Expert Systems With Applications 232 (2023) 120813

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

An interactive ACO enriched with an eclectic multi-criteria ordinal classifier to address many-objective optimisation problems

Gilberto Rivera [a], Laura Cruz-Reyes [b], Eduardo Fernandez [c], Claudia Gomez-Santillan [b,*], Nelson Rangel-Valdez [b,d]

[a] División Multidisciplinaria de Ciudad Universitaria, Universidad Autónoma de Ciudad Juárez, 32579, Cd. Juárez, Chihuahua, Mexico
[b] División de Estudios de Posgrado e Investigación, Tecnológico Nacional de México/Instituto Tecnológico de Ciudad Madero, 89440 Ciudad Madero, Tamaulipas, Mexico
[c] Facultad de Contaduría y Administración, Universidad Autónoma de Coahuila, 27000 Torreón, Coahuila, Mexico
[d] CONACyT-Tecnológico Nacional de México/Instituto Tecnológico de Ciudad Madero, Av. 1o de Mayo esq. Sor Juana Inés de la Cruz S/N, Col. Los Mangos, 89440, Cd. Madero, Tamaulipas, Mexico

A R T I C L E I N F O

Keywords:
Swarm intelligence
Many-objective optimisation
Preference incorporation
Multiple criteria ordinal classification
Interactive methods

A B S T R A C T

Despite the vast research on many-objective optimisation problems, the presence of many objective functions is still a challenge worthy of further study. A way to treat this kind of problem is to incorporate the preferences of the decision maker (DM) into the optimisation process. In this paper, we introduce an interactive ant colony optimisation combined with an ordinal classification method in which classes are described by characteristic profiles. Through several interactions, the DM is supposed to identify some representative solutions of the classes 'satisfactory' and 'dissatisfactory,' which are used to initialise an ordinal classifier that increases the selective pressure by discriminating in favour of the 'satisfactory' class. This method can work with any either asymmetric or symmetric binary preference relation, a feature that confers a very wide generality. As another advantage, the interaction with the DM has minimal cognitive demands, which is an advisable feature for any approach based on interaction. Although the preference model is quite general, the proposal was tested using an eclectic model which combines compensatory preferences, veto conditions, and interval numbers to handle imprecise values; those preferences are aggregated in an asymmetric preference relation. Our approach performs particularly well in 10-objective problems according to the standards in the state-of-the-art literature. Numerical results and tests for statistical significance on the DTLZ and WFG test suites support this claim.

1. Introduction

Most real-world optimisation problems involve considering simultaneously multiple objective functions. The conflicting character of these functions makes it impossible to obtain a single solution that optimises all the objectives. In multi-objective optimisation, a crucial step in solving a Multi-Objective Problem (MOP) is to identify a set of conflicting solutions in which improving one objective worsens the performance of others. This set of solutions in the objective space is known as the Pareto front or frontier (Zitzler & Thiele, 1998). Multi-Objective Evolutionary Algorithms (MOEAs) have been widely applied to solve MOPs. They have proven their ability to solve complex problems with two or three objectives (Okola et al., 2023). However, these MOEAs have shown that the quality of the solutions deteriorates severely when addressing problems that include more than three conflicting objectives (Bechikh et al., 2017; Bezerra et al., 2018). 'Many-objective Optimisation Problems' (MaOPs) is the term used to refer to this type of MOPs.

To deal with the challenges of MaOPs, Many-Objective Evolutionary Algorithms (MaOEAs) have been developed as a natural extension of MOEAs (Wang et al., 2023). Several survey papers have proposed categories to organise the works on MaOEAs (Von Lücken et al., 2019). Coello Coello et al. (2020) described the three main types commonly adopted to affront some of the challenges for solving MaOPs:

(i) approaches based on improving convergence; that is, they try to increase the selection pressure of Pareto-based MOEAs (e.g.,

* Corresponding author.
E-mail addresses: gilberto.rivera@uacj.mx (G. Rivera), lauracruzreyes@itcm.edu.mx (L. Cruz-Reyes), eddyf171051@gmail.com (E. Fernandez), claudia.gs@cdmadero.tecnm.mx (C. Gomez-Santillan), nelson.rv@cdmadero.tecnm.mx (N. Rangel-Valdez).

https://doi.org/10.1016/j.eswa.2023.120813
Received 23 February 2023; Received in revised form 6 June 2023; Accepted 9 June 2023
Available online 20 June 2023
0957-4174/Published by Elsevier Ltd.

Xiang et al., 2017; Qiu et al., 2021; Zhang et al., 2023; Guo et al., 2023; Li et al., 2023);
(ii) decomposition-based approaches, which convert MaOPs into several sub-problems (e.g., Ge et al., 2019a; Ge et al., 2019b; Liu et al., 2022, Bao et al., 2023); and
(iii) performance indicator-based approaches, which promote convergence or maintain diversity by calculating indicator values (Falcón-Cardona & Coello Coello, 2020; Gu, Zhou, Wang, & Xiong, 2023; Huang & Wang, 2021; Liu, Wang, Yao, & Peng, 2023).

Typically, MaOEAs focus on finding a convergent, representative, and well-distributed sample of the Pareto frontier (Gu, Xu, & Li, 2022; Zhao et al., 2019). Although such a sample is essential, it is insufficient to solve the problem entirely in practice. Since all the Pareto solutions are mathematically equivalent, the Decision Maker (DM) must provide information about their preferences to choose the most preferred one (Fernandez et al., 2011). In the presence of many objectives, the DM could hardly accomplish this task without an adequate method from the Multi-Criteria Decision Analysis (MCDA). This fact represents a major challenge for addressing real-world problems. Preference incorporation in MOEAs is a promising way to cope with the difficulties arising from the presence of many objectives.

According to Li et al. (2015), the DM's preferences have been incorporated in MOEAs that are not designed for solving MaOPs. These algorithms use preference information to focus the search on the Region of Interest (RoI), the subset of the Pareto front most in agreement with the DM's preferences (Adra et al., 2007). The solution finally chosen by the DM, called the 'best compromise,' is supposed to belong to the RoI (Fernandez et al., 2011).

The ways to incorporate preferences can be distinguished as follows:

a) According to the stage in which preferences are articulated,
b) According to the issues which the DMs should address to express their preferences.

Concerning Point a), the DMs can provide their preference information before (*a priori* way), after (*a posteriori* way), or during (interactive way) the approximation of the Pareto frontier (Hwang and Masud, 1979). Most of the multi-objective metaheuristic approaches proposed in the literature have considered the participation of the DM through *a posteriori* way; that is, they direct the search toward a representative (and usually very large) subset of the Pareto front. However, this way of preference articulation faces severe difficulties when the number of objective functions increases, challenging the cognitive limitations of the human mind, as first stated by Miller (1956). In addition to these cognitive limitations, the selective pressure of most metaheuristic algorithms toward the true Pareto frontier is degraded in MaOPs. There is fundamental evidence showing that either *a priori* or interactive articulation of preferences reduces the search space and helps to find the RoI (Branke & Deb, 2005; Said et al., 2010). Nevertheless, the *a priori* incorporation of preferences has been criticised because, at the beginning of the search, the DM has not yet completely understood the problem that is faced, nor of the complex interactions among the objective functions and the trade-offs they will have to make. Therefore, most DMs cannot make *a priori* direct elicitation of their preference model parameters and further relevant information concerning their preferences.

The interactive articulation of preferences can provide significant advantages. Interactive methods assume that, initially, the DMs do not entirely understand their problem and the complex trade-offs among the objective functions in the Pareto frontier. It is commonly accepted that interactive methods help DMs 'learn' about the problem they face. In each interaction step, the DM's preferences are updated, contributing to identifying better solutions. Furthermore, since the DMs are involved in the algorithmic search and have systematically accepted new solutions as good ones, they feel more comfortable with the results of the interactive procedure (Cheng et al., 2017; Li et al., 2020; Cruz-Reyes et al., 2020, Rivera et al., 2021).

Nevertheless, the progressive articulation of preferences is not free of inconveniences. These methods demand more involvement in the DM and more cognitive effort. When the DM compares solutions that result from different iterations, their judgments should be fully transitive, which could be a big concern in real-world problems, especially when the number of objectives increases and the cognitive limitations of the human mind become relevant.

Concerning Point b), the most common ways used by DMs to express their preferences are:

- comparison of objective functions (e.g., Branke & Deb, 2005; Brockhoff et al., 2013);
- reference points (e.g., Lahdelma et al., 2005; Qi et al., 2018; Abouhawwash & Deb, 2021);
- desirability thresholds (e.g., Wagner & Trautmann, 2010; He et al., 2021);
- ranking of solutions (e.g., Cvetkovic & Parmee, 2002; Yuan et al., 2021; Kadziński & Szczepański, 2022);
- pairwise comparison of solutions (e.g., Branke et al., 2016; Tomczyk and Kadziński, 2020);
- scoring of solutions (e.g., Saldanha et al., 2019; Li et al., 2019);
- classification of solutions (e.g., Greco et al., 2010; Cruz-Reyes et al., 2020; Corrente et al., 2021);
- characterisation of the preferred region (e.g., Gong et al., 2017); and
- preference relations that replace Pareto dominance (e.g., Molina et al., 2009; Fernandez et al., 2010; Fernández et al., 2022a).

Most of these ways can be used either *a priori* or interactively; however, they usually require a remarkable cognitive effort from the DM, and transitivity in the DM's preference judgments. This is critical when the number of objective functions overcomes a particular threshold.

In this paper, we propose an interactive algorithm based on ACO (Ant Colony Optimisation) that incorporates preferences through binary classification of solutions. Binary classification in classes such as 'Good' and 'No Good' is less cognitively demanding than, for instance, ranking, pairwise comparisons, and solution scoring. No transitive judgments are demanded. The cognitive effort required for the DM depends on the number of solutions that should be classified. But if they are only a few, the effort would also be small. Here, we use a multi-criteria ordinal classification method that even works with a single object (solution, action) per class.

The novelties of our approach are the following:

- It demands minimal cognitive effort from the DM during each interaction by using a binary multi-criteria ordinal classification.
- A many-objective metaheuristic approach that can embed a wide range of preference models, ranging from fully compensatory (e.g., the normalised weighted sum) to non-compensatory (e.g., outranking) relations, opening a vast space to represent preference models.
- An eclectic preference model that can handle partially compensatory preferences, veto, and incomparability, which are existing situations in real-life decision-making problems.
- It tolerates imprecision in the parameter values of the preference model by using interval numbers, which improve the credibility and robustness of a many-objective metaheuristic.
- It is competitive in addressing continuous MaOPs while preserving the benefits of the innovations above.

As far as we know, our ACO algorithm is the single metaheuristic algorithm combining all these features. In this way, our contribution is valuable. The rest of this paper is organised as follows. Section 2 reviews the related literature. Section 3 presents the foundations necessary to introduce our contribution. Section 4 introduces the proposed ACO, describing each step of the algorithm elaborately. Section 5 presents the computational experiments that back the advantages of our proposal. Lastly, Section 6 discusses some conclusions and directions for future research.

2. Related literature

This paper deals with combining multi-criteria ordinal classification methods and multi-objective metaheuristics. In the scientific literature, such a combination is manifested in two directions:

1) using metaheuristics to infer the parameters of multi-criteria ordinal classification methods, and
2) incorporating preferences to increase selective pressure toward the RoI into the metaheuristic search process.

As examples of Point 1) we can mention the papers by Fernandez et al. (2009), Fernandez et al. (2019a), Fernandez et al. (2019b), Fernández et al. (2023), Doumpos et al. (2009), Covantes et al. (2016), Douissa & Jabeur (2020), Balderas et al. (2022) and Kadziński & Szczepański (2022). Nonetheless, bear in mind that the present paper is focused on Point 2).

To our knowledge, only a few studies have combined multi-criteria ordinal classification methods and multi-objective metaheuristics, mainly oriented to incorporate the DM's preference information to increase selective pressure toward the best compromise in evolutionary multi-objective optimisation. Examples of this approach are the following studies.

Greco et al. (2010) considered the combination of the Dominance-based Rough Set Approach (DRSA) with Evolutionary Multi-objective Optimisation (EMO). DRSA-EMO included the application of decision rules in EMO, which are induced by the DM's preferences; these preferences are elicited by asking the DM to sort some solutions according to their preferences into two classes: 'relatively good' and 'others.' The obtained rules are used to rank solutions in the current population of EMO, impacting the selection and crossover operators. DRSA-EMO is proposed as a methodological framework for interactive EMO algorithms. A computer implementation of this methodology is called DARWIN (Dominance-based rough set Approach to handling Robust Winning solutions in INteractive multi-objective optimisation).

Oliveira et al. (2013) proposed the Evolutionary Algorithm Based on an Outranking Relation (EvABOR). This was the first study using a multi-criteria ordinal classification method based on outranking, in this case, the ELECTRE TRI method. This method assigns a solution to a category based on the limits established by the DM for the category. In EvaBOR, the DM's preferences are incorporated in all the genetic operators, e.g., in the crossover operator, when two solutions compete to be one of the parents, the one that belongs to the best class is chosen. Three distinct EvABOR approaches were developed. The main differences between these approaches lie in selecting the individuals that go to the next generation. EvABOR-I uses the outranking relation only. The other two approaches use an interaction between the outranking and the non-dominance relations. For the evaluation, five ordinal classes and three instances of the reactive power compensation problem with three objective functions were considered; EvABOR-III obtained the best performance in these conditions.

Cruz-Reyes et al. (2017) addressed incorporating preferences in an MOEA to characterise the RoI through the THESEUS multi-criteria classification method, creating a Hybrid Genetic Algorithm for multi-criteria classification composed of two phases. First, a metaheuristic generates a small set of solutions classified into ordered categories by the DM. Consequently, the DM's preferences are indirectly reflected in this set. In the second phase, THESEUS is combined with an evolutionary algorithm. The first method is used to classify new solutions. Those solutions classified as 'satisfactory' are used to create selective pressure toward the RoI.

