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RESEARCH ARTICLE

Stress concentration reduction in an axially loaded rectangular bar with an elliptical hole

Reducción de concentración de esfuerzos en una barra rectangular con un agujero elíptico cargado axialmente

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ABSTRACT

Two-dimensional finite element analysis was used for stress concentration factors estimation in a wide range of bar-ellipse height and height-width elliptical hole ratios in a tensile bar. Least squares polynomial equations were fit to the design points. These equations agreed and tension stress concentration factor for elliptical holes was described. Such equation reproduces the curves and through its derivative the optimal values that minimize the concentration factor are estimated. The path of the minimal concentration factor was illustrated in a contour plot.

KEYWORDS: stress concentration factors; ellipse; stress reduction; nominal stress; least squares polynomial.

RESUMEN

Se utilizaron análisis bidimensionales de elementos finitos para estimar los factores de concentración de tensión en una amplia gama de proporciones de altura de barra-elipse y de altura-anchura de agujeros elípticos en una barra de tracción. Las ecuaciones polinomiales de mínimos cuadrados se ajustaron a los puntos de diseño. Estas ecuaciones coincidieron y se describió el factor de concentración de esfuerzos en tensión para agujeros elípticos. Dicha ecuación reproduce las curvas y a través de su derivada se estiman los valores óptimos que minimizan el factor de concentración. La trayectoria del factor de concentración mínimo se ilustró en un gráfico de contorno.

PALABRAS CLAVE: factores de concentración de esfuerzos; elipse; reducción de esfuerzos; esfuerzo nominal; polinomio de mínimos cuadrados.

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I. INTRODUCTION

Stress concentration is defined as the accumulation of stress in a specific area due to an abrupt transition in its geometry. This phenomenon is characterized by significantly higher stress values near the region of geometric change [1]. The term "stress concentration" applies to a structure when there is a relatively large stress gradient in an area of the structure compared to the nominal stress. In a bar subjected to tension, the geometric discontinuities (circular holes, elliptical holes, notches, grooves, fillets, etc.) modify the stress distribution causing relatively higher stresses in certain areas. Stress concentration causes strength degradation and premature failure of structures due to fatigue cracking and plastic deformation frequently occurs at these points [2]; but we can reduce it to an extent through various methods.

Ernst Gustav Kirsch was the one who began to analyze the stresses that occur around a hole in an infinite plate, presenting us with the linear elastic solution for the analysis of stress concentrations in these geometric circumstances [3]. Kirsch's solution contains the well-known factor-of-three stress concentration at the hole under uniaxial loading but can vary from two to four for more complex loading condition like biaxial tension or multiple forces interacting with the geometry in different directions [3].

The flow analogy concept, often called "streamlining", helps in reduction of stress concentration in mechanical components. It came to be used as a golden tip for study of stress concentration of a component. It is the basic analysis of behavior of a system using flow constituents. Velocity distribution due to a fluid flow in a channel seemed like the stress distribution in an axially loaded plate. When the cross section of the channel is reduced abruptly, the velocity of flow increases in order to maintain the flow rate and as a result, the streamlines come closer and the whole streamline path becomes narrower. Same case is observed in stress induced plate. Moreover, equation for both fluid flow and stressed system also came out similar. Lorente *et al.* [4] show that it exists a complete analogy between the configuring of heat and fluid flow and the configuring of the "flow of stresses" in mechanically loaded elastic solid parts by using the simplest possible examples that have analogs in fluid flow and convective heat transfer: single ducts, bifurcated ducts, fins with heat tubes, and single and conjugate boundary layers. To facilitate the flow of stresses, fluid, and heat is to generate the configuration of the flow system [4].

The geometry of an ellipse is defined by the dimensions corresponding to the 2 axes of symmetry. If the axes of the ellipse are equal then a circle is obtained; by making one of the axes very small, a geometry that can be considered a crack is obtained and starting from this analysis is where Inglis [5] used elliptical coordinates to solve the problem of elliptical holes. Elliptical coordinates are a generalization of 2D polar coordinates that are ideally suited to problems involving ellipses. The solution of Inglis is that by relating the parameters a (distance from the origin to the tip on the horizontal axis) and P (radius of curvature at the tip of the horizontal axis) gets maximum stress at the tip of the ellipse. The maximum stresses in a material are limited by its yield or failure strength. However, in a crack the radius of curvature is zero, so it produces an infinite stress concentration at the tip, which means that the result of Inglis is correct when the behavior of the material is governed by Hooke's Law and has no limits on the mechanical properties. In this way, having an infinite yield stress makes the problem linear, enabling its analytical solution.

