, including zero or small relative velocity, if desired. One advantage of the method is that with a single coupling constant, the kinematic properties of the trajectory can be modified to obtain different performances. Additionally in this sense the approach of the equations is used to find the state variables initial condition at each time instant.

To validate the approach to a simulation study in space is created to recreate the orbital maneuvers on the vicinity of the celestial small body, the simulation results reveal the advantages of the methods used. Orbital parameters such as disturbances (potential gravitational and solar pressure radiation) and spacecraft parameters and control parameters are considered. The dynamics of the orbital trajectory are simulated in a tracking trajectory control based on PD-PWPF controller (Valenzuela Najera R., 2020).

2. PROBLEM STATEMENT.

Consider an orbital scenario in which a spacecraft performs close-proximity operations within the gravitational influence of a small celestial body to approach and reach a hovering position. In Figure 2, vector $\delta\mathbf{s}(t_i)$ represents the initial conditions of relative position and relative velocity required to perform each rendezvous operation.

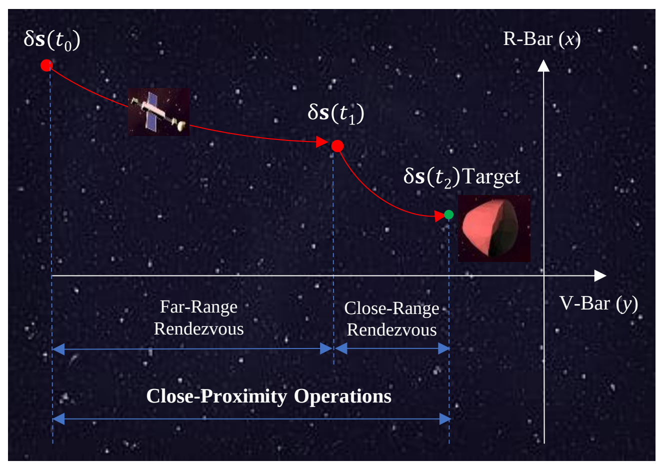

Figure 2. Rendezvous operations plan with the celestial body.

Small celestial bodies near Earth (NEA's) draw the attention of scientists in the study of their geography and orbital behavior around the sun. One of the asteroids that has attracted more interest in this regard is the asteroid 99942 Apophis whose physical and geometric properties are published in (Valenzuela Najera R., 2020).

Table 1. Celestial body properties.

Property	Value
Mean diameter.	d = 0.320 km
Mass	m_a = 4.3x10^{10} kg
Ellipsoid semi-major axis	α = 0.370 km
Ellipsoid semi-axis medium	β =1.06Υ
Ellipsoid semi-axis minor	Υ = α /1.5

Orbit eccentricity	e = 0.19119530
Semi-major axis	a = 137,994,806 km

3. REFERENCE FRAME

The dynamics of the spacecraft motion over the asteroid, is based on linearized movement about a circular orbit, which is valid for celestial bodies (Scheeres, 2014) with eccentricity values $0 < e < 1$ described for elliptic orbits (Schaub, John & Junkins, 2009). Figure 3 represents the reference frame for the spacecraft relative motion which is called orbit fixed frame.

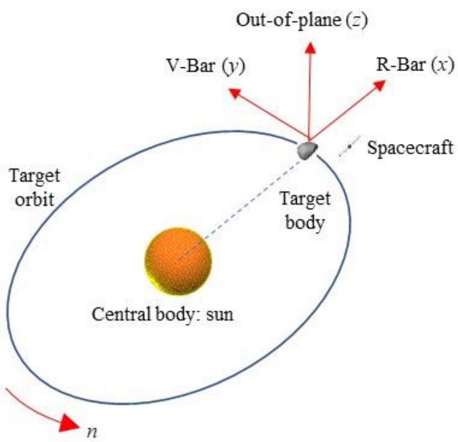

Figure 3. Orbit fixed reference frame.

4. TAU THEORY-BASED GUIDANCE METHOD.

An important action in the planning of space maneuvers is the trajectory that the spacecraft will follow towards the target, in this sense, a bio-inspired method of trajectory generation based on the contact time principle of the Tau theory is selected to demonstrate its usefulness in the generation of short - and long-range trajectories in orbital operations. The Tau Theory developed by Lee (2009) explains how the movements of a living being are controlled through sensory feedback from its own physical abilities. An extensive bibliographical collection about the Tau theory can be found in (Lee, 1976, Lee 1998, Lee, 2009) where the development of the equations of motion based on the 3 fundamental principles of the theory is explained (Lee, 1998)

Applications of Tau theory in motion control have been developed to control robots' motion (Zhang, Cheng & Zhao, 2017) and for similar tasks such as perching, docking, braking, or landing in unmanned aircraft systems (Kendoul &Ahmed, 2010; Kendoul, 2013; Zhang, Xie & Ma, 2014), a summary table of application of the tau theory is described by Yang, Fang & Li, (2016). The work done by Valenzuela Najera (2020), describes an application of the Tau theory in soft landing maneuvers on a celestial body. Starting from the action gap-closing equations of motion, the mathematical principles of Coupling-Tau and Intrinsic-Tau are used to develop landing trajectories in cases of non-zero initial velocity and zero initial velocity respectively and final velocity equal to zero in both cases.

In this sense, for our study, the improved Intrinsic-Tau principle will be used to perform proximity operations within the gravitational influence of a celestial body. The time tau $\tau(t)$ of each spatial and/or force gap be expressed as the first-order time-to-closure of the action-gap at the current rate of closure, in V-Bar direction.

$$\tau(t) = \frac{y(t)}{\dot{y}(t)} \tag{1}$$

4.1. Improved Intrinsic-Tau G.

Operations where the final velocity must be non-zero, an initial velocity V_G is included to the original intrinsic movement equation (Lee, 1998; Zhang, Xie & Ma, 2014), which derives a new improved formulation of the Intrinsic-tau movement strategy given by (Yang, Fang & Li, 2016)

$$\begin{aligned} G_v(t) &= -\frac{1}{2}at^2 + V_G t + G_0 \\ \dot{G}_v(t) &= -at + V_G \\ \ddot{G}_v(t) &= -a \end{aligned} \tag{2}$$