Cruz-Reyes et al. (2020) introduced a new hybrid evolutionary algorithm whose main feature is incorporating the DM's preferences through the THESEUS and ELECTRE-TRI multi-criteria ordinal classification methods in the early stages of the optimisation process, being progressively updated. Seven problems from the DTLZ test suite and three instances of the project portfolio optimisation problem, both with three and eight objectives, were used to analyse the proposal performance concerning convergence, objectives increasing, and the used classification method. Compared to MOEA/D and MOEA/D-DE, the proposed strategy obtained a better convergence toward the RoI and a better characterisation of that region.

Castellanos-Alvarez et al. (2021) proposed a new MOEA called NSGA-III-P (Non-dominated Sorting Genetic Algorithm III with Preferences). The distinctive characteristic of NSGA-III-P is a new DM's preference incorporation method based on the INTERCLASS-nC ordinal multi-criteria classification approach to guide the algorithm toward the RoI. Besides, the algorithm uses interval numbers to express preferences with imprecision. NSGA-III-P showed a better approximation to the RoI than the original NSGA-III in solving seven problems from the DTLZ test suit with three objectives.

Corrente et al. (2021) presented a new interactive method for evolutionary multi-objective optimisation to incorporate a decision rule preference model in the search for the best compromise solution. During the search, the DM is requested interactively to classify a small sample of solutions from the current population into two classes, namely 'Good' and 'Bad.' The Dominance-based Rough Set approach (DRSA) uses the assignment examples to approximate the classes and induces a rule-based decision model representing the DM's preferences. In each generation, the set of solutions is separated into non-dominated fronts; within these fronts, the solutions are scored by considering the number of rules supporting the assignment to each class. Such score and the non-domination rank are used to make selective pressure toward the RoI. In XIMEA-DRSA (explainable multi-objective optimisation evolutionary approach), the decision rules are presented to explain the impact of the DM's answers. Its effectiveness has been tested on both continuous and discrete multi-objective optimisation problems.

Castellanos et al. (2022) introduced a strategy to enhance two swarm intelligence algorithms with the preferences of the DM expressed in classes for the INTERCLASS-nC ordinal classifier based on interval outranking. This hybridising strategy was tested in two algorithms, i.e., the Multi-objective Grey Wolf Optimisation and the Indicator-based Multi-objective Ant Colony Optimisation (ACO) for continuous domains; the corresponding extended versions were called GWO-InClass and ACO-InClass. These algorithms, validated on nine problems in the DTLZ test suit with three, five, and ten objective functions, are appropriate when many objective functions are considered and can reach the RoI better than the original metaheuristics.

Except for the papers by Greco et al. (2010) and Corrente et al. (2021), the other studies have used the relational paradigm—precisely, outranking relations—to assign solutions to classes. Outranking methods use majority rules and veto thresholds to build an outranking relation that is not transitive (cf. Roy, 1991). These relations help model non-transitive and non-compensatory preferences, in which lower values of some objective functions cannot be compensated by very high values of the remaining objectives. This is particularly important to handle ordinal and qualitative criteria performance levels. In outranking methods, the intensity of opposition in some criteria can produce a veto effect, but the intensity of preference does not increase the credibility degree of outranking. Hence, outranking methods may not be a good model of partially compensatory preferences, in which, to a certain extent, relatively poor values of some objectives are compensated by improved values of others.

The functional paradigm is the main alternative to the outranking-based relational paradigm. In the functional paradigm, a real value function is defined on the decision set, representing the DM's preferences. In comparing two actions of the decision set, greater values of the preference function mean preference. So, preference relations derived from the functional paradigm are transitive, and each pair of actions fulfils either strict preference or indifference. The model is fully compensatory. Incomparability and veto are prohibited. As a norm for the rational behaviour of an ideal DM, the functional paradigm is acceptable. However, there is much evidence against its general validity for real DMs. Ill-defined preferences, threshold effects, and veto situations produce incomparability and intransitivity in real-life decision-making (Roy, 1991; Bouyssou et al., 2006).

To address those inconveniences, we propose ACO-eMOC: an *ACO* algorithm enriched with an *E*clectic *M*ulti-*C*riteria *O*rdinal *C*lassification model that combines some features of both the functional and relational paradigms. This choice was made because Particle Swarm Optimisation (PSO) and ACO are the most popular swarm intelligence algorithms. Although PSO performs better in most continuous problems, ACO clearly outperforms PSO in discrete problems (cf. Selvi and Umarani, 2010; Wu et al., 2021a). Even in continuous problems, when the number of evaluations of the objective function increases, ACO becomes competitive with PSO and Differential Evolution (Ezugwu et al., 2020). In general, ACO and PSO have shown to be promising techniques for addressing realistic problems (Kuo et al., 2023; Wu et al., 2021b; Wu et al., 2019).

The ordinal classification method embedded in ACO-eMOC is based on a value function that permits partial compensation, thus allowing, to a certain extent, to model the intensity of preference. The approach supports veto and incomparability. A strict preference relation is derived from the model, which is not necessarily transitive. The imprecision in model parameter values is represented by using interval numbers. To assign solutions to ordered classes (ordered in the sense of preferences), we adapt the general method proposed by Fernández et al. (2022b) instantiated with an asymmetric preference relation. Since the non-symmetry is the single property this relation should fulfil, we use the strict preference relation derived from the eclectic multiple criteria decision model, which combines value function with interval numbers and veto capacity. Although previous studies used ordinal classification, ACO-eMOC is quite comprehensive and is the single many-objective metaheuristic incorporating the DM's preferences through an eclectic model that supports both compensatory and non-compensatory features simultaneously.

3. Background

This section presents: (1) an overview of interval numbers; (2) a description of the ordinal classification method by Fernández et al. (2022b); and (3) a description of the multiple criteria preference model, whose preference relation is used by the ordinal classification method.

3.1. Interval numbers

As a subset of the real line $\mathbb{R}$, the interval numbers are an expansion of real numbers (cf. Moore, 1979). Some authors have defined them as a range of values between two limits. They are a straightforward way to model the imprecision derived from inaccurate measurements or the variability of the DM's judgments and beliefs (Balderas et al., 2019; Gnansounou, 2017).

In this article, interval numbers will be denoted by bold italic letters, e.g., $\boldsymbol{B} = \left[\underline{B}, \overline{B}\right]$, where $\underline{B}$ and $\overline{B}$ correspond to the lower and upper limits. Among several operations that are defined for interval numbers, we will only overview the following basic arithmetic operations:

- $\boldsymbol{A} + \boldsymbol{B} = \left[\underline{A} + \underline{B}, \overline{A} + \overline{B}\right]$,
- $\boldsymbol{A} - \boldsymbol{B} = \left[\underline{A} - \overline{B}, \overline{A} - \underline{B}\right]$,
- $\boldsymbol{AB} = \left[\min\left\{\underline{AB}, \underline{A}\,\overline{B}, \overline{A}\underline{B}, \overline{AB}\right\}, \max\left\{\underline{AB}, \underline{A}\,\overline{B}, \overline{A}\underline{B}, \overline{AB}\right\}\right]$, and
- $\boldsymbol{A}/\boldsymbol{B} = \left[\underline{A}, \overline{A}\right]\left[1/\overline{B}, 1/\underline{B}\right]$,

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are interval numbers.

The order relations on interval numbers are defined using the possibility function $Poss(\boldsymbol{B} \geq \boldsymbol{A})$. This function is defined by Equation (1) as follows (Yao et al., 2011):

$$Poss(\boldsymbol{B} \geq \boldsymbol{A}) = \begin{cases} 1 & \text{if } p_{BA} > 1, \\ 0 & \text{if } p_{BA} \leq 0, \\ p_{BA} & \text{otherwise,} \end{cases} \tag{1}$$

$$p_{BA} = \frac{\overline{B} - \underline{A}}{\left(\overline{B} - \underline{B}\right) + \left(\overline{A} - \underline{A}\right)}$$

In the case when $\boldsymbol{A}$ and $\boldsymbol{B}$ are degenerate intervals (i.e., $\underline{A} = \overline{A}$ and $\underline{B} = \overline{B}$), they are treated as real numbers A and B; consequently, $Poss(\boldsymbol{B} \geq \boldsymbol{A}) = 1$ if and only if $B \geq A$; otherwise, $Poss(\boldsymbol{B} \geq \boldsymbol{A}) = 0$. A realisation of an interval $\boldsymbol{B}$ is a real number b within $\boldsymbol{B}$ (cf. Fliedner and Liesiö, 2016). Balderas et al. (2019) interpreted $Poss(\boldsymbol{B} \geq \boldsymbol{A}) = \alpha$ as: 'α is the degree of credibility of the following statement: once two realisations are obtained from $\boldsymbol{B}$ and $\boldsymbol{A}$, the realisation a will be smaller than or equal to the realisation b.' Consequently, the relational operators $\geq$ and $>$ can be extended to compare interval numbers as follows:

$$\boldsymbol{B} \geq \boldsymbol{A} \Rightarrow Poss(\boldsymbol{B} \geq \boldsymbol{A}) \geq 0.5, \tag{2}$$

and

$$\boldsymbol{B} >_{\alpha} \boldsymbol{A} \Rightarrow Poss(\boldsymbol{B} \geq \boldsymbol{A}) \rangle \alpha. \tag{3}$$

On the set of interval numbers, the relation $\geq$ is reflexive and transitive, and $>_{\alpha}$ is asymmetric. The credibility of $>_{\alpha}$ increases with α ($0.5 < \alpha \leq 1$). Of course, these relations can also compare real numbers (degenerate intervals). Balderas et al. (2019) described interval numbers and the properties of the possibility function.

3.2. Classification method

Fernández et al. (2022b) recently proposed an ordinal classification method with far-reaching features. This classifier can work supposing any of the two following relations: $\mathscr{P}$ (that is asymmetric) and $\mathscr{S}$ (that is reflexive). In this paper, we focus on the former.

Suppose that O is a set of potential actions (in our context, feasible solutions to a MOP). Let $\mathscr{P}$ denote an asymmetric binary preference relation defined on a subset of $O \times O$; $x\mathscr{P}y$ denotes 'x is preferred to y.' This method has broad generality because $\mathscr{P}$ can be derived from any preference model; the single requirement is that $\mathscr{P}$ must be asymmetric. We present below an adaptation of the method by Fernández et al. (2022b) to the case of only two classes.

Requirements on the set of characteristic profiles

Suppose a set of two classes: $C = \{C_1, C_2\}$, being C_2 preferred to C_1. Each class C_k is characterised by a reference set R_k, which is composed of characteristic profiles ($R_k \subset O$). The reference sets must fulfil the following conditions:

i. For each profile $y \in R_2$, there is no profile $x \in R_1$ such that $x\mathscr{P}y$
ii. For each profile $x \in R_1$, there is at least one profile $y \in R_2$ such that $y\mathscr{P}x$
iii. For each profile $y \in R_2$, there is at least one profile $x \in R_1$ such that $y\mathscr{P}x$
iv. There is no pair $(x, y) \in R_k \times R_k$ such that $x\mathscr{P}y$ for $k = 1, 2$

Definition 1. *The relation* $\mathscr{P}$ *between actions and reference sets.*

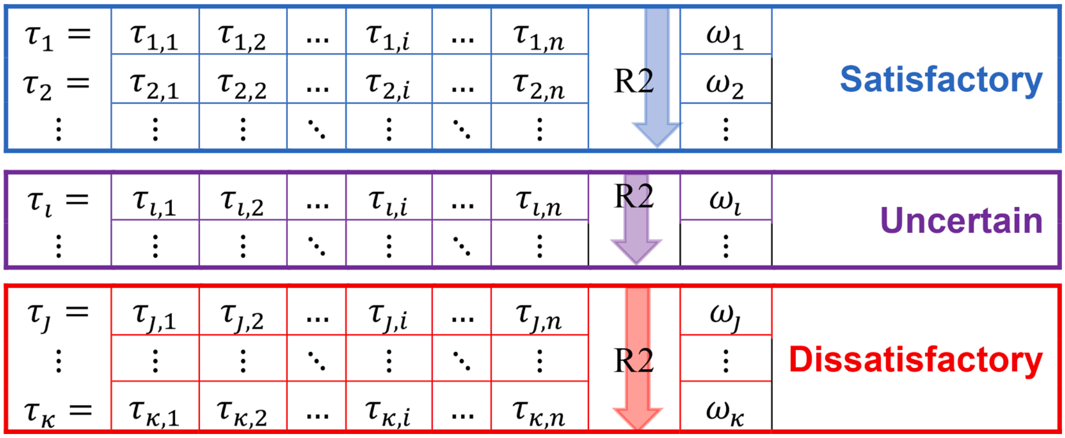

Fig. 1. Structure of the pheromone matrix in ACO-eMOC.

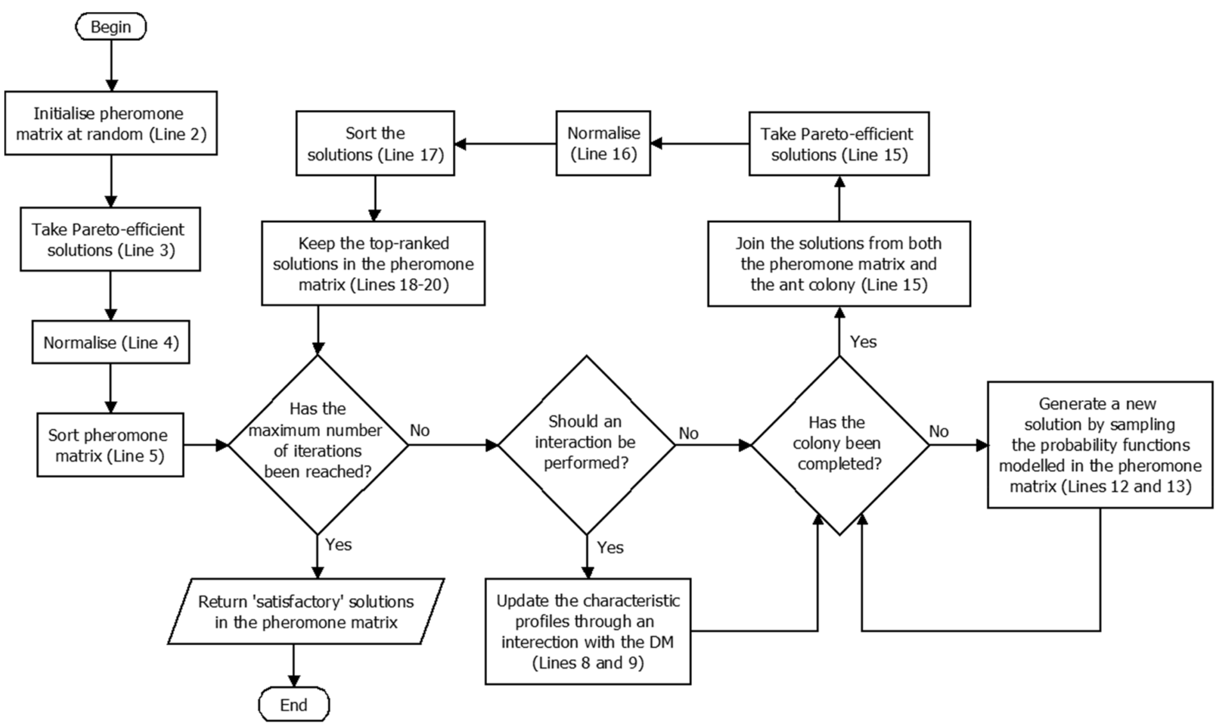

Fig. 2. Flowchart of ACO-eMOC.