Knowledge of stress concentration near holes is necessary to ensure reliable design of structural components. Generally, the use of high-strength materials is used in the design of structural parts with high mechanical performance to reduce the stress concentration factor and this brings with it the requirement of a better understanding and modeling of the behavior of these structures [6]. In the design stage of a mechanical component with geometric discontinuities subjected to traction, it must be ensured that the stress concentrations present in the critical areas do not exceed the capacities of the material and to achieve this there are different applicable methods pursuant to the case. Some of the methods used for stress concentration reduction are the removal of nearby material by adding auxiliary circular holes, optimization of the shape of the main hole and auxiliary holes, gradual transitions in changes of cross-sectional area, etc. [2], [7].

Pilkey [8] and Young [9] have published figures and tables about the stress concentration factor in which they consider an immense variety of possible configurations of dimensional ratios in study samples where many of these sources of information consider only a two-dimensional solution of the elasticity theory.

The theory of elasticity shows that the value of the maximum stress of a circular hole in an infinite plate under tension is three times the applied stress when the material remains elastic. Stowell [10] obtained experimental data on the stress concentration of a circular hole in a large, wide plate of aluminum alloy 24S-T3 under tension. He found experimental values of stress concentration factors close to 3 when no section of the plate was deformed beyond the elastic range, however when a higher axial tension force was applied the plate entered the plastic range and the stress concentration factor decreased to 1.4.

Troyani *et al.* [11] have used the Finite Element Method to determine the theoretical values of stress concentration factors in rectangular plates that are subjected to uniform traction with a central hole representing a geometric discontinuity.

The stress concentration factors at the root of the notch or hole in the surface of the plate are inversely proportional functions of its thickness. The factor decreases rapidly and tends to each constant related to Poisson's ratio with increasing plate thickness [6]. Sternberg [12] obtained an approximate solution for 3D stress distributions near a circular hole in an infinite plate of arbitrary thickness based on a modification of the Ritz method in the theory of elasticity. The results of his study critically illuminate the importance of two-dimensional analysis in problems of stress concentration factors. It also reports a slight difference between the stresses obtained in the 2D deformation theory and its 3D solutions. The increase in stress concentration factor for an infinite plate with a cylindrical hole under uniaxial load is less than 3%. Because of this, Folia's and Wang [13] developed a 3D solution using Navier's equation for plates of uniform thickness and stress-free plate faces. The solution results show that the stress concentration factor reaches its maximum value in the middle of the thin plate, while when the plate is thick, the maximum stress concentration factor is obtained near the plate surfaces. In conclusion, the stress concentration factor is sensitive to the thickness of the plate and the Poisson's ratio.

She and Guo [14] used finite element method to analyze the through-the-thickness variation of the stress concentration factor along the wall of elliptic holes in isotropic plates subjected to a remote tensile stress.

In 2010, A. Kotousov published an article that focuses on demonstrating the importance of considering the thickness of the plate being analyzed. Three-dimensional effects, such as the influence of the plate thickness on stress components, are largely ignored or considered as negligible for all practical purposes [15].

Jain and Mittal presented a finite element method study on both isotropic and orthotropic plates with central circular hole subjected to transverse static loading to study the effect of the hole diameter to plate width ratio on the stress concentration factor. They concluded that the stress concentration factor for all stresses plays an important role in all cases of plate boundaries, for a loading at the hole boundary [16].

Darwish and his team presented in their article a precise and modified equation to calculate the stress concentration factor of a uniaxially loaded isotropic plate with a countersunk hole in the center. They built a finite element model using the ANSYS parametric design language to run the analysis [17].

Zhou [18] implemented a finite element analysis to obtain the stress distribution to develop an optimization in the Young's modulus algorithm. Said algorithmic optimization results in obtaining an effective approximation for the optimal design of functionally graded material for stress concentration reduction.

Jaiswal presented in a recent paper a methodology to model a functionally graded material (FGM) using FEM based on ANSYS to reduce the stress concentration by applying an FGM ring around the rectangular slot that is subjected to a tensile load, obtaining as a result that the stress reduction by this method is reduced compared to the case of homogeneous material and by modifying the parameters of the FGM ring the magnitude of the reduction can be controlled [19].