G_0 represents the initial intrinsic gap and is the acceleration. Considering that our spacecraft moves to change its position in V-Bar and R-Bar direction, a transfer maneuver is performed from time $t = 0$ to T in a specific position (x_0, y_0) and initial velocity $(\dot{x}_0, \dot{y}_0)$ to reach a position (x_T, y_T) and nonzero final velocity $(\dot{x}_T, \dot{y}_T)$, the equations that describe the movement in y-direction are defined for the Tau-G improved strategy (Yang, Fang & Li, 2016):

$$\begin{aligned} y(t) &= y_T + \dot{y}_T(t-T) - \frac{Yy_0}{G_0^{1/k_{yG}}} G_v^{1/k_{yG}} \\ \dot{y}(t) &= \dot{y}_T - \frac{Yy_0}{k_{yG} G_0^{1/k_{yG}}} \dot{G}_v G_v^{1/k_{yG}-1} \\ \ddot{y}(t) &= -\frac{Yy_0}{k_{yG} G_0^{1/k_{yG}}} G_v^{1/k_{yG}-2} \left(\frac{1-k_{yG}}{k_{yG}} \dot{G}_v^2 + G_v \ddot{G}_v \right) \end{aligned} \tag{3}$$

where k_{yG} is a constant parameter controlling gap convergence along the y- axis. By substituting G_v and $\dot{G}_v$ in the position and velocity Eqns. (3) respectively, it is determined that:

$$\begin{aligned} Yy_0 &= y_T - y_0 - \dot{y}_T T \\ V_G &= \frac{k_{yG} \Delta\dot{y}_0 G_0}{Yy_0} \end{aligned} \tag{4}$$

$\Delta\dot{y}_0 = \dot{y}_T - \dot{y}_0$ represents the final target state at time T. In this sense the substitution of Eqns. (4) in the position Eqn. (2), leads to the definition of the parameters G_0 and V_G, which are identified as bonding of intrinsic movements due to gravitational effects (Yang, Fang & Li, 2016) denoted by:

$$G_0 = \frac{Yy_0 gT^2}{2\left(Yy_0 + k_{yG}\Delta\dot{y}_0 T\right)}$$
$$V_G = \frac{k_{yG}\Delta\dot{y}_0 gT^2}{2\left(Yy_0 + k_{yG}\Delta\dot{y}_0 T\right)} \tag{5}$$

By knowing the components of position and velocity both initial and final, Eqns. (2-5) can be applied to analyze the final state of the rendezvous trajectory, which will depend on the value defined for the constant k_{yG} . Figure 4 shows the trajectory time-history evolution using different values of k_{yG} which shows that $0 < k_{yG} < 0.5$, $(y_0, \dot{y}_0, \ddot{y}_0) \rightarrow (\dot{y}_T, \ddot{y}_T, 0)$ when $t \rightarrow T$ (Yang, Fang & Li, 2016).

The final state of the movement with the improved Tau G strategy will depend on the initial conditions and the values of the constant k_{yG}, we can conclude that the final conditions for the improved tau G theory are reached only when when $0 < k_{yG} < 0.5$. See table "*Relations between final movement states and k-value, guided by the improved tau-G strategy*" in Yang, Fang & Li, (2016).

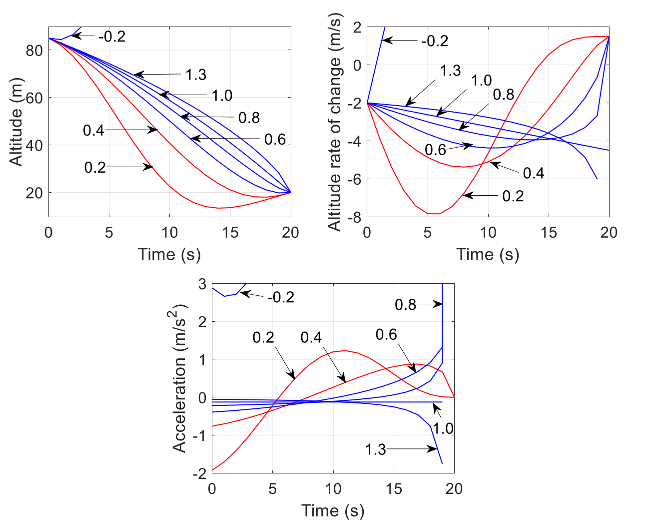

Figure 4. Improved Intrinsic Tau strategy at different k_{yG}-values.

5. ORBITAL DYNAMICS.

The analysis of the operating conditions of the spacecraft allows us to define and establish the forces that intervene in the space maneuver and that can affect the expected result. This section describes the motion equations for rendezvous maneuvers in addition to disturbances due to the asteroid's gravity force and solar radiation pressure as they affect the performance of the spacecraft in the vicinity of a celestial body. These equations can be used to model the effects of a small-body's orbit around the Sun on the spacecraft dynamics when the eccentricity of the orbit of the two gravitating primaries is nearly zero. The Hill three-body problem is a valid approximation of the circular restricted three-body problem for spacecraft motion near the small-body if spacecraft motion occurs inside the small-body's Hill sphere (Nazakawa & Ida, 1988), whose radius is given by:

$$R_{HILL} = a\sqrt[3]{\frac{m_a}{3M}} \tag{6}$$

M is the center body mass (Sun).

5.1 Hill's Equations.

By considering disturbances affecting the celestial body and the motion to achieve proximity operations for circular orbits, equations of relative motions called Hill's equation (Hintz, 2015) are describes as:

$$\begin{aligned} \ddot{x} - 2n\dot{y} - 3n^2 x &= \mathbf{f}_x + \nabla \boldsymbol{u}_x + \boldsymbol{a}_{SRP_x} \\ \ddot{y} + 2n\dot{x} &= \mathbf{f}_y + \nabla \boldsymbol{u}_y + \boldsymbol{a}_{SRP_y} \\ \ddot{z} + n^2 z &= \mathbf{f}_z + \nabla \boldsymbol{u}_z + \boldsymbol{a}_{SRP_z} \end{aligned} \tag{7}$$

where:

$n = \sqrt{\frac{\mu_s}{a^3}}$: celestial body mean motion.

$\mu_s = GMs$: gravitational parameter, where G is the gravitational constant and M the mass of the central body.

$a =$ radius (semi-axis major) of the target orbit.

$\mathbf{f}_x, \mathbf{f}_y, \mathbf{f}_z$: spacecraft vehicle input thruster acceleration forces,

$\nabla \boldsymbol{u} \in \mathbb{R}^{3\times1}$: gradient of the gravitational potential function

$\boldsymbol{a}_{SRP} \in \mathbb{R}^{3\times1}$:disturbance vector caused by solar radiation pressure.