Table 1
Parameters for the benchmark problems.

Test suite	Problems	m	K	N
DTLZ	1	{3, 5, 10}	5	$m+k-1$
	2–6	{3, 5, 10}	10	$m+k-1$
	7	{3, 5, 10}	20	$m+k-1$
WFG	1–9	3	2(m-1)	24
	1–9	5	2(m-1)	47
	1–9	10	2(m-1)	105

a) $x\mathscr{P}R_k$ if and only if there is at least one profile $y \in R_k$ such that $x\mathscr{P}y$
b) $R_k\mathscr{P}x$ if and only if there is at least one profile $y \in R_k$ such that $y\mathscr{P}x$

Definition 2. *Descending assignment rule.*

Consider that R_1 and R_2 fulfil the above-listed requirements (numbered i–iv). Consider also a fictitious R_0 such that $y\mathscr{P}R_0$ for all $y \in O$. The class of a new action x can be suggested by the following heuristic (called 'descending rule'):

1. For $k = 2, 1, 0$, determine the first k such that $x\mathscr{P}R_k$
2. If $k = 2$, assign x to C_2
3. If $k = 0$, assign x to C_1
4. If $k = 1$, select C_1 and C_2 as possible classes to assign x.

Definition 3. *Ascending assignment rule.*

Consider that R_1 and R_2 fulfil the requirements i–iv. Consider also a fictitious R_3 such that $R_3\mathscr{P}y$ for all $y \in O$. The ascending rule can suggest the class of a new action x:

1. For $k = 1, 2, 3$, determine the first k such that $R_k\mathscr{P}x$
2. If $k = 1$, assign x to C_1

Table 2
Results on the DTLZ test suite.

		COMPARISON WITH RVEA-iGNG Problems in which ACO-eMOC		Indicator	Problems in which ACO-eMOC		Indicator	COMPARISON WITH FDEA II Problems in which ACO-eMOC		Indicator	Problems in which ACO-eMOC		Indicator
m	***p***	**(a)** outperforms RVEA-iGNG	**(b)** is outperformed by RVEA-iGNG		**(a)** outperforms RVEA-iGNG	**(b)** is outperformed by RVEA-iGNG		**(a)** outperforms FDEA II	**(b)** is outperformed by FDEA II		**(a)** outperforms FDEA II	**(b)** is outperformed by FDEA II	
3	3	2, 4	1, 3, 6, 7		4	1–3, 6		4	1–3, 5, 6		4, 7	1–3, 5, 6	
	5	2, 4, 7	1, 6		4, 5	1, 3, 6		4, 7	1–3, 5, 6		4, 7	1–3, 5, 6	
	7	2–5, 7	6		2–5, 7	1, 6		3–5, 7	1, 2, 6		2–4, 7	1, 5, 6	
	9	1–5, 7	6		2–5, 7	1, 6		1, 3–5, 7	2, 6		2–4, 7	1, 5, 6	
	11	1–5, 7			2–5, 7	1		1, 3–7			2–4, 7	1	
5	3		2, 3, 4, 5, 6	**Euclidean**		2–6	**Chebyshev**		1–6	**Euclidean**	7	1–3, 6	**Chebyshev**
	5	1	2, 4–6		1, 7	2–6			1–6		7	1, 3, 6	
	7	1, 7	2, 4, 5		1, 6, 7	2–5		7	1–5		6, 7	1, 3	
	9	1, 3, 5–7	2, 4		1, 3, 6, 7	2		2, 6, 7	1, 4		2, 4–7		
	11	1, 3, 5–7	2		1, 3, 6, 7	2		2, 4, 6, 7			2, 4–7		
10	3	5, 7	1, 4, 6	**Minimum**	3, 5	1, 2, 4, 6	**Minimum**	4, 5, 7	1, 2, 3, 6	**Minimum**	4, 5	1–3, 6	**Minimum**
	5	5, 7	4, 6		3, 5, 7	1, 2, 4, 6		4, 5, 7	2, 3, 6		4, 5, 7	1–3, 6	
	7	2, 3, 5, 7	6		3, 5–7	2, 4		4, 5, 7	2, 3, 6		4–7	1–3	
	9	1–7			1, 3, 5–7			1, 3–5, 7	2		1, 3–7		
	11	1– 7			1–3, 5–7			1, 3–5, 7			1–7		
3	3	4	1, 3, 6		4, 7	1, 2, 6		4	1–3, 5–7		4, 7	1–3, 5, 6	
	5	2, 4	3, 6		2, 4, 5, 7	1, 6		4	1–3, 5–7		4, 7	1–3, 5, 6	
	7	1, 2, 4, 5, 7	3, 6		2–5, 7	1		4, 7	2, 3, 5, 6		4, 6, 7	1–3, 5	
	9	1–5, 7			2–7	1		1, 4, 6, 7	3		2, 4, 6, 7	1, 3, 5	
	11	1–5, 7			2–7			1, 4, 6, 7	3		1, 2, 4, 6, 7	3	
5	3		2–4, 6	**Euclidean**	7	2, 3, 5, 6	**Chebyshev**		1–6	**Euclidean**	7	1, 2, 5, 6	**Chebyshev**
	5	1, 5	2, 4, 6		1, 7	2, 3, 5			1–6		7	1, 5, 6	
	7	1, 5–7	2, 3		1, 3–5, 7	2		7	1, 3, 5, 6		4, 5, 7	1, 6	
	9	1, 4–7			1, 3–7	2		1, 2, 4, 7	3, 6		1, 2, 4, 5, 7	6	
	11	1, 4–7			1, 3–7			1, 2, 4, 7	3, 6		1, 2, 4, 5, 7	6	
10	3	3, 7	2, 4	**Average**	3, 7	1, 2, 4, 6	**Average**	4, 7	1–3, 5, 6	**Average**	4, 7	1–3, 5, 6	**Average**
	5	1, 3, 5, 7	2, 4		3, 5, 7	1, 2, 4, 6		4, 7	1–3, 6		4, 7	1–3, 5, 6	
	7	1, 3, 5–7	2, 4		1, 3, 5, 7	2, 4, 6		4, 5, 7	1, 2, 6		1, 4, 5, 7	2, 3, 6	
	9	1, 3, 5–7	2		1, 3, 5–7	2		4, 5, 7	2		1, 3–7		
	11	1, 2, 5–7			1, 3, 5–7			1, 4, 5, 7			1–7		

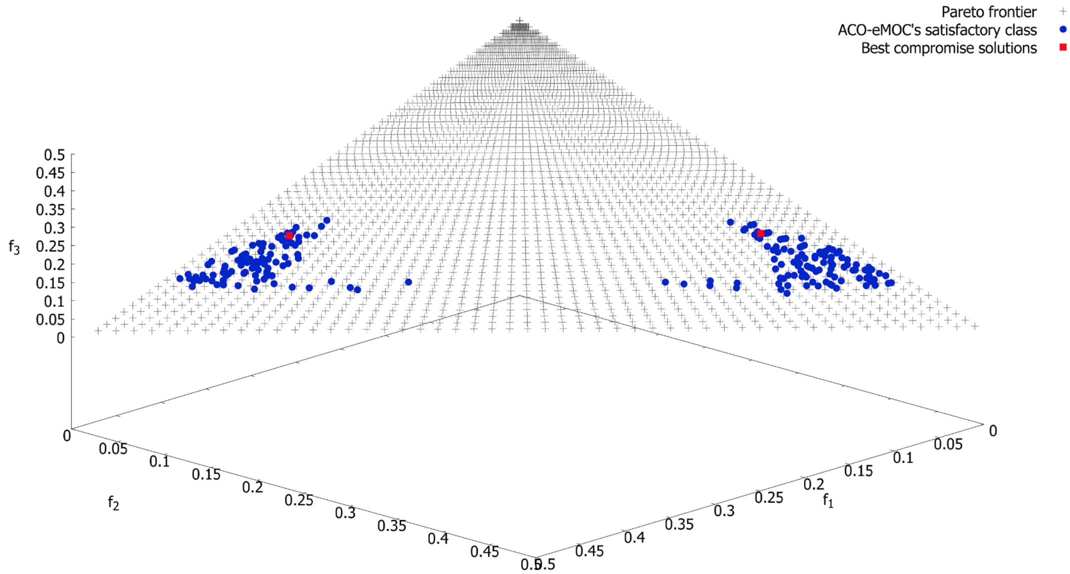

Fig. 3. Results of ACO-eMOC on DTLZ1 ($m = 3$).

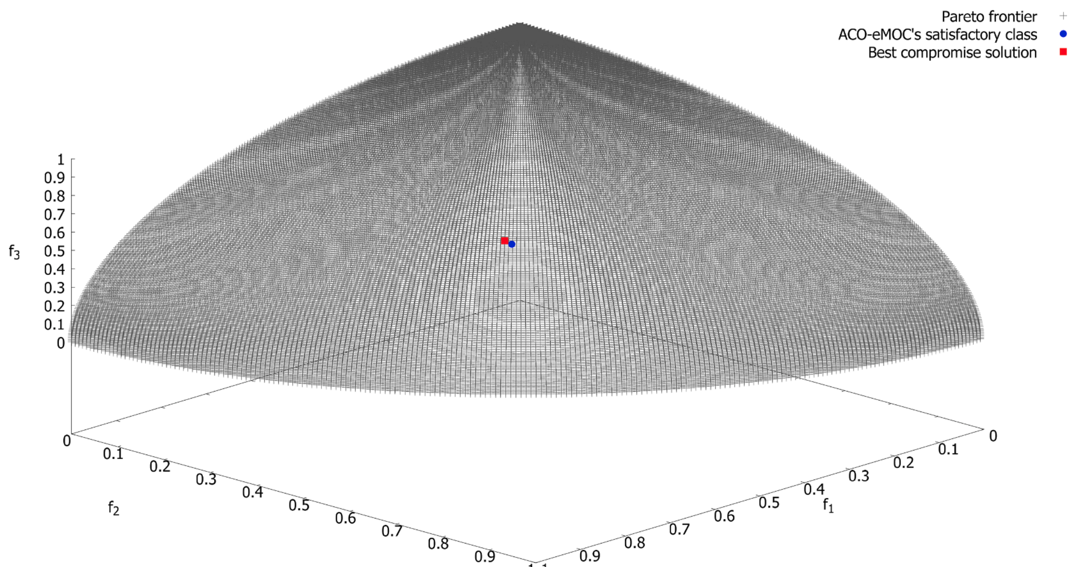

Fig. 4. Results of ACO-eMOC on DTLZ2 ($m = 3$).

3. If $k = 3$, assign $x$to C_2
4. If $k = 2$, select C_1 and C_2 as possible classes to assign x.

Definition 4. *Conjoint assignment rule.*

Fernández et al. (2022b) argued in favour of the conjoint use of both the ascending and descending rules as follows (conjoint rule):

a) If x is classified into C_2 by the descending rule and C_1 by the ascending rule, take the range C_1–C_2 as the possible class of x.
b) If x is suggested for the range C_1–C_2 by both the descending and ascending rules, take the same range of classes as possible assignments for x.
c) If x is suggested for C_k by both the ascending and descending rules, take the same class C_k as the class of x.

3.3. Decision model

In this subsection, we describe a multiple criteria decision model to define the asymmetric preference relation $\mathscr{P}$, whose parameters have been extended with intervals to improve credibility and robustness.