Subsequently, Rani presented a work that focused on numerically analyzing the concentration of stresses around the central elliptical inclusion coated by FGM using Extended Finite Element Method by preparing a MATLAB code. The result of the study shows that the FGM coating helps to significantly reduce the stress concentration factor. "Higher coating thickness was found better to reduce SCF significantly. The aspect ratio of ellipse was noticed as less significant for FGM coated inclusions; however, it is quite substantial for uncoated inclusions" [20].

Zappalorto [21] proposed a formula to obtain stress concentration factors in plates with central notches under tensile loading. Said formula results in a useful tool for calculating the stress concentration factor in notched orthotropic plates, composite orthotropic laminae, orthotropic unidirectional laminates and homogenized orthotropic composite laminates.

In the review of these previous works, we did not find evidence of information that defines the relationship between the variables involved in the reduction of the axial stress concentration factor in the rectangular section bar with an elliptical central hole, whose development is the main objective of this work. For this, an analysis of the stress concentration reduction methods that have been used with verifiable results was carried out and it was decided to reduce bar material by modifying the geometry of the hole through variable parameters using the finite element method. In this article, various proposed parameters of the hole geometry are analyzed, looking for the minimum stress concentration, finding the best of them, managing to quantify the reduction of said concentration in relation to the initial geometry.

Since stress concentration factor is a dimensionless value, it is useful to use the procedures and results of this work to adapt them to the geometric characteristics of a rectangular bar with a central hole subjected to an axial load to which it is desired to reduce the concentration of stresses.

II. METHODOLOGY

In the present work, the finite element analysis was used, generating 2D models in ANSYS Workbench software. The discretization was carried out using finer refinement in the areas of stress concentration [22].

Several finite element models of rectangular bars with a central elliptical hole, as stress concentration, were created. Maximum normal stresses were obtained for 144 parameter variations: height and width of the ellipse. The stress concentration factors were obtained for every design point and polynomial equations to estimate them were obtained by the least squares method.

STRESS CONCENTRATION FACTOR

According to [8], the stress concentration factor is a theoretical factor, and it is governed by the equation (1).

$$k_t = \frac{\sigma max}{\sigma nom} \quad (1)$$

Where σmax is the maximum stress to be expected in the member under the actual load and σnom is the nominal stress also known as the reference stress.

TENSILE BAR WITH ELLIPTICAL HOLE IN THE CENTER

The model is the classical rectangular bar with height w, constant thickness, a central hole, and a tension force P, as shown in the Figure 1, where the dotted lines represent the ellipse that causes stress concentration reduction. The ellipse geometry is defined by its width b and its height a. The parameters $w/2a$ and b/a define the stress concentration factor k_t. According to [9], for the present case the nominal stress is defined by the equation (2).

$$\sigma nom = \frac{P}{w - 2a} \quad (2)$$

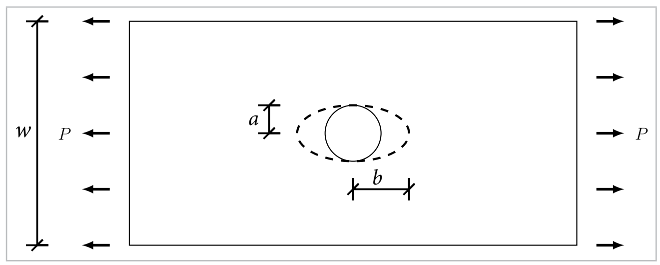

Figure 1. Rectangular bar with central hole.

FINITE ELEMENT MODEL RESULTS

In this section, the stress concentration factors for a wide range of ellipse parameter configurations were generated with finite element method. The bar has horizontal and vertical symmetry. Because of these conditions, only one-fourth of the whole bar and the full thickness was modeled. It is shown in the Figure 2. Several mesh configurations were analyzed, a simple 2D modeling discretization was selected to generate the FE mesh.

Figure 2. Finite element model (quarter of the whole bar).

In the Figure 3 it is shown the model refinement in the zones of stress concentration. The traction force P was included in the boundary conditions of the model. A table with 144 parameters combinations was introduced as design points in the FE software and the maximal stresses were obtained. Then, the stress concentration factors k_t are determined according to the equation (2) for every parameter combination (Figure 4). In the plot lines of b/a = 1, 1.2, 1.4, 1.6, it is clear that the minimal stress concentration factor lies in the leftmost at $w/2a$ = 2, it is in the minimal $w/2a$ ratio. However, it is of interest the behavior of the stress concentration factors for the cases of b/a > 1.6 where the exact location of the minimum stress concentration factor value is not in the leftmost. For example, in the lines corresponding to b/a = 2, the minimum is approximately at $w/2a$ = 3; however, in the line for b/a = 3, the minimum is approximately at $w/2a$ = 4.5.