For distances between chaser(spacecraft) and target that are very small compared with the distance to the center of the central body , a linearized solution of the equations of relative motion has been derived from the Hill equations by W. H. Clohessy and R. S. Wiltshire (Fehse, 2003). The resulting equations of motion for constant input forces are Eqns. (8).

$$\begin{aligned} x(t) &= (4-3c)x_0 + \frac{s}{n}\dot{x}_0 + 2\left(\frac{1-c}{n}\right)\dot{y}_0 \ldots \\ &+ \frac{\mathbf{f}_x}{n^2}(1-c) + \frac{2}{n^2}\mathbf{f}_y(s-nt) \\ y(t) &= 6(s-nt)x_0 + y_0 - 2\left(\frac{1-c}{n}\right)\dot{x}_0 + \left(\frac{4}{n}s - 3t\right)\dot{y}_0 \ldots \\ &+ \frac{2}{n^2}\mathbf{f}_x(nt-s) + \mathbf{f}_y\left(\frac{4}{n^2}(1-c) - \frac{3}{2}t^2\right) \\ z(t) &= z_0 c + \frac{\dot{z}_0}{n}s + \frac{\mathbf{f}_z}{n^2}(1-c) \end{aligned} \tag{8}$$

$s = \sin(nt)$ and $c = \cos(nt)$

A complementary solution for CWH (Clohessy-Wiltshire-Hill) equations where $\mathbf{f} = 0$ (Hintz, 2015) represent relative motion of the satellite without thrust forces is given by:

$$\begin{aligned} x(t) &= (4-3c)x_0 + \frac{s}{n}\dot{x}_0 + 2\left(\frac{1-c}{n}\right)\dot{y}_0 \\ y(t) &= 6(s-nt)x_0 + y_0 - 2\left(\frac{1-c}{n}\right)\dot{x}_0 + \left(\frac{4}{n}s - 3t\right)\dot{y}_0 \\ z(t) &= z_0 c + \frac{\dot{z}_0}{n}s \end{aligned} \tag{9}$$

By differentiating Eqns. (9) with respect to time, the relative velocity components for the force-free motion are obtained as follows:

$$\begin{aligned}\dot{x}(t) &= 3nsx_0 + c\dot{x}_0 + 2s\dot{y}_0 \\ \dot{y}(t) &= 6n(c-1)x_0 - 2s\dot{x}_0 + (4c-3)\dot{y}_0 \\ \dot{z}(t) &= -nsz_0 + c\dot{z}_0\end{aligned} \tag{10}$$

To facilitate solution for orbital rendezvous it is convenient to express the analytical solution in a matrix-vector form as:

$$\delta\mathbf{s}(t) = [\delta\mathbf{r}(t) \quad \delta\mathbf{v}(t)]^{\mathrm{T}} = [x_0 \quad y_0 \quad z_0 \quad \dot{x}_0 \quad \dot{y}_0 \quad \dot{z}_0]^{\mathrm{T}} \tag{11}$$

$$\delta\mathbf{s}(t) = \mathbf{\Phi}(t)\delta s(0), \mathbf{\Phi}(0) = \mathbf{I}_6 \tag{12}$$

Component denotes the state transition matrix for the Eqn. (9-10):

$$\mathbf{\Phi}(t) = \begin{bmatrix} (4-3c) & 0 & 0 & \frac{s}{n} & \frac{2}{n}(1-c) & 0 \\ 6(s-nt) & 1 & 0 & -\frac{2}{n}(1-c) & 4s-3nt & 0 \\ 0 & 0 & c & 0 & 0 & \frac{s}{n} \\ 3ns & 0 & 0 & c & 2s & 0 \\ -6n(1-c) & 0 & 0 & -2s & (4c-3) & 0 \\ 0 & 0 & -ns & 0 & 0 & c \end{bmatrix} \tag{13}$$

An advantage of the linearized, CWH frame is that the standard orbital calculations are simplified. The partition of the 6x6 transition matrix to four 3x3 submatrices as denoted as:

$$\mathbf{\Phi}(\mathrm{t}) = \begin{bmatrix}\mathbf{M}(t) & \mathbf{N}(t) \\ \mathbf{S}(t) & \mathbf{T}(t)\end{bmatrix} \tag{14}$$

From Eqn. (14), the relative position vector $\delta\mathbf{r}(t)$, and the relative velocity $\delta\mathbf{v}(t)$vector are given by:

$$\begin{aligned}\delta\mathbf{r}(\mathrm{t}) &= \mathbf{M}(t)\delta\mathbf{r}(0) + \mathbf{N}(t)\delta\mathbf{v}(0) \\ \delta\mathbf{v}(\mathrm{t}) &= \mathbf{S}(t)\delta\mathbf{r}(0) + \mathbf{T}(t)\delta\mathbf{v}(0)\end{aligned} \tag{15}$$

If a rendezvous maneuver is required to perform from an arbitrary initial state $\delta\mathbf{s}(0)$ to the origin of CWH reference frame with a specified time (Craig, 2018), the required velocity vector at initial time $t = 0$ is:

$$\delta\mathbf{v}(0)_{req} = -\left(\mathbf{N}(t)\right)^{-1}\mathbf{M}(\mathrm{t})\delta\mathbf{r}(0) \tag{16}$$

With the spacecraft's current relative initial velocity $\delta\mathbf{v}(0)$ and the required velocity $\delta\mathbf{v}(0)_{req}$, the $\Delta\mathbf{v}_0$ provided by the thrusters at t = 0 is given by:

$$\Delta\mathbf{v}_0 = \delta\mathbf{v}(0)_{req} - \delta\mathbf{v}(0) \tag{17}$$

The initial velocity change $\Delta\mathbf{v}_0$ places the spacecraft on a trajectory that will intercept the origin at time *t*. After starting the movement with a relative initial velocity $\delta\mathbf{v}(0)_{req}$ the spacecraft reaches the origin of the reference frame CWH $\delta\mathbf{r}(t) = \mathbf{0}$, where the final speed of the maneuver is given by:

$$\delta\mathbf{v}(t) = \mathbf{S}(t)\delta\mathbf{r}(0) + \mathbf{T}(t)\delta\mathbf{v}(0)_{req} \tag{18}$$

When the final velocity is equal to zero, a relative velocity required at the end of the movement at time *t*, therefore the second impulse is given by:

$$\Delta\mathbf{v}_f = -\delta\mathbf{v}(t) \tag{19}$$

This second impulse simply cancels out the spacecraft's residual relative velocity when it arrives at the origin or target. The magnitude of the total velocity increment for the two-impulse maneuver

$$\Delta\mathbf{V} = |\Delta\mathbf{v}_0| + |\Delta\mathbf{v}_f| \tag{20}$$

The above equations will help us determine the initial conditions to start a close-proximity maneuver, in addition to calculating the necessary $\Delta\mathbf{V}$ provided by the thrusters.
In space maneuvers the consideration of the amount of effort (**ΔV**) to carry out a transfer trajectory is of great importance for fuel optimization. Thrust maneuvers to a first approximation can be treated as impulses, i.e. as instantaneous changes of velocity at the time of maneuver. Because limitations of thrust level are available, such ideal impulsive maneuvers do not exist, and constant thrust forces must be applied over a time to perform the maneuver. In real applications all thrusters have a finite force level (Fehse, 2003), for this reason, a constant force must be applied over a certain time *t* to achieve a desired **ΔV.**
The determination of the amount of effort to perform a space maneuver by using *improved intrinsic Tau* strategy is given by:

$$\Delta\mathbf{V}_{\mathbf{Subtotal}} = |\Delta\mathbf{v}_0| + |\Delta\mathbf{v}_f| - |\Delta\mathbf{v}_G| \tag{21}$$

$\Delta\mathbf{v}_G$ represents the inertial velocity vector of the probe before applying the $\Delta\mathbf{v}_0$ impulse vector for the start of the space maneuver.