Let x be an action (alternative/object) of a decision problem. In the context of a MOP, they are points in the n-dimensional space of the independent variables (there are n independent variables). Here, $f(x) = \langle f_1(x), f_2(x), f_3(x), \cdots, f_m(x) \rangle$ is the vector objective function of x (there are m objective functions). One of the most popular value functions for modelling the DM's preferences about $f(x)$ is the normalised weighted sum. Let's suppose that the objectives are minimising; that is, the preference increases as the values of the objectives decrease; we have extended the value function to interval weights as follows:

$$\boldsymbol{U}(x) = \sum_{j=1}^{m} \boldsymbol{w}_j \left[\frac{f_j^{\max} - f_j(x)}{f_j^{\max} - f_j^{\min}} \right], \tag{4}$$

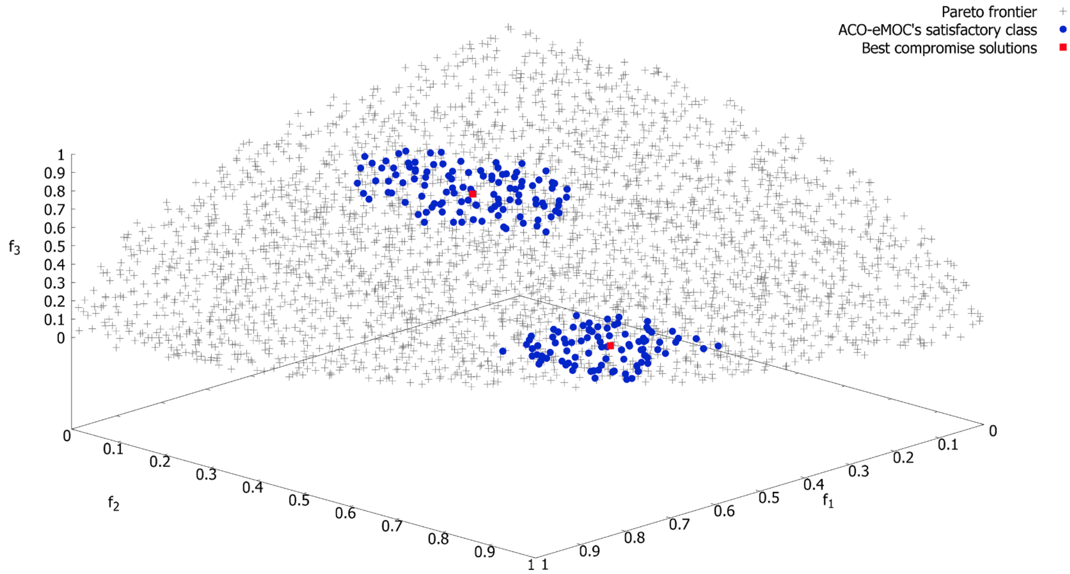

Fig. 5. Results of ACO-eMOC on DTLZ3 ($m = 3$).

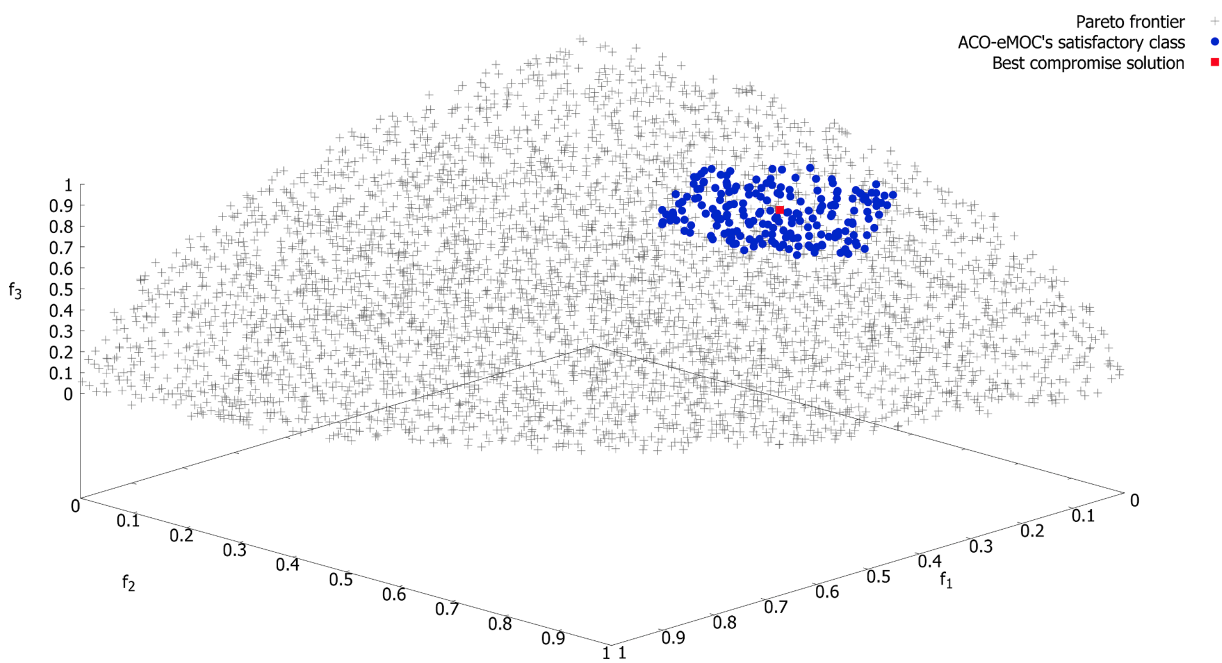

Fig. 6. Results of ACO-eMOC on DTLZ4 ($m = 3$).

where $f_j^{\max} = \max_{x \in O}\left\{f_j(x)\right\}, f_j^{\min} = \min_{x \in O}\left\{f_j(x)\right\}$, and $\boldsymbol{w}_j = \left[\underline{w_j}, \overline{w_j}\right]$ is the interval weight of the jth criterion. Each weight expresses the importance of its related criterion. Unfortunately, imprecisions in setting the weights, to a great extent, are unavoidable. An adequate decision support approach should handle imprecisions in the preference parameters. In Equation (4), interval numbers are used to model that imprecision. As a side benefit, interval numbers make the DM feel more comfortable when their preferences are elicited (compared to precise values). By extension, the value function $\boldsymbol{U}$ is also an interval number.

A preference function, as in Equation (4), models fully compensatory and transitive preferences. Nevertheless, the model in Equation (4) can be extended to handle partially compensatory preferences and veto effects, conferring this model the character 'eclectic.' Let $\mathscr{P}_\alpha$ be the binary preference relation defined as follows:

$$x \mathscr{P}_\alpha y \Leftrightarrow \boldsymbol{U}(x) >_\alpha \boldsymbol{U}(y) \wedge \neg y \mathscr{V} x, \quad (5)$$

where

$$y \mathscr{V} x \Leftrightarrow \bigvee_{j=1}^{m} f_j(x) - f_j(y) \geq \boldsymbol{v}_j. \quad (6)$$

According to Equation (5), x is preferred to y if and only if x has a higher value in function $\boldsymbol{U}$ than y and there is no veto in favour of y. In Equation (6), for the veto occurs in favour of y, it is necessary that at least one of the differences in the objectives between x and y exceeds $\boldsymbol{v}_j$; in this equation, $y \mathscr{V} x$ is a binary relation modelling the statement 'y vetoes x,' and $\boldsymbol{v}_j$ is an interval number representing the veto threshold of the jth criterion. The presence of veto thresholds becomes $\mathscr{P}_\alpha$ a relation that can be parametrised to have different properties, ranging from fully compensatory to non-compensatory. This degree of generalisation is possible because partial compensation is allowed within the ranges of

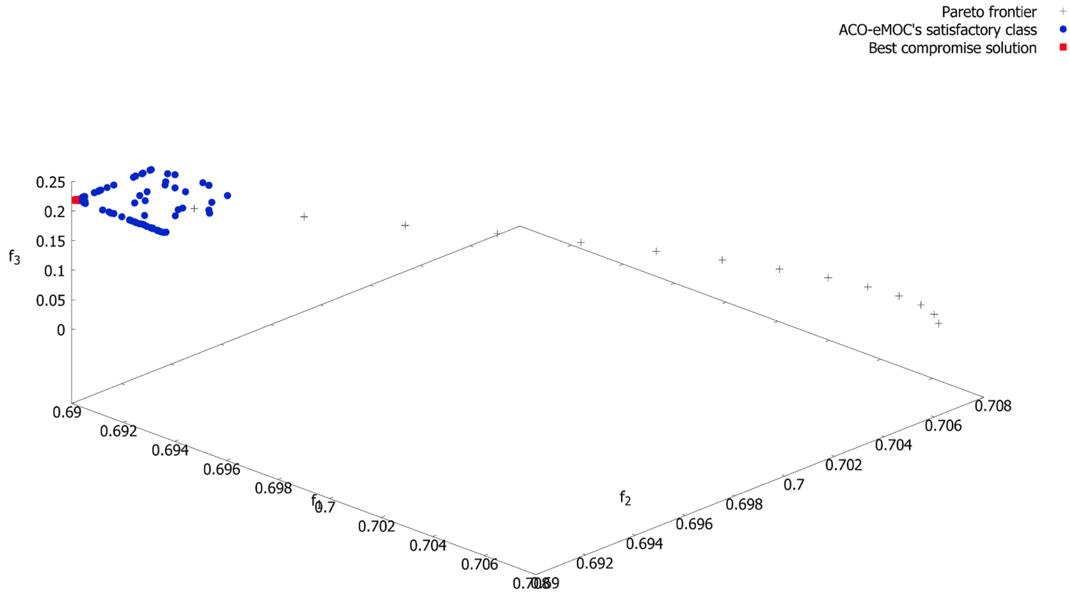

Fig. 7. Results of ACO-eMOC on DTLZ5 ($m = 3$).

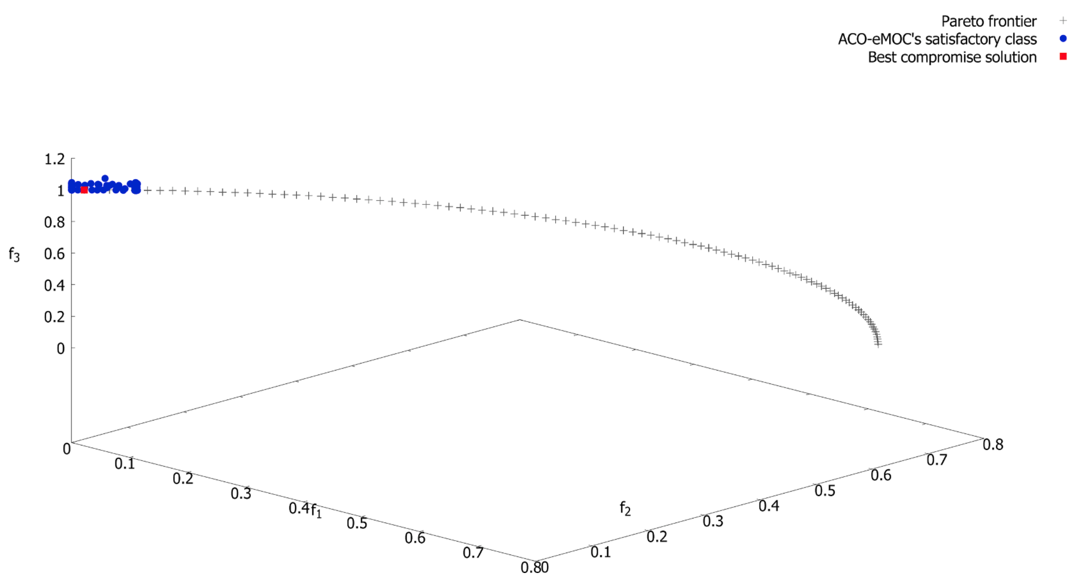

Fig. 8. Results of ACO-eMOC on DTLZ6 ($m = 3$).

the veto thresholds; even the veto effect could be deactivated for some specific criteria by using $\underline{v_j} = \overline{v_j} = \infty$.

The subscript in $\mathscr{P}_\alpha$ explicitly denotes the dependence of the preference relation on the credibility threshold α. The preference relation becomes more certain as α increases; mandatorily, $0.5 < \alpha \leq 1$. $\mathscr{P}_\alpha$ is non-reflexive and asymmetric. In general, $\mathscr{P}_\alpha$ is not transitive because of the veto conditions (Equation (6)).

Lastly, regarding computational complexity, $\mathscr{P}_\alpha$ is $O(m)$ because Equations (4) and (6) can be programmed using a loop with m iterations. Note that $\mathscr{P}_\alpha$ is the costliest operation of the assignment rules (indeed, in our implementation, the number of classes and the number of characteristic profiles are constant); therefore, the assignment rules are also $O(m)$.

4. Our proposal: ACO-eMOC

ACO-eMOC is inspired by some strategies of previous ACO algorithms for many-objective optimisation (Castellanos et al., 2022; Rivera et al., 2022) and extends them to embed the eclectic ordinal classifier presented in Section 3.2.

In ACO algorithms, the pheromone matrix (commonly denoted by τ) is a model of pivotal importance. In ACO-eMOC, the pheromone matrix acts as an archive that store the best-so-far solution vectors. Fig. 1 depicts the structure of the pheromone matrix used in ACO-eMOC. Let's consider the following:

- κ is the size of the pheromone matrix; it is the number of solutions stored in τ.
- $\tau_\iota = \langle \tau_{\iota,1}, \tau_{\iota,2}, \tau_{\iota,3}, \ldots, \tau_{\iota,n} \rangle$ is the vector with the values of the decision variables in the ι th solution archived in τ.
- The κ solutions are sorted considering two criteria: the class suggested by the conjoint assignment rule and the R2 scores. R2 (Brockhoff et al., 2012) is an indicator that measures the distribution of solutions. Here, we use the R2 version implemented in MOMBI II (Hernández Gómez and Coello Coello, 2015). The primary sorting criterion is the class in the following order: 'satisfactory,' 'uncertain,'

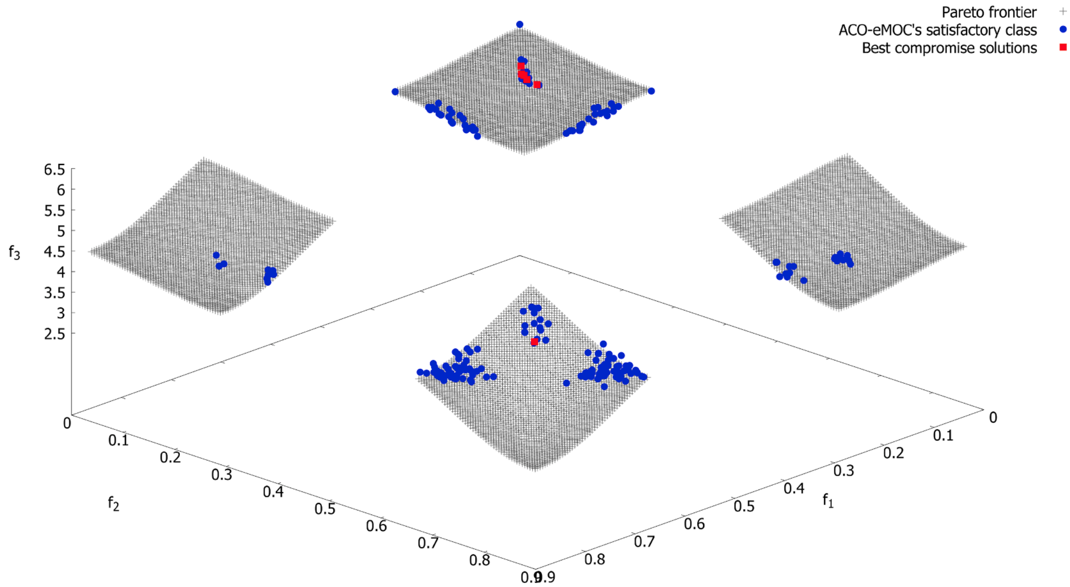

Fig. 9. Results of ACO-eMOC on DTLZ7 ($m = 3$).

and 'dissatisfactory.' A solution is satisfactory if the conjoint rule suggests C_2, it is dissatisfactory if the conjoint rule suggests C_1, and it is uncertain if the rule suggests the range C_1–C_2. The secondary criterion is the R2 score; in this regard, the solutions are sorted in increasing order to favour more uniformly distributed samples.