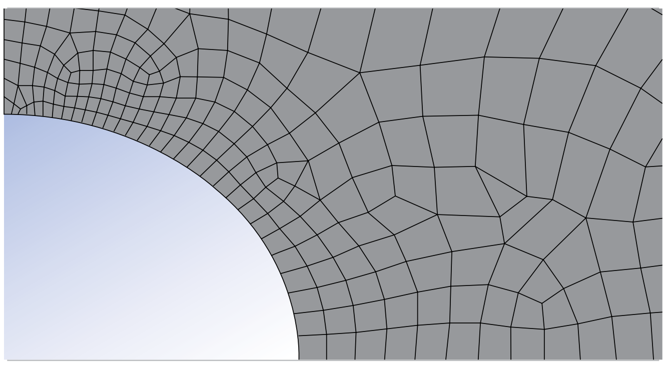

Figure 3. Finite element model (mesh refinement).

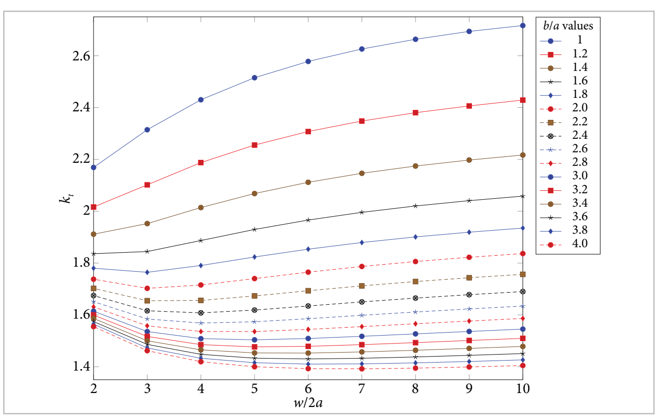

Figure 4. Stress concentration factors k_t for the ellipse cut off in the rectangular bar.

STRESS CONCENTRATION FACTOR EQUATIONS BY LEAST SQUARES POLYNOMIAL FITTING

The parameter values obtained in the previous section are used herein to interpolate polynomials to calculate stress concentration at any other ellipse parameter configuration along the boundaries $1 \leq b/a \leq 4$ and $2 \leq w/2a \leq 10$.

The stress concentration factors equations are developed by fitting the FE results to second and third order polynomial equations. Separated equations are developed for the $w/2a$ and b/a parameters. Multi parameter least squares equation fits were performed with spreadsheets in Excel. This procedure of solving the linear system of equations with more equations than variables minimize the error for the stress concentration factor. The system of equations to fitting is represented in the equation (3).

$$\begin{bmatrix} 1 & \left(\frac{w}{2a}\right)_1 & \left(\frac{w}{2a}\right)_1^2 & \left(\frac{w}{2a}\right)_1^3 \\ 1 & \left(\frac{w}{2a}\right)_2 & \left(\frac{w}{2a}\right)_2^2 & \left(\frac{w}{2a}\right)_2^3 \\ 1 & \left(\frac{w}{2a}\right)_3 & \left(\frac{w}{2a}\right)_3^2 & \left(\frac{w}{2a}\right)_3^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \left(\frac{w}{2a}\right)_7 & \left(\frac{w}{2a}\right)_1^2 & \left(\frac{w}{2a}\right)_1^2 \\ 1 & \left(\frac{w}{2a}\right)_8 & \left(\frac{w}{2a}\right)_1^2 & \left(\frac{w}{2a}\right)_1^2 \\ 1 & \left(\frac{w}{2a}\right)_9 & \left(\frac{w}{2a}\right)_1^2 & \left(\frac{w}{2a}\right)_1^2 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} k_{t\,1} \\ k_{t\,2} \\ k_{t\,3} \\ k_{t\,4} \\ k_{t\,5} \\ k_{t\,6} \\ k_{t\,7} \\ k_{t\,8} \end{Bmatrix} \qquad (3)$$