5.2. Gravitational Potential.

Acceleration force exerted by the asteroid due its gravitational field on the spacecraft in the rendezvous maneuver is considered by using the general form for the spherical harmonic potential, the gravity field is given as (Scheeres, 2012):

$$u(r,\phi,\theta) = \frac{\mu}{r}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{r_o}{r}\right)^l P_{lm}(\sin\phi)\left[C_{lm}\cos m\theta + S_{lm}\sin m\theta\right] \tag{22}$$

where:
r,ϕ,θ: radius vector, latitude, and longitude, respectively.
$\mu = GM$: asteroid gravitational parameter, where *G* is the gravitational constant and *M* the mass of the body.

r : distance from the mass center of small body to the spacecraft,
r_o : normalizing radius (often chosen as either the maximum radius or mean radius of the body),
P_{lm} : associated Legendre Functions,
C_{lm} and S_{lm} : gravity field harmonic coefficients.

Eqn. (22) is obtained considering the asteroid as a constant density ellipsoid with semi-major axes $\gamma \le \beta \le \alpha$, the simplified form of the gravity potential leads to (Scheeres, 2012) the following considerations:

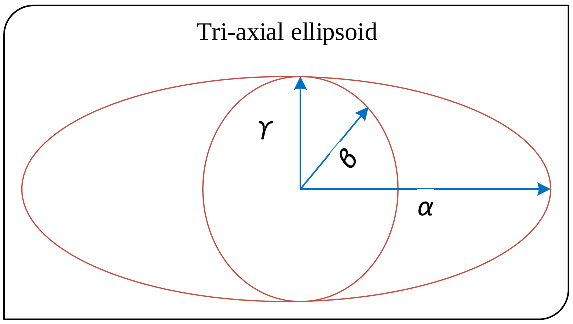

Figure 5. Hemispherical tri-axial model of a celestial body.

The gravitational potential is given by:

$$u = \frac{\mu}{r}\left[1 + \left(\frac{r_0}{r}\right)^2 \left\{C_{20}(1 - \frac{3}{2}\cos^2\phi) + 3C_{22}\cos^2\phi\cos(2\theta)\right\}\right] \quad (23)$$

for which, the second order harmonic coefficients are:

$$C_{20} = \frac{1}{5\alpha^2}\left(\gamma - \frac{\alpha^2 - \beta^2}{2}\right)$$
$$C_{22} = \frac{1}{20\alpha^2}\left(\alpha^2 - \beta^2\right) \quad (24)$$

and the partial derivatives of the potential are expressed as:

$$\frac{\partial u}{\partial x} = u_x = -\frac{3\mu x}{2}\int_{\lambda(r)}^{\infty}\frac{du}{\left(\alpha^2 + u\right)\Delta(u)}$$
$$\frac{\partial u}{\partial y} = u_y = -\frac{3\mu y}{2}\int_{\lambda(r)}^{\infty}\frac{du}{\left(\beta^2 + u\right)\Delta(u)} \quad (25)$$
$$\frac{\partial u}{\partial z} = u_z = -\frac{3\mu z}{2}\int_{\lambda(r)}^{\infty}\frac{du}{\left(\gamma^2 + u\right)\Delta(u)}$$

5.3. Solar Radiation Pressure.

The solar radiation pressure is another important orbital disturbance for missions to small bodies. This work considers a simple model to calculate the acceleration due to solar radiation supposing that the spacecraft presents a constant area perpendicular to the solar line and the incident radiation is reflected (Scheeres, 2012) which is represented in Figure 6.

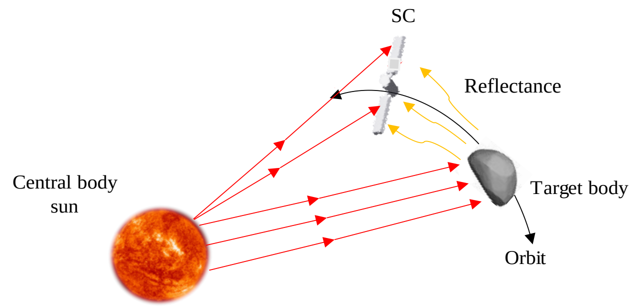

Figure 6. Solar radiation pressure *SRP*.

Since the spacecraft is very close to the target asteroid the magnitude of the solar radiation pressure acceleration can be expressed as:

$$\mathbf{a}_{SRP} = -\frac{(1+\rho)P_o}{B_{sc}\mathbf{d}^2} \quad (26)$$

ρ: Total reflection of the spacecraft,
P_0: Solar constant approximately equal to $1\text{x}10^8 \text{ kg km}^3/\text{s}^2\text{m}^2$
A_{sc}: The reflection area of the spacecraft
M_{sc}: The mass of the spacecraft
$B_{sc} = M_{sc} / A_{sc}$
$\mathbf{d}$: Distance vector from the asteroid to the sun.

6. ORBITAL MANEUVER PLANNING.

The use of equations (1-21) leads us to define a space probe trajectory plan within the gravitational influence of the celestial body. Figure 7 shows the planning performance of the spacecraft in a far and close- range rendezvous operation after a hovering state. The position initial vector is defined as $\delta\mathbf{r}(t_0) = (2890, 3750)$ m and $\delta\mathbf{v}(t_0) = (0,0)$ m/s at orbital vector $\delta\mathbf{s}(t_0)$. By using of the Eqn. (16) the initial velocity required is defined $\delta\mathbf{v}(t_0)_{req} = (-4.8158, -6.2506, 0)\ m/s$, and by Eqn. (17) $\Delta\mathbf{v}(t_0) = (-4.8158, -6.2506)\ m/s$, substituting $\delta\mathbf{r}(t_0) = (x_0, y_0)$, $\Delta\mathbf{v}(t_0) = (\dot{x}_0, \dot{y}_0)$ and the assumed parameters of the final position vector $\delta\mathbf{r}(t_1) = (x_{T_1}, y_{T_1}) = (1000, 1000)\ m$ and the final velocity vector $\delta\mathbf{v}(t_1) = (\dot{x}_{T_1}, \dot{y}_{T_1}) = (-1, -2)\ m/s$, which correspond to the desired final position and final velocity in Eqns. (2-5), it is possible to verify the trajectory evolution and velocity profile with respect to the first orbital maneuver time T_1.