- ω_ι is the weight associated to τ_ι based on its position. These weights are values of a Gaussian function with mean 1.0, argument ι, and standard deviation $\varsigma{\cdot}\kappa$, which is defined as:

$$\omega_\iota = \frac{e^{-\varphi(\iota)}}{\varsigma{\cdot}\kappa\sqrt{2\pi}}, \text{where } \varphi(\iota) = \frac{(\iota - 1)}{2\varsigma^2\kappa^2}. \tag{7}$$

In Equation (7), ς is a parameter defining the balance between exploration and exploitation. With small values, the weights are distributed to favour the best-evaluated solutions exponentially; contrarily, with high values, the weights are more linearly distributed ($0 < \varsigma \leq 1$).

Sorting the pheromone matrix implies the iterated application of:

(i) the conjoint assignment rule, with the resulting complexity $O(\kappa m)$, and
(ii) the R2 ranking algorithm, with complexity $O(\kappa(\log\kappa + m))$ according to Hernández Gómez and Coello Coello (2015).

Integrating (i) and (ii), the computational complexity of the sorting method is $O(\kappa(\log\kappa + m))$, where κ is the size of the pheromone matrix and m is the number of objectives.

Ants use the pheromone matrix to construct solutions during each iteration. Let $x_\iota = \langle x_{\iota,1}, x_{\iota,2}, x_{\iota,3}, \ldots, x_{\iota,n}\rangle$ be the solution associated with the ι th ant, where $\tau_{\iota,i}$ is the value of the ith decision variable ($1 \leq i \leq n$). The ι th ant constructs x_ι following these steps:

1. A row of the pheromone matrix is chosen following a roulette wheel-based technique. The probability of choosing τ_ι is p_ι, defined as:

$$p_\iota = \frac{\omega_\iota}{\sum_{j=1}^{\kappa}\omega_j}, \tag{8}$$

where ω_ι is the weight associated to τ_ι, and κ is the size of the pheromone matrix (number of rows).

2. A Gaussian probability function with mean $x_{\iota,i}$ is defined for each decision variable ($1 \leq i \leq n$), which is used for sampling new values. The normal function for the selected $x_{\iota,i}$ is defined as:

$$g_\iota^i(x) = \frac{e^{-\phi_i(\iota)}}{s_\iota^i\sqrt{2\pi}}, \text{ where } \phi_i(\iota) = \frac{(x - x_{\iota,i})^2}{2(s_\iota^i)}, \tag{9}$$

where s_ι^i is the standard deviation of $g_\iota^i(x)$, which is estimated while ants sample solutions in each iteration as follows:

$$s_\iota^i = \xi\sum_{j=1}^{\kappa}\frac{|x_{j,i} - x_{\iota,i}|}{\kappa - 1}. \tag{10}$$

Equation (10) is the distance from the selected vector x_ι to the others (x_j) in the pheromone and scaled by ξ in all the dimensions ($0 < \xi \leq 1$). This parameter acts like the evaporation rate in the classic ACO algorithm. With low values, ξ biases the search toward the solution space around the top-ranked solutions in the archive.

3. Each ant assigns the decision variables of a new solution x_j by sampling the probability functions as follows:

$$x_{j,i} \sim g_\iota^i(x)\forall i \in \{1, 2, 3, \cdots, n\}, \tag{11}$$

where n is the number of decision variables, and ι is the row chosen from the pheromone archive in Step 1.

During each iteration, ants construct κ solutions sampling the regions near the solutions in the archive, especially the top-ranked solutions. This process is repeated until a maximum number of iterations is reached ($iter_{max}$). The computational complexity of constructing solutions is determined by Equations (9)–(11), whose complexity function is $O(\kappa n)$ during each iteration, where κ is the colony's size and n is the number of independent variables of an action x.

Additionally, in ACO-eMOC, p interactions are performed throughout the optimisation process to update the preferences on the vector solutions. In each interaction, the DM is supposed to identify the sets of characteristics profiles, R_1 and R_2. These sets are critical to the descending, ascending, and conjoint assignment rules. These rules make up an ordinal classifier, which is progressively updated to reflect the current DM's preferences.

Algorithm 1 presents an algorithm outline for ACO-eMOC. Lines 1 and 2 initialise the main variables. Here, p is the number of interactions, which are distributed throughout the run (an interaction is performed every $\Delta\ell$ iterations). Next, the pheromone matrix is initialised at random within the ranges of the decision variables. Then, the Pareto-efficient solutions are kept (Line 3) and normalised (Line 4). Solutions are normalised because multiple criteria (objectives) are involved in the

Table 3
Results on the WFG test suite.

m	*p*	**COMPARISON WITH RVEA-iGNG** **Problems in which ACO-eMOC** **(a)** outperforms RVEA-iGNG	**(b)** is outperformed by RVEA-iGNG	**Indicator**	**Problems in which ACO-eMOC** **(a)** outperforms RVEA-iGNG	**(b)** is outperformed by RVEA-iGNG	**Indicator**	**COMPARISON WITH FDEA II** **Problems in which ACO-eMOC** **(a)** outperforms FDEA II	**(b)** is outperformed by FDEA II	**Indicator**	**Problems in which ACO-eMOC** **(a)** outperforms FDEA II	**(b)** is outperformed by FDEA II	**Indicator**
3	3	3	1, 2, 4–6, 8, 9			1, 2, 4–6, 8, 9		3	1, 2, 4–9			1–6, 8	
	5	3, 7	1, 2, 4–6, 8, 9		7	1, 2, 4–6, 8, 9		3	1, 2, 4–6, 8		7	1–6, 8	
	7	3, 7, 8	1, 2, 4–6, 9		3, 5, 7, 8	1, 2, 6, 9		3, 7	1, 2, 4–6		3, 5, 7, 9	1, 2, 4, 6	
	9	3, 4, 6–9	1, 2		3–9	1, 2		3, 7–9	1, 2, 4, 6		3, 5–7, 9	1, 2, 4	
	11	3–9	2		3–9	1		3, 5, 7–9	4		2, 3, 5–9	1, 4	
5	3	3, 9	1, 2, 4–6, 8	**Euclidean**	3, 4	1, 2, 5–8	**Chebyshev**	9	1–7	**Euclidean**	8, 9	1, 3, 4–7	**Chebyshev**
	5	3, 7–9	1, 2, 4–6		3, 4, 7–9	1, 2, 5, 6		8, 9	1, 2, 4–6		2, 8, 9	1, 4–6	
	7	3, 5, 7–9	1, 2, 6		3–9	1		3, 5, 7–9	1, 2, 4, 6		2, 3, 5, 7–9	1, 4	
	9	3–9	1, 2		3– 9	1		3, 5–9	1, 2		2, 3, 5–9	1, 4	
	11	3–9	1		2–9	1		3, 5–9	1		2, 3, 5–9	1, 4	
10	3	3, 7–9	1, 2, 4–6	**Minimum**	8, 9	1, 2, 4–7	**Minimum**	5, 7–9	1–6	**Minimum**	8, 9	1–7	**Minimum**
	5	3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4–6	
	7	3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4, 6		3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4–6	
	9	3–9	1, 2		3, 5–9	2		3, 6–9	1, 5, 2		3, 6–9	2, 4	
	11	2–9	1		1–9			2, 3, 5–9	1		1–3, 5–9	4	
3	3	3, 7	1, 2, 4–6, 8, 9			1, 2, 4–6, 8, 9		3	1, 2, 4–6, 8, 9			1, 2, 4–6, 8, 9	
	5	3, 7	1, 2, 4–6, 8, 9			1, 2, 4–6, 8, 9		3, 7	1, 2, 4–6, 8		3	1, 2, 4–6, 8	
	7	3, 7, 8	1, 2, 4–6, 9		3, 8	1, 2, 4–6, 9		3, 7	1, 2, 4–6		3, 7, 9	1, 2, 4–6	
	9	3, 7–9	1, 2, 4, 6		3–5, 7–9	1, 2, 6		3, 5, 7–9	1, 2, 4, 6		3, 5, 7–9	1, 6	
	11	3, 7–9	2		3–5, 7–9	1, 6		3, 5, 7–9	2, 4		2, 3, 5, 7–9	1, 6	
5	3	3, 7, 9	1, 2, 4–6, 8	**Euclidean**	3, 9	1, 2, 5, 6	**Chebyshev**	8, 9	1–7	**Euclidean**	8	1–6	**Chebyshev**
	5	3, 7–9	1, 2, 4–6		3, 8, 9	1, 2, 5, 6		8, 9	1–6		8, 9	1, 2, 4–6	
	7	3, 7–9	1, 2, 4, 5		3–5, 7–9	1, 2, 6		8, 9	1, 2, 4, 5		3, 7–9	1, 2, 4, 6	
	9	3, 5–9	1, 2		3–5, 7–9	1, 2, 6		3, 5–9	1, 2, 4		3, 5, 7–9	1, 2, 4, 6	
	11	3, 5–9	2		3–5, 7–9	2		3, 5–9	4		3, 5–9	4	
10	3	3, 7–9	1, 2, 4–6	**Average**	3, 8, 9	1, 2, 4–6	**Average**	3, 7–9	1, 2, 4–6	**Average**	8	1, 2, 4–6	**Average**
	5	3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4–6		7–9	1, 2, 4–6	
	7	3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4, 6		3, 7–9	1, 2, 4–6		3, 7–9	1, 2, 4, 6	
	9	3–9	1, 2		3, 5, 7–9	1, 2, 4		3, 5–9	1, 4		3, 5–9	1	
	11	3–9	1, 2		1, 3, 5, 7–9	2		3, 5–9	1, 4		1, 3, 5–9		

Table 4
Extended experiments on the WFG1, WFG2 and WFG4 problems.

m	Indicator	COMPARISON WITH RVEA-iGNG Problem	Number of interactions in which ACO-eMOC (a) outperforms RVEA-iGNG	(b) is outperformed by RVEA-iGNG	COMPARISON WITH FDEA II Problem	Number of interactions in which ACO-eMOC (a) outperforms FDEA II	(b) is outperformed by FDEA II
3	Min. Euclidean	WFG1	17, 19, 21		WFG1	17, 19, 21	
	Avg. Euclidean		15, 17, 19, 21			17, 19, 21	
	Min. Chebyshev		19, 21	13, 15, 17		19, 21	13, 15
	Avg. Chebyshev					19, 21	13, 15
5	Min. Euclidean		19, 21	13, 15		19, 21	13, 15, 17
	Avg. Euclidean		15, 17, 19, 21			15, 17, 19, 21	
	Min. Chebyshev		19, 21	13, 15		19, 21	13
	Avg. Chebyshev		17, 19, 21			15, 17, 19, 21	
10	Min. Euclidean		13, 15, 17, 19, 21			17, 19, 21	13
	Avg. Euclidean		13, 15, 17, 19, 21			15, 17, 19, 21	
	Min. Chebyshev		13, 15, 17, 19, 21			13, 15, 17, 19, 21	
	Avg. Chebyshev		13, 15, 17, 19, 21			13, 15, 17, 19, 21	
3	Min. Euclidean	WFG2			WFG4	19, 21	13
	Avg. Euclidean		19, 21	13, 15		19, 21	
	Min. Chebyshev		17, 19, 21			19, 21	13
	Avg. Chebyshev		15, 17, 19, 21			17, 19, 21	
5	Min. Euclidean		13, 15, 17, 19, 21			15, 17, 19, 21	
	Avg. Euclidean		17, 19, 21	13, 15		19, 21	13, 15
	Min. Chebyshev		13, 15, 17, 19, 21			17, 19, 21	
	Avg. Chebyshev		17, 19, 21			19, 21	
10	Min. Euclidean		13, 15, 17, 19, 21			13, 15, 17, 19, 21	
	Avg. Euclidean		17, 19, 21			17, 19, 21	
	Min. Chebyshev		13, 15, 17, 19, 21			15, 17, 19, 21	
	Avg. Chebyshev		13, 15, 17, 19, 21			13, 15, 17, 19, 21	

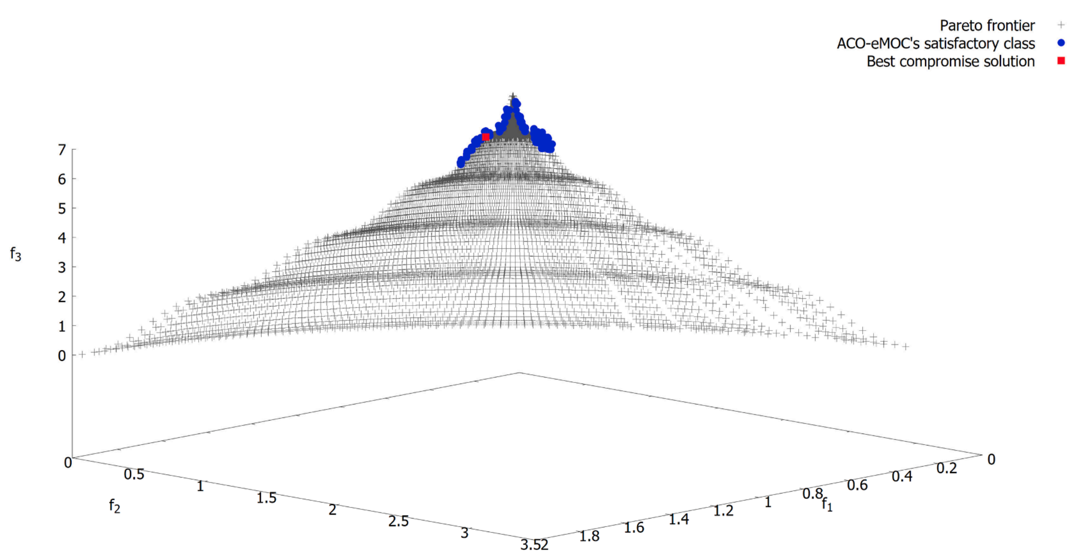

Fig. 10. Results of ACO-eMOC on WFG1 ($m = 3$).

decision-making process and are measured on different scales; normalisation provides a common scale that allows the ordinal classifier to weigh the criteria in function of the preference model alone.