Finally, the equation obtained to determine the stress concentration factor for the geometry of the Figure 1 is

$$k_t = c_1 + c_2\left(\frac{w}{2a}\right) + c_3\left(\frac{w}{2a}\right)^2 + c_4\left(\frac{w}{2a}\right)^3 \qquad (4)$$

where

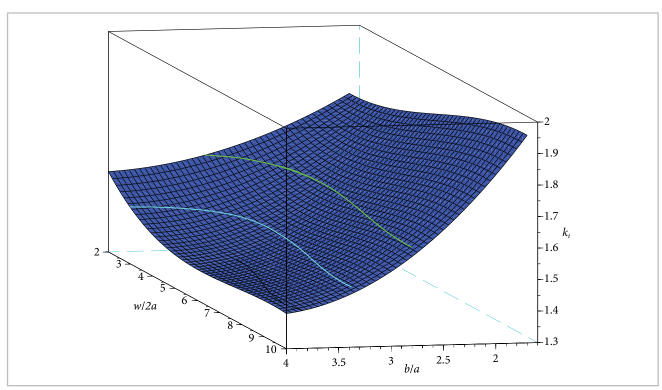

Figure 5. Stress concentration factors k_t given by the equation 4.

$$c_i = d_{1i} + d_{2i}\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2 \quad (5)$$

$$d_{i,j} = \begin{bmatrix} 1.7724..\\..00639 & 1.1241..\\..57758 & -0.1347..\\..6981 & 0.0055..\\..87083 \\ -0.0663..\\..71077 & -1.1662..\\..93746 & -0.1486..\\..86635 & -0.0063..\\..63178 \\ 0.0567..\\..0029 & 0.2883..\\..42389 & -0.0372..\\..34428 & 0.0015..\\..99727 \end{bmatrix} \quad (6)$$

$$1 \leq \frac{b}{a} \leq 1.7$$

$$d_{i,j} = \begin{bmatrix} 1.6917..\\..32954 & 0.3053..\\..76814 & -0.0253..\\..72987 & 0.0007..\\..44539 \\ -0.1137..\\..44483 & -0.2679..\\..64657 & 0.0299..\\..7906 & -0.0011..\\..63792 \\ -0.0202..\\..86378 & 0.0368..\\..20132 & -0.0044..\\..08681 & 0.0001..\\..79595 \end{bmatrix} \quad (7)$$

$$1.7 \leq \frac{b}{a} \leq 4$$

III. RESULTS AND DISCUSSION

In the Figure 5 it is a 3D plot of the equation (4). In this plot, it can be seen some contours in green, cyan, and blue color. It can be seen how the polynomial reproduce the changes in the minimum k_t along $w/2a$. This procedure gives good fits to the equally spaced data points; there is no evidence of higher order oscillations. Table 1 and Table 2 were obtained and store the results of k_t for the different variations in the dimensional ratios b/a and $w/2a$.

TABLE 1
DIFFERENCE (%) BETWEEN FITTED AND FE VALUES (FOR $w/2a$ = 6-10)

b/a	$w/2a$				
	10	9	8	7	6
1	0.007	0.135	0.067	0.001	0.014
1.2	0.086	0.214	0.23	0.179	0.089
1.4	0.275	0.033	0.034	0.184	0.356
1.6	0.12	0.275	0.264	0.04	0.195
1.8	0.977	0.467	0.497	0.795	1.08
2	0.06	0.485	0.442	0.118	0.178
2.2	0.311	0.886	0.832	0.486	0.182
2.4	0.315	0.913	0.849	0.487	0.181
2.6	0.098	0.713	0.642	0.269	0.035
2.8	0.22	0.408	0.33	0.047	0.346
3	0.535	0.099	0.018	0.36	0.65
3.2	0.763	0.128	0.211	0.587	0.867
3.4	0.83	0.202	0.285	0.654	0.926
3.6	0.675	0.06	0.143	0.504	0.77
3.8	0.246	0.347	0.265	0.086	0.348
4	0.502	1.066	0.983	0.642	0.38

TABLE 2
DIFFERENCE (%) BETWEEN FITTED AND FE VALUES (FOR $w/2a$ = 2-5)

b/a	$w/2a$			
	5	4	3	2
1	0.055	0.124	0.031	0.026
1.2	0.035	0.13	0.433	0.007
1.4	0.395	0.148	0.324	0.406
1.6	0.222	0.121	0.664	0.279
1.8	1.077	0.606	0.07	0.776
2	0.165	0.304	0.87	0.314
2.2	0.208	0.678	1.13	0.074
2.4	0.222	0.698	1.12	0.102
2.6	0.023	0.505	0.932	0.195
2.8	0.272	0.219	0.679	0.324
3	0.565	0.064	0.424	0.489
3.2	0.772	0.274	0.243	0.575
3.4	0.827	0.333	0.206	0.598
3.6	0.679	0.209	0.284	0.484
3.8	0.287	0.149	0.637	0.322
4	0.412	0.764	1.095	0.007