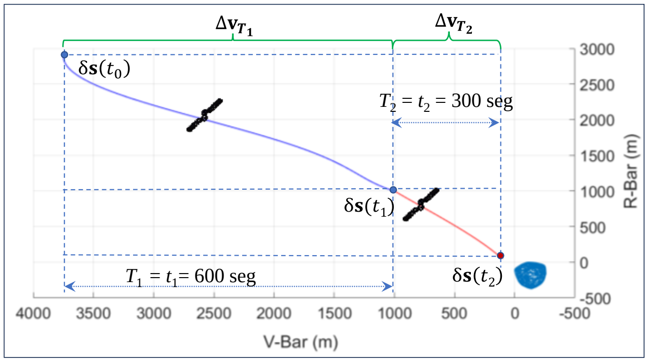

Figure 7. Proximity operations performance planning.

Figure 8 shows the evolution of the position and velocity trajectory vectors over time, where the goals in the planned initial and final conditions are reached.

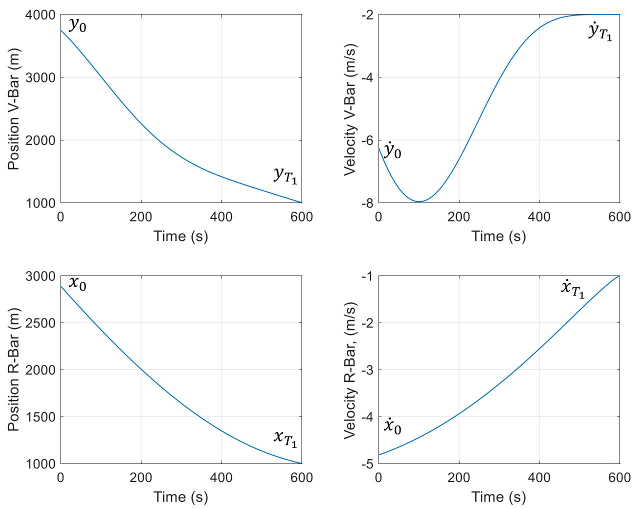

Figure 8. Evolution of the position and velocity in first maneuver, with $k_{xG} = 0.45$, $k_{yG} = 0.15$.

In a CWH inertial reference frame, the final conditions of operation 1 become the initial conditions of operation 2, $\delta\mathbf{r}(t_1) = (1000, 1000)\ m$ and $\delta\mathbf{v}(t_1) = (-1, -2)\ m/s$ at $\delta\mathbf{s}(t_1)$. The new required velocity is, $\delta\mathbf{v}(t_1)_{req} = (-3.3331, -3.3336)\ m/s$. The new initial velocity change $\Delta\mathbf{v}(t_1) = (-2.3331, -1.3336)\ m/s$ is calculated to perform the next proximity operation toward the position vector $\delta\mathbf{r}(t_2) = (x_{T_2}, y_{T_2}) = (55, 95)\ m$. The final desired velocity vector , $\delta\mathbf{v}(t_2) = (\dot{x}_{T_2}, \dot{y}_{T_2}) = (-0.2, -0.8)\ m/s$. The evolution of the position and velocity with respect to time are shown in Figure 9.

7. ORBITAL MANEUVER TRACKING SIMULATION STUDY.

The information generated in the previous section results from the application of the *Improved Intrinsic-Tau* strategy, which represents the *Tau theory-based guidance method* proposed for this present study. $\Delta\mathbf{v}_0$ initial conditions necessary for the first impulse in both first and second orbital maneuvers are calculated using Eqns. (16-17). Such information is used to carry out a trajectory simulation in a coupled controller PD-PWPF (Valenzuela Najera R., 2020) to track the rendezvous trajectory of the spacecraft with the celestial object.

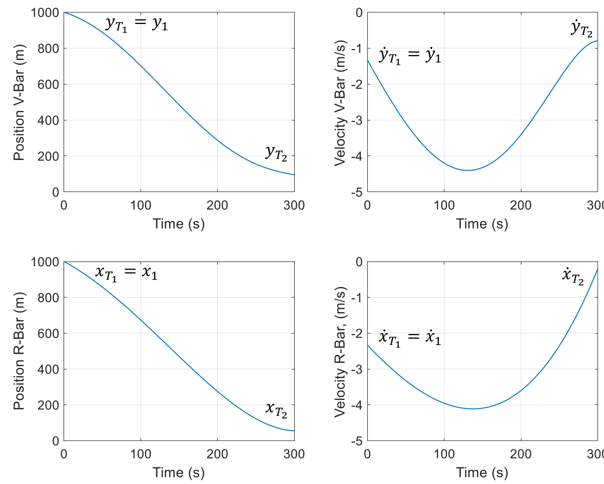

Figure 9. Evolution of the position and velocity in second maneuver, with $k_{xG} = 0.5$, $k_{yG} = 0.35$

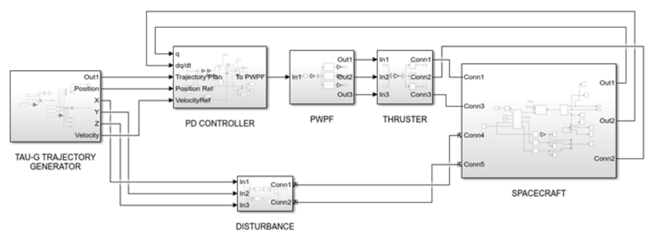

Figure 10. Tau G trajectory guidance and control strategy in Simulink™ environment.

The data generated for the planning of the proximity operations trajectories, allows to estimate the Delta-**V** theoretical necessary to carry out the orbital operations shown in Table 2, with which it is possible to determine the fuel consumption of the spacecraft in the maneuvers.

Table 2. Delta-V determination.