Afterwards, the pheromone matrix is sorted by class and R2 scores (Line 5); note that the classifier has not been initialised with the sets of characteristic profiles; consequently, the conjoint rule always returns 'uncertain' as the suggested class, being R2 scores the single criterion of sorting. This fact is valid for all iterations before the first interaction with the DM. Therefore, ACO-eMOC searches for a representative sample of the complete Pareto frontier during the first $\imath - 1$ iterations.

The main iterated process is represented in Lines 6–21. Here, $iter_{max}$ is the maximum number of iterations. Lines 7–10 represent an interac-

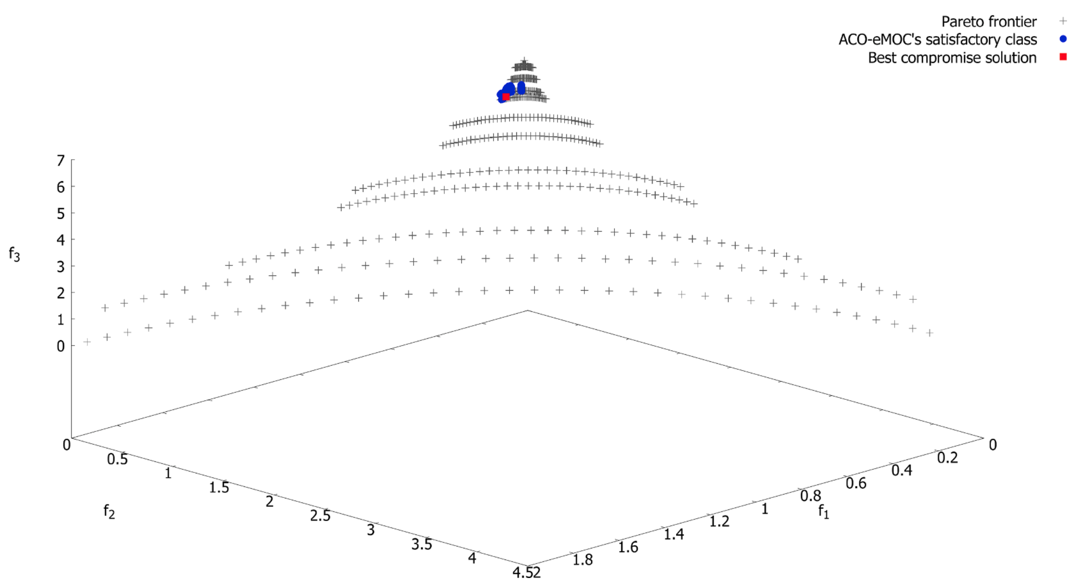

Fig. 11. Results of ACO-eMOC on WFG2 (m = 3).

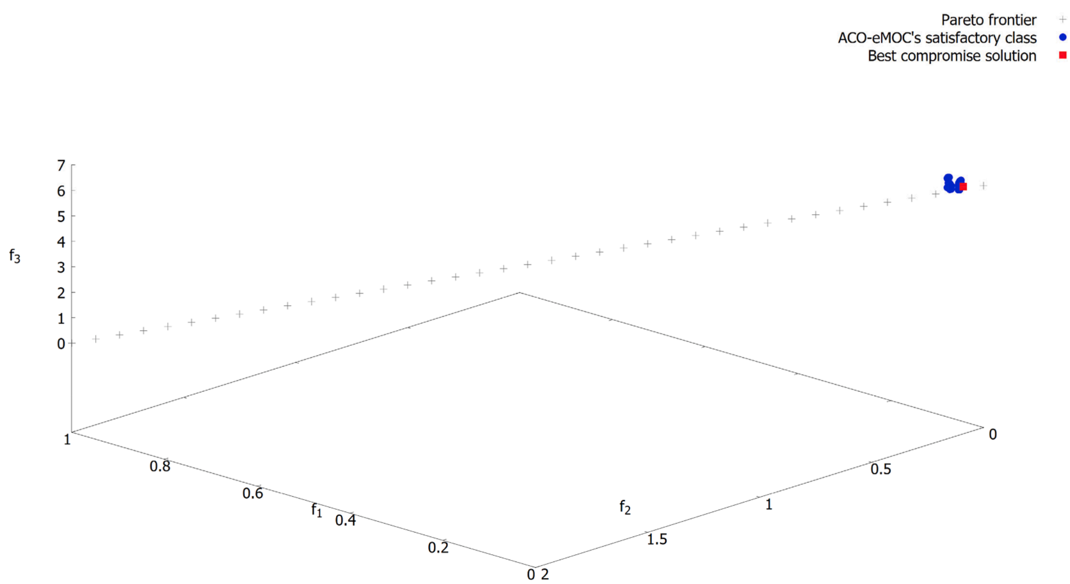

Fig. 12. Results of ACO-eMOC on WFG3 (m = 3).

tion with the DM, which aims to update the sets of characteristic profiles. These sets are critical to favour a bias toward the 'satisfactory' class. When these sets are identified, the pheromone matrix is accordingly sorted, taking the result of the ordinal classifier as the main criterion, which increases the selective pressure toward the RoI.

Algorithm 1. Ant Colony Optimisation enriched with an Eclectic Multi-criteria Ordinal Classifier

Input: Data of the problem (m, n) and the preference model ($\boldsymbol{v}$, $\boldsymbol{w}$, α)
Output: An approximation of the RoI
1.initialise $\iota \leftarrow 1$, $\Delta\ell \leftarrow \frac{iter_{\max}}{p+1}$, $\ell \leftarrow \Delta\ell$, $R_1 = \varnothing$, $R_2 = \varnothing$ *p is the number of interactions*
2.initialise τ at random
3. $\tau \leftarrow PS(\tau)$ *only take the Pareto-efficient solutions*
4.`normalise`(τ)
5. `sorting`(τ) *See* Fig. 1
6.while $t \leq iter_{\max}$ **do**
7. if round(ℓ) = ι **then** *Check if an interaction should be performed*
R_1 and R_2 are the sets of characteristics profiles identified by the DM during the interaction
8. $(R_1, R_2) \leftarrow$interaction_with_the_DM(τ)

(*continued on next column*)

(*continued*)

Algorithm 1. Ant Colony Optimisation enriched with an Eclectic Multi-criteria Ordinal Classifier

9. $\ell \leftarrow \ell + \Delta\ell$
10 end if
11. for each ant $x_j \in A$ **do** *A is the ant colony, which has κ ants*
12. $\iota \leftarrow$roulette_wheel(τ) *See Step 1, Equation* (8)
13. $x_{j,i} \sim g_i^{\iota}(x) \forall i \in \{1, 2, 3, \cdots, n\}$ *See Steps 2 and 3, Equation* (9)–(11)
14. end for
15. $O \leftarrow PS(\tau \cup A)$ *only take the Pareto-efficient solutions*
16. `normalise`(O)
17. `sorting`(O) *See* Fig. 1
18. $\tau \leftarrow \varnothing$
19. Copy into τ the first κ elements of O
20.$t \leftarrow t + 1$
21. end while
22. return 'satisfactory' solutions archived in τ

In Lines 11–14, the ants construct solutions following Steps 1–3 (Equations (8)–(11)). Lines 16–20 update the control variables. Finally,

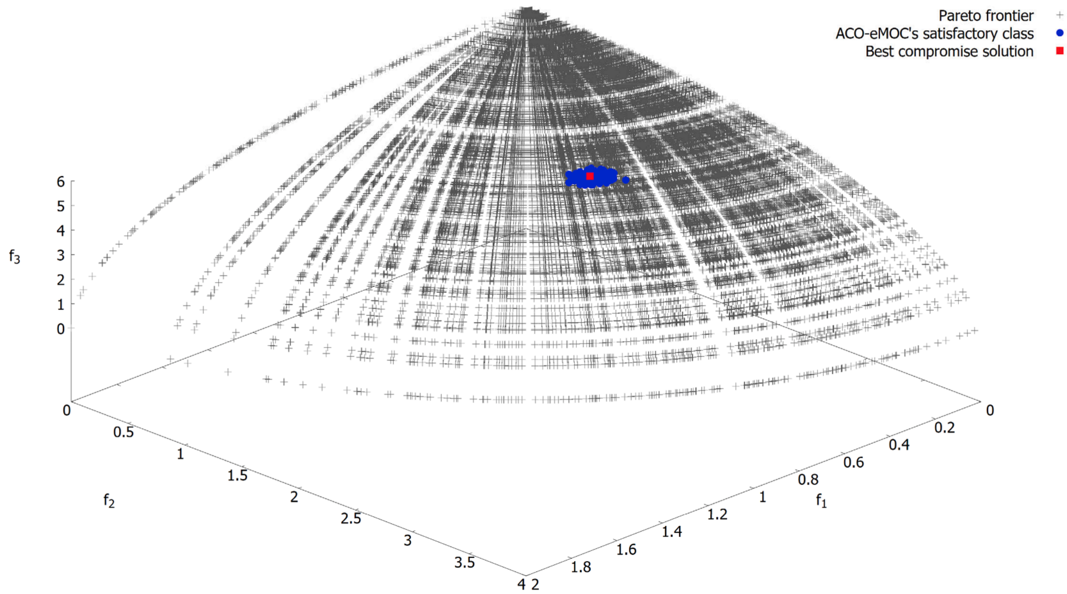

Fig. 13. Results of ACO-eMOC on WFG4 ($m = 3$).

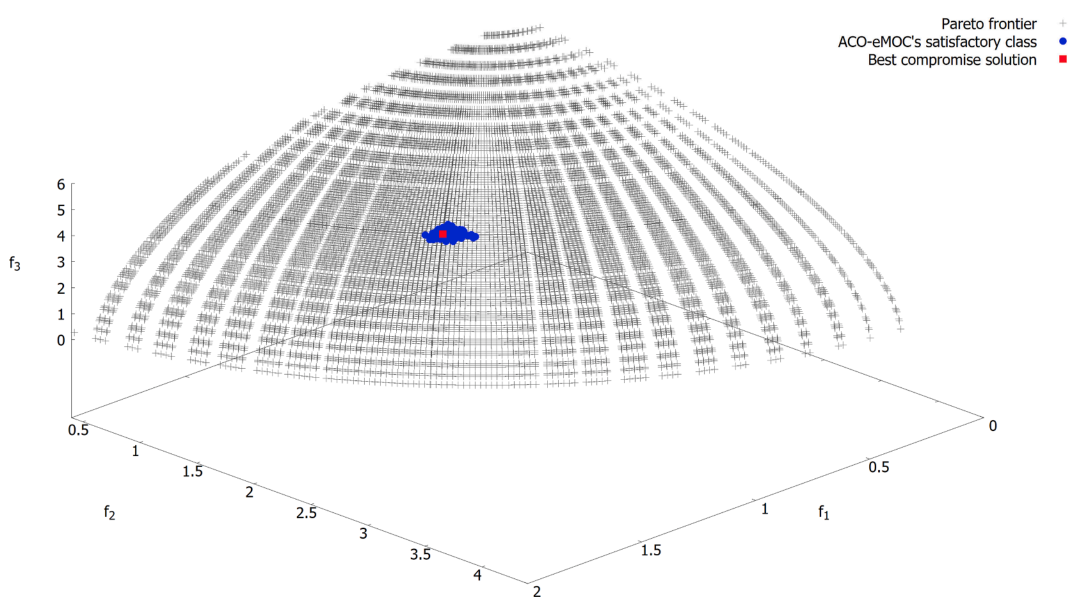

Fig. 14. Results of ACO-eMOC on WFG5 ($m = 3$).

the solutions classified as 'satisfactory' are returned as an approximation to the RoI (Line 22).

Regarding the complexity of ACO-eMOC, the following remarks on the costliest operations should be considered:

(i) Identifying the Pareto-efficient solutions (Lines 3 and 15) can be performed in $O(\kappa^2 m)$, where κ is the size of both the pheromone matrix and the colony ($\kappa = |\tau| = |A|$) and m is the number of objective functions.
(ii) Normalising the solution vectors (Lines 4 and 16) can be performed with an algorithm whose complexity function is $O(\kappa m)$.
(iii) Sorting the pheromone matrix (Lines 5 and 17) belongs to $O(\kappa(\log\kappa + m))$.
(iv) Constructing solutions (Lines 11–14) belongs to $O(\kappa n)$.

The overall complexity of Algorithm 1 at each iteration is determined by linearly integrating Operations (i)–(iv). Therefore, Algorithm 1 belongs to $O(\kappa(\kappa m + n))$, considering that Operations (i) and (iv) are the dominating terms.

Lastly, Fig. 2 presents the flowchart for Algorithm 1. Here, the main processes are represented in a more abstract way to favour the overall understanding of ACO-eMOC.

5. Experimental validation

In this section, we present the experimental results that evidence the advantages of our ACO-eMOC. Section 5.1 describes how we have simulated the interaction with the DM, which is necessary for extensive experimentation. Section 5.2 describes the conditions of all the experiments conducted in this section. Sections 5.3 and 5.4 present the results using the DTLZ and WFG test suites. Lastly, Section 5.5 presents an overall evaluation of ACO-eMOC on both suites.