MINIMUM STRESS CONCENTRATION PATH

In the present work, it is considered b/a as the independent parameter and $w/2a$ as the dependent parameter. So, the minimum of k_t for the b/a curves can be obtained equaling to zero the derivative of the equation (4), it is

$$\frac{dk_t}{d(w/2a)} = c_2 + 2c_3\frac{w}{2a} + 3c_4\left(\frac{w}{2a}\right)^2 = 0 \quad (8)$$

$$\frac{w}{2a} = \frac{\left[\begin{matrix} e_2 - e_3\left(\frac{b}{a}\right) + e_4\left(\frac{b}{a}\right)^2 + \\ e_5\sqrt{-e_6 + e_7\left(\frac{b}{a}\right) - e_8\left(\frac{b}{a}\right)^2 + e_9\left(\frac{b}{a}\right)^3 - e_{10}\left(\frac{b}{a}\right)^4} \end{matrix}\right]}{744539 - e_{11}\left(\frac{b}{a}\right) + 179595\left(\frac{b}{a}\right)^2} \quad (9)$$

$$1.7 \leq \frac{b}{a} \leq 4$$

$$e = 10^{46}\begin{Bmatrix} 4.3655 \times 10^{-56}, & 1.93738 \times 10^{-30},... \\ 2.2890 \times 10^{-30}, & 3.36629 \times 10^{-31},... \\ 2.2052 \times 10^{-54}, & 4.59261,... \\ 17.1923, & 7.17828,... \\ 1.03011, & 0.0481565,... \\ 1.1637 \times 10^{-40}, & \end{Bmatrix} \quad (10)$$

The equation (9) is helpful to determine the $w/2a$ values that cause a minimum stress concentration factor

k_t for b/a parameter values within the interval $1.7 < b/a \leq 4$. It is important to remind that, for values $b/a < 1.7$ the minimal k_t value is located at the minimum $w/2a$ parameter value, the equation (9) does not apply since then. In the Figure 6 there is a 2D contour plot of the equation (4) for the values k_t = 1.4, 1.42, 1.44, 1.46 and 1.48. Also, it is plotted with dotted line the optimal path that minimize the k_t values of the equation (9).

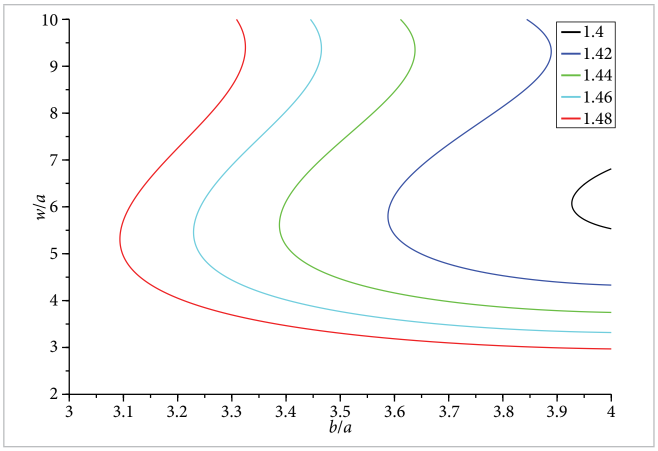

Figure 6. Contour plot of the equation (4).

IV. CONCLUSIONS

Two-dimensional finite element analysis was formed and stress concentration factors for wide ranges of bar-ellipse height ratios $w/2a$ and height-width ellipse ratios b/a were generated.

The b/a ratio was varied from 1 to 4, while the $w/2a$ ratio was varied from 2 to 10. Polynomial equations were fit to the finite element results. These equations generally agreed within around 1 percent difference with the finite element results. Tension stress concentration factor for a circular hole, $b/a = 1$, was higher than the elliptical hole, $b > a$, in general.

A least squares polynomial fitting was implemented with the design points; achieving an equation that describes the relation with good enough accuracy to reproducing the curves and to finding analytically the optimal $w/2a$ values that minimize the concentration factor. The path of the minimal concentration factor was illustrated in a contour plot.

In the references [8] and [9], there are various equations that approximate the maximum stresses in various cases of plane geometries with load combinations, but not the geometries analyzed herein. Nor are optimal geometry parameters shown to minimize stress.

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