Operation 1	(m/s)
$\lvert \Delta\mathbf{v}_0 \rvert$	7.8907
$\lvert \Delta\mathbf{v}_f \rvert$	7.8907
$\lvert \Delta\mathbf{v}_{T1} \rvert$	2.236
$\Delta\mathbf{v}_{\text{Subtotal 1}}$	13.5454
Operation 2	(m/s)
$\lvert \Delta\mathbf{v}_0 \rvert$	2.6873
$\lvert \Delta\mathbf{v}_f \rvert$	4.714
$\lvert \Delta\mathbf{v}_{T2} \rvert$	0.8246
$\Delta\mathbf{v}_{\text{Subtotal 2}}$	6.5767
$\Delta\mathbf{V}_{\text{Subtotal}} = 20.1221\ m/s$	

Table 3 shows the simulation parameters entered to coupled controller for the simulation of proximity operations.

Table 3. Simulation parameters.

Spacecraft operation 1		
$\delta\mathbf{r}(t_0)$	$(2890, 3750)$	m
$\delta\mathbf{v}(t_0)$	$(0, 0)$	m/s
$\Delta\mathbf{v}(t_0)$	$(-4.8158, -6.2506)$	m/s
$\delta\mathbf{r}(t_1)$	$(1000, 1000)$	m
$\delta\mathbf{v}(t_1)$	$(-1, -2)$	m/s
k_{xG_0}, k_{yG_0}	0.45, 0.15	-
Spacecraft operation 2		
$\delta\mathbf{r}(t_1)$	$(1000, 1000, 0)$	m
$\delta\mathbf{v}(t_1)$	$(-1, -2)$	m/s
$\Delta\mathbf{v}(t_1)$	$(-2.3331, -1.3336)$	m/s
$\delta\mathbf{r}(t_2)$	$(55, 95)$	m
$\delta\mathbf{v}(t_2)$	$(-0.2, -0.8)$	m/s
k_{xG_2}, k_{yG_2}	0.45, 0.45	-
PD – Control Gains		
$Kp_x\ Kp_y$	0.0000012 0.000021	-
$Kv_x\ Kv_y$	53 7.5	-

The PWPF control, spacecraft and asteroid parameters are the same as defined in (Valenzuela Najera R., 2020). Simulation results are shown below.

7.1 Position and velocity trajectory tracking.

A tracking of the position and velocity of the spacecraft on the trajectory during time t_1 is shown in Figure 11 and for time t_2 in Figure 12.

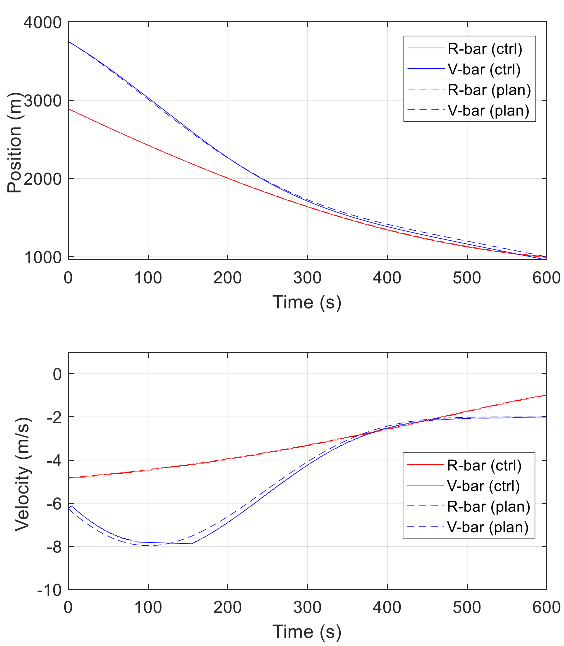

Figure 11. Orbital maneuver No 1 time-history controlled.

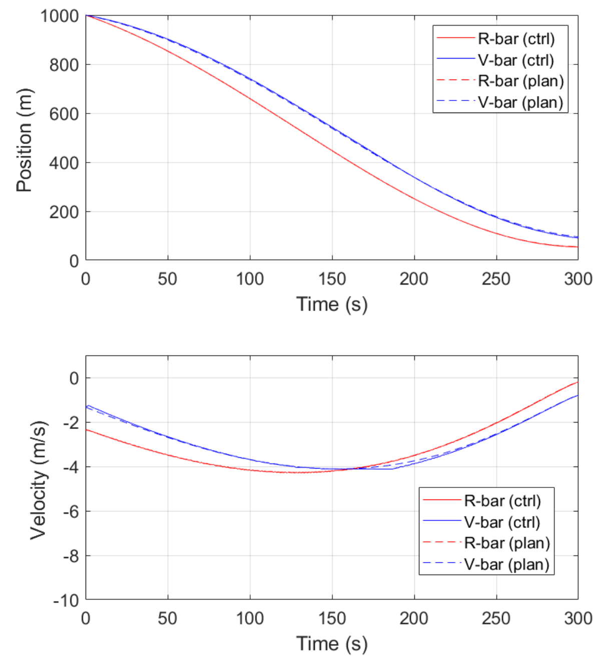

Figure 12. Orbital maneuver No 2 time-history controlled.

7.2 Along track V-bar position and velocity - Radial X-bar position and velocity.

The following Figures 7.13 and 7.14 show the controlled position and velocity along-track axis and radial axis, whose initial and final conditions are met in each maneuver.

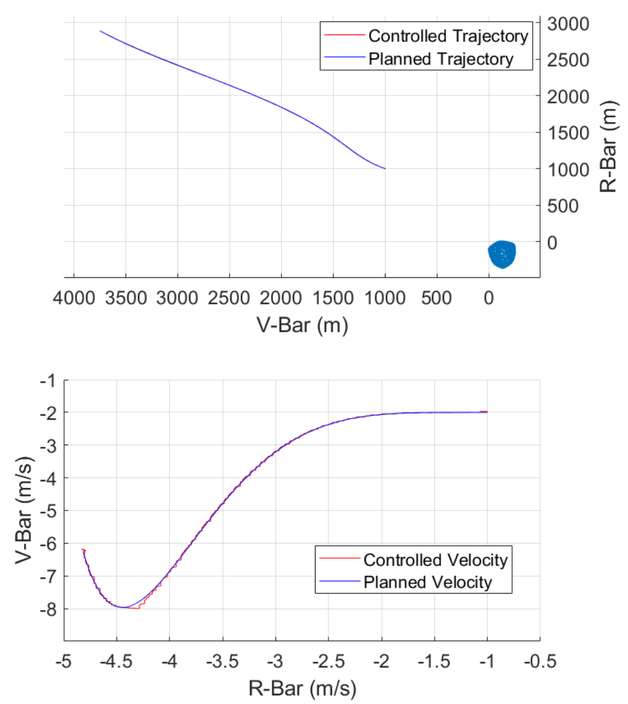

Figure 13. Orbital maneuver No. 1 time-history controlled.