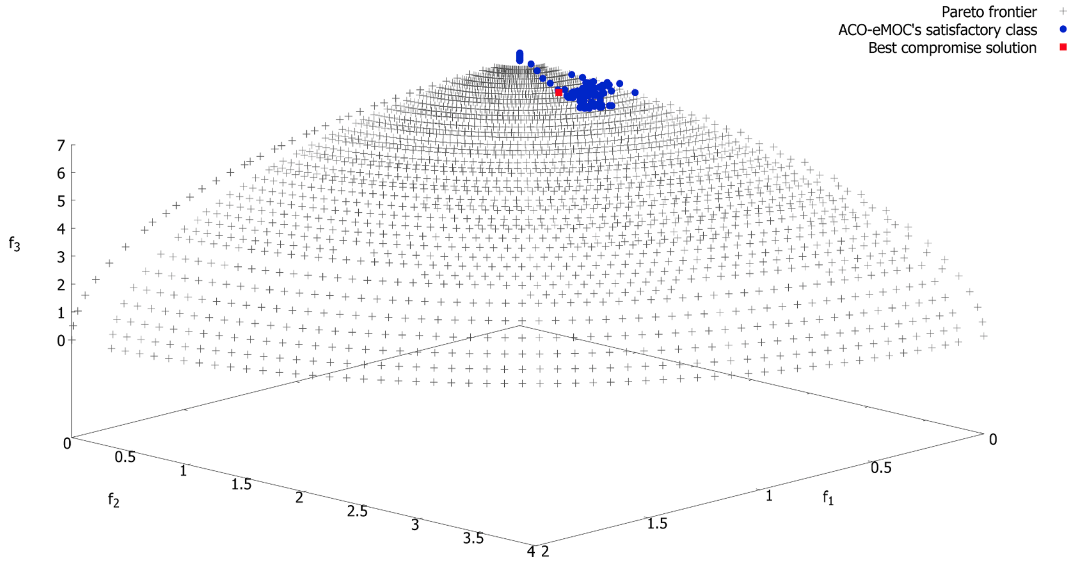

Fig. 15. Results of ACO-eMOC on WFG6 ($m = 3$).

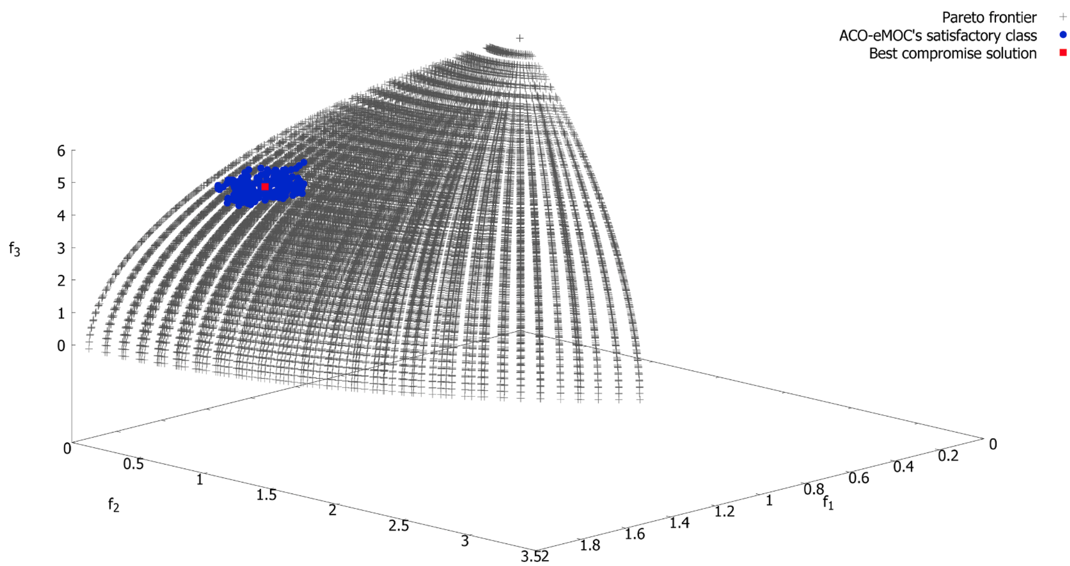

Fig. 16. Results of ACO-eMOC on WFG7 ($m = 3$).

5.1. Simulating the interaction with the DM

There are two directions in which an interaction with the DM could enrich the preference model:

(a) Updating the model parameters $\boldsymbol{w}$ and $\boldsymbol{v}$, which define the preference relation $\mathscr{P}_\alpha$.
(b) Updating the sets of characteristic profiles R_1 and R_2, which define the notion of what 'satisfactory' and 'dissatisfactory' solutions mean.

In Case (a), the model parameters could be elicited during the interaction, and both direct and indirect elicitation methods could be employed. However, this way would increase the cognitive effort invested by the DM during each interaction. To avoid this potential difficulty in our approach, these parameters are modelled as interval numbers, which is a robust way to tolerate uncertainty, coping with slight fluctuations in $\boldsymbol{w}$ and $\boldsymbol{v}$ arising from the interaction. The DM is supposed to provide these interval parameters with an adequate extent only once.

In Case (b), R_1 and R_2 can be updated with a low level of cognitive demand, which is the most advisable feature of an interactive approach. The number of classes is minimum, and the eclectic ordinal classifier can work with $|R_1| = |R_2| = 1$. Therefore, the DM could only identify one characteristic profile per class. So, the DM can certainly perform this task in each interaction. In ACO-eMOC, we focus on this type of progressive preference incorporation. Below, we explain how the identification of R_1 and R_2 can be simulated.

Given a set of solutions O, the subset of solutions in the class 'satisfactory' should fulfil the following conditions:

1. They are optimal solutions in the sense of Pareto dominance. Let's $\times$ and y two solution vectors, the binary relation 'x dominates y' is defined as:

$$x \preccurlyeq y = \left\{ (x,y) : f_j(x) \leq f_j(y) \forall j \in \{1,2,3,\cdots,m\} \wedge f_k(x) \langle f_k(y) \exists k \in \{1,2,3,\cdots,m\} \right\}. \quad (12)$$

A vector solution $x \in O$ is Pareto optimal (a.k.a. non-dominated) if

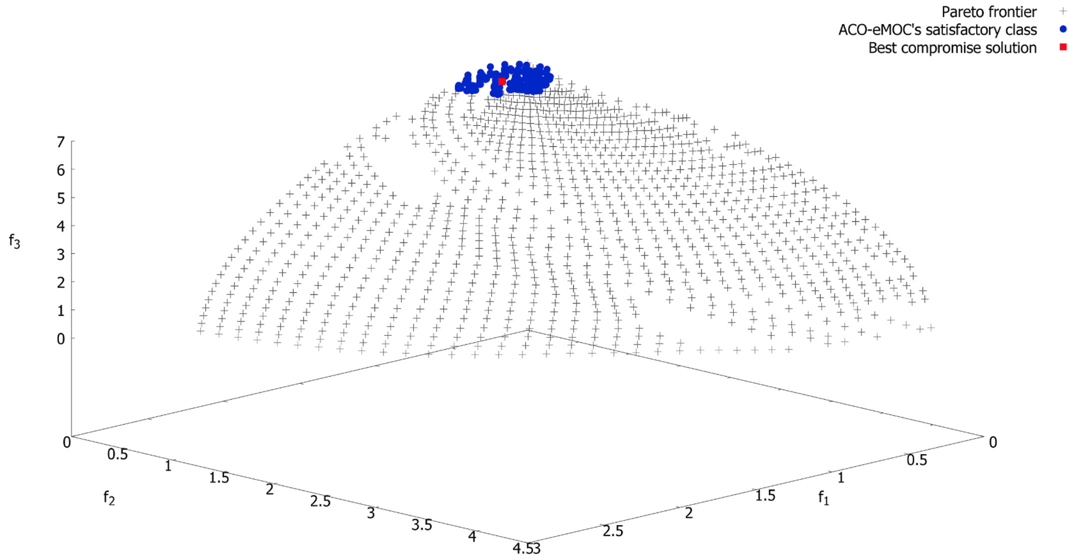

Fig. 17. Results of ACO-eMOC on WFG8 ($m = 3$).

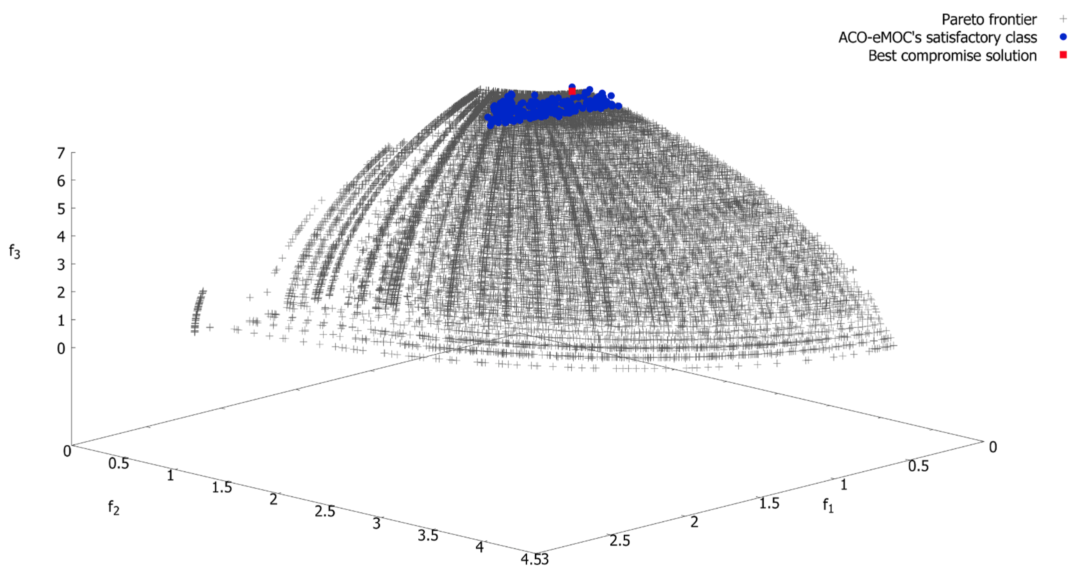

Fig. 18. Results of ACO-eMOC on WFG9 ($m = 3$).

no solution dominates x. The Pareto set consists of the optimal solutions, expressed as $PS(O) = \{x \in O : yx \forall y \in O\}$. The image of the Pareto set is termed 'the Pareto front,' defined as follows:

$$PF(O) = \{\langle f_1(x), f_2(x), f_3(x), \cdots, f_m(x)\rangle : x \in PS(O)\}. \tag{13}$$

In a broad sense, the non-dominated solutions are preferred to dominated ones. This set can be determined without knowing the preference model of the particular DM facing the problem. So far, no solution is preferred over others; in the next step, the preference model will play a key role in identifying the best compromise solution.

2. The 'weakness' of a solution × is the number of solution vectors preferred to x; that is, the cardinality of the set:

$$W(x, O) = \{y \in O : y\mathscr{P}_a x\}. \tag{14}$$

Any solution $x \in O$ with $|W(x, O)| = 0$ is considered satisfactory. The class 'satisfactory' is defined as follows:

$$C_2(O) = \{x \in PS(O) : W(x, O) = \varnothing\}, \tag{15}$$

and the class 'dissatisfactory' is:

$$C_1(O) = O \backslash C_2. \tag{16}$$

Then, the DM should identify the most representative solutions for each class. To simulate this task, we propose calculating the 'strength' of a solution. The strength of x is the number of solutions over which x is preferred. Expressly, the strength of x is the cardinality of the set:

$$S(x, O) = \{y \in O : x\mathscr{P}_a y\}. \tag{17}$$

The sets of characteristic profiles, $R_2(O)$ and $R_1(O)$, are defined as:

$$R_2(O) = \left\{x \in C_2(O) : |S(x, \mathrm{O})| = \underset{y \in C_2(O)}{\mathrm{median}}\{|S(y, \mathrm{O})|\}\right\}, \tag{18}$$

and

Table 5
Borda count grouped by indicator.

m	Algorithm	Borda scores grouped by indicator (a) Minimum Euclidean	(b) Average Euclidean	(c) Minimum Chebyshev	(d) Average Chebyshev
3	RVEA-iGNG	70.0	67.5	65.5	69.0
	FDEA II	58.5	54.5	61.5	58.5
	ACO-eMOC with $p = 3$	$101.0^{\downarrow}$	$100.5^{\downarrow}$	$101.5^{\downarrow}$	$101.0^{\downarrow}$
	ACO-eMOC with $p = 5$	$86.0^{\downarrow}$	$88.5^{\downarrow}$	$92.0^{\downarrow}$	$87.5^{\downarrow}$
	ACO-eMOC with $p = 7$	67.0	66.5	$54.5^{\uparrow}$	63.5
	ACO-eMOC with $p = 9$	$40.0^{\uparrow}$	$39.5^{\uparrow}$	$43.5^{\uparrow}$	$40.0^{\uparrow}$
	ACO-eMOC with $p = 11$	$25.5^{\uparrow}$	$31.0^{\uparrow}$	$29.5^{\uparrow}$	$28.5^{\uparrow}$
5	RVEA-iGNG	67.5	71.0	71.5	72.5
	FDEA II	59.0	54.5	70.0	66.5
	ACO-eMOC with $p = 3$	$101.0^{\downarrow}$	$100.5^{\downarrow}$	$101.5^{\downarrow}$	$100.0^{\downarrow}$
	ACO-eMOC with $p = 5$	$88.0^{\downarrow}$	$87.5^{\downarrow}$	$82.5^{\downarrow}$	$85.0^{\downarrow}$
	ACO-eMOC with $p = 7$	66.5	67.5	$58.0^{\uparrow}$	$56.0^{\uparrow}$
	ACO-eMOC with $p = 9$	$41.0^{\uparrow}$	$36.0^{\uparrow}$	$35.5^{\uparrow}$	$39.5^{\uparrow}$
	ACO-eMOC with $p = 11$	$25.0^{\uparrow}$	$31.0^{\uparrow}$	$29.0^{\uparrow}$	$28.5^{\uparrow}$
10	RVEA-iGNG	76.0	77.5	73.5	68.5
	FDEA II	71.0	65.0	71.0	73.0
	ACO-eMOC with $p = 3$	$95.0^{\downarrow}$	$94.0^{\downarrow}$	$101.0^{\downarrow}$	$97.0^{\downarrow}$
	ACO-eMOC with $p = 5$	$83.0^{\downarrow}$	$82.0^{\downarrow}$	$83.5^{\downarrow}$	$85.0^{\downarrow}$
	ACO-eMOC with $p = 7$	$62.5^{\uparrow}$	$62.5^{\uparrow}$	$60.0^{\uparrow}$	$61.5^{\uparrow}$
	ACO-eMOC with $p = 9$	$36.5^{\uparrow}$	$39.0^{\uparrow}$	$35.5^{\uparrow}$	$35.5^{\uparrow}$
	ACO-eMOC with $p = 11$	$24.0^{\uparrow}$	$28.0^{\uparrow}$	$23.5^{\uparrow}$	$27.5^{\uparrow}$

↓: the ACO-eMOC's ranking is worse than both references algorithms
↑: the ACO-eMOC's ranking is better than both references algorithms

$$R_1(O) = \left\{ x \in C_1(O) \, : |S(x,O)| = \underset{y \in C_1(O)}{\text{median}}\{|S(y,O)| - |W(y,O)|\} \right\}. \tag{19}$$

$R_2(O)$ is limited to contain up to three solutions, and $R_1(O)$ up to five (that is, $R_2 \leq 3$ and $R_1 \leq 5$). If more than three/five solutions fulfil Equations (18) and (19), the exceeding solutions with the worst R2 scores are removed. The R_2 score (Brockhoff et al., 2012) is an indicator that measures uniformity in the distribution of solutions; here, it favours the more equidistantly distributed vectors. This strategy promotes representativeness in $R_1(O)$ and $R_2(O)$.