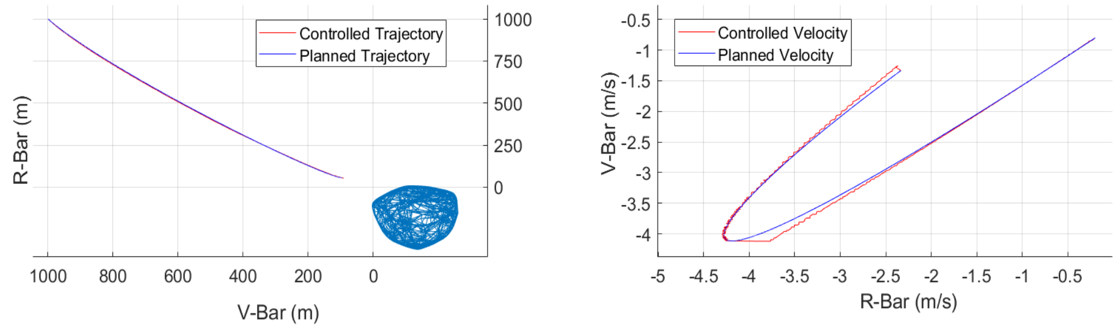

Figure 14. Orbital maneuver No. 2 time-history controlled.

7.3. On/off thruster control signals.

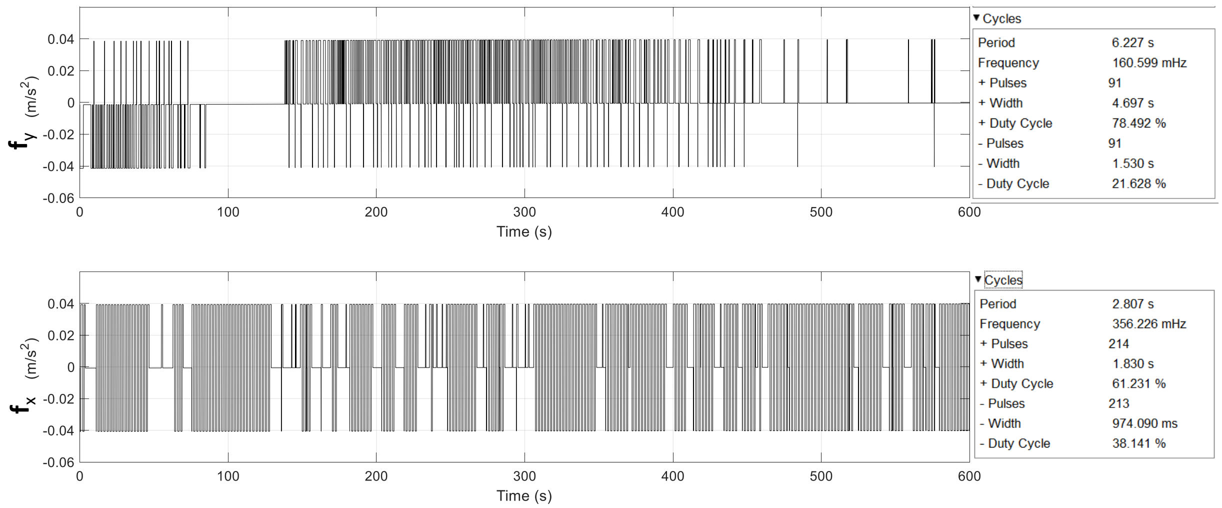

Figure 15: Thruster control signal. Maneuver No. 1

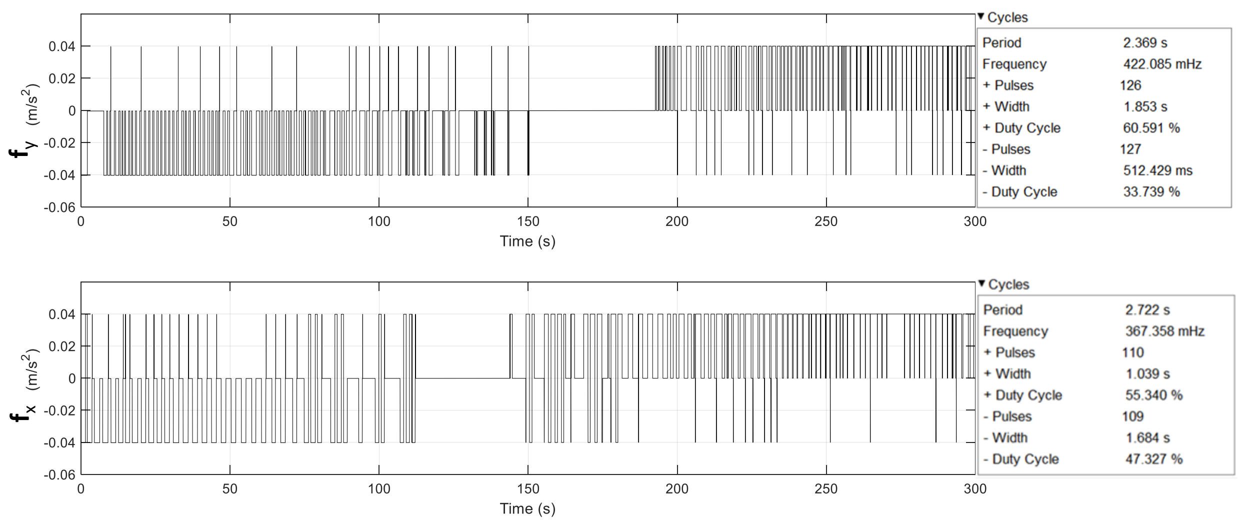

Figure 16: Thruster control signal. Maneuver No.

Considering the force thrust applied on the maneuvers, the total ΔV expenditure for the implementation of the given velocity profile is expressed by:

$$\Delta\mathbf{V}_{\mathrm{Total}} = |\Delta\mathbf{v}_0| + |\Delta\mathbf{v}_f| - |\Delta\mathbf{v}_G| + |\mathbf{f}\Delta t| \qquad (27)$$

f represents the force per unit mass applied to keep the spaceship on the trajectory position and Δt represents the maneuver duration in relation with the on-off thruster control signals**.** From figures 15 and 16 we can define that using the duration of the positive and negative pulses it is possible to determine the percentage of the total time necessary in which the forces of the thrusters will be applied to maintain the trajectory in the target orbit to obtain the total AV , which gives us a result of…….

8. DISCUSSION.

The process of spatial maneuvers is quite a complex spatial operation since all dynamic variables (position, speed, angular rates) are carefully considered. In this work the obtained results show that rendezvous maneuvers can be achieved and at the same time maintain the spacecraft with a certain inertial speed that allows it to remain orbiting within the orbital influence of the celestial body. Furthermore, it is shown that like the information on rendezvous and docking trajectory techniques presented in subchapter 3.3 Fehse (2003), the bio-inspired method of Tau G theory meets the conditions for trajectory planning.: *assumed initial conditions, motion equations, position after a fixed time and duration of the motion to a new position*. Since the movements of the spacecraft in the atmosphere of the small celestial body depend only on the impulses generated by the thrusters, the continuous thrust trajectory generation technique generally used for space maneuvers of rendezvous and docking is analogously compared with the bio-inspired method showed in Table 4.

Table 4. Relationship between space maneuver techniques:

Boundary conditions	Parameter	Continuous thrust Bio-inspired Method
Initial conditions	$\delta\mathbf{r}(t_0)$	- Position of the probe in space with respect to the celestial body
	$\delta\mathbf{v}(t_0)$	- Represents the velocity of the initial state
	$\Delta\mathbf{v}(t_0)$	- It depends on the sum of the initial state velocity and the velocity required to perform the maneuver.
Trajectory performance	Type of maneuver	- Straight line trajectories on V-bar and R-bar. - Station keeping on different altitude and on out-of-plane distance w.r.t. the target. - Transfer by continuous thrust of limited level to a different altitude or to a different position along the target orbit. - Circular fly-around.
Final conditions	$\delta\mathbf{r}(t_f)$	**Continuous thrust** - It depends on Eqn. (9) $\delta\mathbf{r}(t_f) = \delta\mathbf{r}(t_0) + \Delta\mathbf{v}(t_0)t_f$
		Bio-inspired Method - It depends on position Eqns.(3-5) The final position can be known and defined from the beginning of the trajectory
	$\delta\mathbf{v}(t_f)$	**Continuous thrust** - It depends on Eqn. (10)

		Bio-inspired Method - It depends on velocity Eqns.(3-5) The final velocity can be known and defined from the beginning of the trajectory
	$\Delta\mathbf{v}(t_f)$	**Continuous thrust** It depends on: $\Delta\mathbf{V}_{\text{Total}} = \lvert\Delta\mathbf{v}_0\rvert + \lvert\Delta\mathbf{v}_f\rvert + \mathbf{f}\Delta t\rvert$
		Bio-inspired Method Eqn. (27) defines $\Delta\mathbf{V}_{\text{Total}}$

Something interesting to discuss in the profile of the acceleration forces applied to the thrusters to maintain the trajectory of the maneuver shown in figures 15 and 16, is that the real expenditure would be given only by $\Delta\mathbf{V}_{\text{Total}} = \lvert\mathbf{f}\Delta t\rvert$. However, in outer space all possible variables including fuel consumption must be considered to avoid failures in navigation systems.

9. CONCLUSIONS.

With the completion of the presentstudy carried out as part of the project activities, the feasibility of using the Tau G improved Theory in approach space maneuvers and proximity operations has been demonstrated, therefore it is important to note that it would be important to continue applying the concept of the theory in possible future space missions on artificial satellites and/or small celestial bodies inside or outside Earth's orbit.

13.1 Taxonomía de los Roles de Colaborador (con las actividades logradas)

Roles	Definición de los roles	Nombre de él(la) investigador(a)	Figura	Grado de contribución	Actividades a realizar en la ejecución del proyecto
1.Responsabilidad de la dirección del proyecto	- Coordinar la planificación y ejecución de la actividad de investigación.	Dr. René Alberto Valenzuela Nájera	Director del proyecto	Principal	1. Responsable de la planificación de las tareas de investigación y desarrollo del proyecto
2. Responsabilidad de Supervisión	- Elaborar la planificación de las actividades de la investigación (cronogramas y controles de seguimiento), - Describe los roles identificados por el director del proyecto y facilita el apoyo constante a todos los roles para conseguir un trabajo integral, coherente y que llegue a buen término.	Dr. René Alberto Valenzuela Nájera	Supervisor del proyecto	Principal	1. Mantener el seguimiento del as actividades planificadas. 2. Identificar y asignar actividades 3. Comunicar sobre avances del proyecto.
3. Realización y redacción de la propuesta	- Preparación, creación y redacción de la propuesta de investigación, específicamente la redacción, revisión de coherencia del texto, presentación de los datos y la normatividad aplicable para garantizar el cumplimiento de los requisitos.	Dr. René Alberto Valenzuela Nájera	Redactor de la propuesta	Principal	1. Elaboración de la propuesta de Investigación.
4. Desarrollo o diseño de la metodología	- Contribuir con el diseño de la metodología, modelos a implementar y el sustento teórico, empírico y científico para la aplicabilidad de los instrumentos en la ejecución del proyecto.	Dr. René Alberto Valenzuela Nájera	Diseñador de la Metodología	Principal	1. Proponer una metodología basada en procesos, donde las activades a realizar cuenten con una secuencia lógica y /o estén relacionadas entre si
5. Recopilación/ recolección de datos e información	- Ejecuta las estrategias propuestas en acciones encaminadas a obtener la información, haciendo la recopilación de datos y la inclusión de la evidencia en el proceso.	Dr. René Alberto Valenzuela Nájera	Recopilador de datos	Principal / Apoyo	1. Recopila la información relacionada para el diseño y desarrollo de subsistemas incluidos en el proyecto 2. Analiza los datos y aplica estrategias de implementación basados en las metodologías definidas
	- Aplicar métodos estadísticos, matemáticos, computacionales, teóricos u otras técnicas formales	Dr. René Alberto Valenzuela Nájera			1. Aplica métodos, técnicas y estrategias para el desarrollo de

6. Elaboración del análisis formal de la investigación	para analizar o sintetizar los datos del estudio. Verifica los resultados preliminares de cada etapa del análisis, los experimentos implementados y otros productos comprometidos en el proyecto		Analista de datos	Principal / Apoyo	subsistemas del proyecto. 2. Realiza pruebas e interpretación de estas.
7. Preparación, creación y/o presentación de los productos o entregables	- Preparar la redacción del reporté técnico de avance parcial y el reporte técnico final. - Se hace la revisión crítica, la - recopilación de las observaciones y comentarios del grupo de investigación. Y finalmente se procede a la edición del documento a entregar.	Dr. René Alberto Valenzuela Nájera	Editor de reportes técnicos	Principal / Apoyo	1. Prepara la documentación relacionada con: - Informes técnicos CATHI - Resultado final - Realiza productos de investigación (Si aplica) y se encarga de su publicación y divulgación