The fictitious classes used by the descending and ascending rules (respectively, C_0 and C_3) are modelled by the nadir and ideal points. $R_0(O)$ contains the nadir point of $C_1(O)$, and $R_3(O)$ contains the ideal point of $C_2(O)$.

5.2. Experimental conditions

We programmed ACO-eMOC in C under Linux (Ubuntu 18) on an Intel Core i7-6700 3.4 GHz with 16 GB of RAM. The parameter values of ACO-eMOC are $\xi = 0.05$ and $\varsigma = 0.1$. This setting was experimentally identified as the best combination of $\xi \in \{0.01, 0.05, 0.1, 0.2\}$ and $\varsigma \in \{0.01, 0.05, 0.1, 0.2\}$.

FDEA II (Qiu et al., 2021) and RVEA-iGNG (Liu et al., 2022) have received increasing attention from the scientific community, being considered solid cutting-edge algorithms. On the one hand, FDEA II uses the fractional dominance relation to retain some solutions with promising performance; on the other hand, RVEA-iGNG is a vector-based decomposition MaOEA enriched with a learning algorithm trained with the solutions generated during the search process. Both reference algorithms are highly competitive according to several multi-objective indicators. The experimental results showed that FDEA-II and RVEA-iGNG have conjointly outperformed a plethora of state-of-the-art algorithms, expressly: A-NSGA-III, AdaW, AR-MOEA, DEA-GNG, hpaEA, $I_{SDE}+$, MOEA/D-AWA, MOEA/DD, MOEA/D-SOM, RVEA, RPD-NSGA-II, and VaEA (cf. Liu et al., 2022; Qiu et al., 2021). In line with these results, it is justified to validate the performance of our ACO-eMOC through a comparison with both FDEA-II and RVEA-iGNG simultaneously.

The parameter setting of the reference algorithm was taken from the original sources (Liu et al., 2022; Qiu et al., 2021). All algorithms were limited to 50,000 evaluations of the objective functions to promote a fair comparison. That is, the population size equals 100 with 500 iterations (in our approach, $\kappa = 100$ and $iter_{\max} = 500$).

Furthermore, we have used the DTLZ and WFG test suites to validate ACO-eMOC. DTLZ (Deb et al., 2002) and WFG (Huband et al., 2006) have become two standard test suites to assess the performance of multi-objective metaheuristics. In DTLZ, there are seven unconstrained problems (named DTLZ1–DTLZ7); in WFG, there are nine unconstrained problems (named WFG1–WFG9). Together, they offer a broad range of geometries in their Pareto fronts. Additionally, these benchmark problems are scalable regarding m (number of objectives) and n (number of decision variables).

Each problem has been tested with three, five, and ten objective functions. Consequently, we have 48 'input instances.' Each input instance is customised considering the following parameters: n, m, and k (number of position-related variables). Table 1 presents the configuration of each problem.

For each input instance, 20 parameter settings representing different DMs were randomly generated. In this process, the following conditions are fulfilled: $\alpha = 0.51$, $\sum_{j=1}^{m} \overline{w_j} \geq 1$, $\sum_{j=1}^{m} \underline{w_j} \leq 1$, and $0.01 \leq \underline{v_j} \leq \overline{v_j} \leq 0.1 \forall j \in \{1, 2, 3, \cdots, m\}$. Additionally, we ranged p (number of interactions with the DM) by considering 3, 5, 7, 9 and 11 interactions. ACO-eMOC ran 30 times on each input instance with a different DM using each value of p. Because RVEA-iGNG and FDEA II do not consider any preference model, they only ran 30 times on each input instance.

To validate our proposed ACO-eMOC, we first approximated the RoI per test instance and each DM. The A-RoI (Approximated RoI) considers the solutions satisfying Equation (20) in a representative sample of 100,000 Pareto-optimal points.

$$A\text{-}RoI(O) = \underset{x \in O}{\text{argmin}}\{|W(x,O)|, -|S(x,O)|\}. \tag{20}$$

In Equation (20), O is the sample. Accordingly, the A-RoI is made of the solutions with minimum weakness and maximum strength (as presented in Equations (14) and (17)), with lexicographic priority in favour of $|W(x,O)|$.

We want to highlight that the multi-objective indicators are intended to measure convergence, distribution, and extent of an approximation of the true Pareto front (e.g., hypervolume, inverted generational distance, spacing, and spreading). They are widely accepted because the *a posteriori* approaches aim to approximate the complete Pareto frontier. Contrastingly, suppose the aim is to approximate a subset of the Pareto front (just as in the preference-based algorithms). In that case, these indicators become misleading because they do not measure the performance in terms of the pursued solution set (cf. Li et al., 2018).

According to Li & Yao (2020), ideal quality indicators do not exist to evaluate solution subsets, notably for MOEAs with preferences. They suggested paying close attention to the design (or extension) of performance indicators considering preference incorporation because they should suit a wide range of scenarios. Furthermore, the performance measurement of an algorithm with preferences requires indicators that evaluate how well the algorithm follows the preferences, i.e., if the generated solutions reflect the preferences (Afsar et al., 2021).

In this paper, we have decided to measure the performance through indicators based on the distance to the Region of Interest (RoI). The RoI just contains the Pareto efficient solutions that best match the DM's preferences. Therefore, the best compromise (the solution finally chosen by the DM) is supposed to be a solution belonging to the RoI.

We calculated the distance to the A-RoI to get a notion of the quality of the solutions obtained per run. Let Θ denote the solution set of a run; the following distance indicators are considered:

- Minimum distance: The distance between the closest pair $(x, y) \in A\text{-}RoI(\Theta) \times A\text{-}RoI(O)$. This metric measures the performance in terms of the best solution alone.
- Average distance: The average distance of all pairs $(x, y) \in A\text{-}RoI(\Theta) \times A\text{-}RoI(O)$. This metric measures the performance regarding the trend of the solutions obtained.

Additionally, both indicators can be calculated through different types of distances. In this paper, we focus on the Euclidean distance and the Chebyshev distance. The applicability of these distances depends on the preferences of the DM. For example, the Euclidean distance is more appropriate for DMs with compensatory preferences; contrastingly, the Chebyshev distance would be for non-compensatory preferences.

Hereon, the adjective 'significant' means that a Friedman non-parametric test with a Nemenyi Post-hoc analysis (both with a 0.95-confidence interval) statistically validated the significance of the results. We performed these tests using the platform named STAC: Statistical Tests for Algorithms Comparison (Rodríguez-Fdez et al., 2015).

5.3. Performance on the DTLZ test suite

Table 2 presents the results of ACO-eMOC compared with RVEA-iGNG and FDEA II on the DTLZ test suite. Column 1 indicates the number of objective functions, Column 2 shows the number of interactions, Columns 3–8 compare ACO-eMOC and RVEA-iGNG, and Columns 9–14 provide the comparison against FDEA II. Here, Columns 3, 6, 9 and 12—identified as Point (a)—list the DTLZ problems in which ACO-eMOC significantly outperformed either RVEA-iGNG or FDEA II. Columns 4, 7, 10 and 13—identified as Point (b)—list the problems in which ACO-eMOC is significantly outperformed by either RVEA-iGNG or FDEA II. Lastly, Columns 5, 8, 11 and 14 indicate the distance indicator considered.

After analysing Table 2, we may discuss the following remarks:

- The performance of ACO-eMOC clearly improves as p increases. The number of interactions is decisive; then, the DM should be prepared to interact with the algorithm repeatedly.
- In the instances with the highest dimensionality ($m = 10$), RVEA-iGNG and FDEA II did not outperform ACO-eMOC with $p = 11$ regardless of the indicator considered.
- Considering $p \geq 9$ and $m = 10$, RVEA-iGNG never outperformed ACO-eMOC in the minimum distance indicators, and FDEA II never outperformed ACO-eMOC in the Chebyshev distance indicators.
- The results on DTLZ7 are especially encouraging. This problem is considered one of the most challenging in this benchmark. It is disconnected in both the Pareto set and the Pareto front; also, its geometry has mixed convex/concave regions (Huband et al., 2006).

Lastly, we have plotted the results of some selected runs of ACO-eMOC. Figs. 3–9 depict the bias introduced by the ordinal classifier on the DTLZ problems with three objective functions. The aim is to appreciate the convergence of our approach, which searches for the 'satisfactory class,' the subset of the Pareto set that encloses the most preferred solutions, including the best compromise.

5.4. Performance on the WFG test suite

Table 3 presents the results of ACO-eMOC compared to RVEA-iGNG and FDEA II on the WFG test suite. Its columns should be interpreted with the same meaning as Table 2.

According to these results, the following partial conclusions may be drafted:

- The best performance of ACO-eMOC is reached when $m = 10$ and $p = 11$. In these circumstances, ACO-eMOC statistically outperformed the reference algorithms in most problems.
- The performance of our algorithm is especially high in terms of the Chebyshev distance. ACO-eMOC with $p = 11$ and $m = 10$ consistently outperformed RVEA-iGNG in minimum Chebyshev distance, and it was never outperformed by FDEA II in average Chebyshev distance. Then, we strongly recommend using ACO-eMOC when the DM's preferences are worst case-oriented (which is plausible).
- WFG1 and WFG2 were the most difficult problems for ACO-eMOC compared to RVEA-iGNG. Interestingly, these are the only two problems with a non-concave Pareto front. The geometry of WFG1 is mixed, and the geometry of WFG2 is convex and disconnected (Zapotecas-Martínez et al., 2019).
- WFG1 and WFG4 were the most challenging problems for our algorithm compared to FDEA II. Regarding WFG4, the geometry of the Pareto front is concave, separable, and multi-modal.

Considering the difficulty ACO-eMOC had in addressing WFG1, WFG2 and WFG4, we extended the experimentation considering $p = \{13, 15, 17, 19, 21\}$. Table 4 summarises the results. Here, we want to emphasise the following remarks:

- These results are especially encouraging, taking the highest dimensionality ($m = 10$); ACO-eMOC outperformed both reference algorithms on a regular basis. Indeed, the best behaviour was observed when the Chebyshev indicators were considered.
- Regardless of the number of objectives (m) and the indicator, ACO-eMOC was never outperformed using $p \geq 19$. Using $p = 19$, our approach obtained high-quality results on the WFG test suite. We did not appreciate any statistical difference using $p = 21$; consequently, $p = 19$ seems to be an adequate upper bound of the number of interactions for this benchmark.

Again, we have plotted the ACO-eMOC's results on the 3-objective WFG problems. Figs. 10–18 present the satisfactory class obtained by our approach and its convergence to the Pareto frontier. We selected the runs in which the satisfactory class is most clearly defined.

5.5. Overall performance

Table 5 presents a ranking based on the Borda count for RVEA-iGNG, FDEA II and the five different settings of ACO-eMOC (arising from the different values of p). For each input instance, the algorithms are sorted according to the Friedman test and the Nemenyi post-hoc analysis. The worst algorithm gets the seventh position, and the best one gets the first position; the place is averaged in case of a draw. The cumulative sum of such positions over every instance is termed the Borda count. Consequently, an overall ranking of the metaheuristics can be suggested following the Borda sum. This ranking would describe the average performance of the algorithms.

According to Table 5, ACO-eMOC outperformed the reference algorithms—regardless of the number of objectives and the indicator—when a suitable number of interactions is reached. Analysing the Borda scores, we recommend using nine interactions. This is the lowest number for which the ACO-eMOC's Borda count is better than those of RVEA-iGNG and FDEA II. This fact means that ACO-eMOC is likely to obtain higher quality solutions (in comparison with the reference algorithms) if the

DM is well disposed to interact nine times at least (lower bound).

In general, the larger the number of objectives, the shorter the Borda score of ACO-eMOC. Therefore, the advantages of our approach are remarked as m increases; so, ACO-eMOC is especially suitable for MaOPs ($m \geq 5$). Lastly, the ACO-eMOC's Borda scores on the Chebyshev indicators are regularly better than those on the Euclidean indicators in the highest dimensionality (see the rows with $p \geq 9$ and $m = 10$). This analysis confirms that our approach is even more favourable for DMs with worst case-oriented preferences when many objective functions are present.

6. Conclusions and directions for future research

In this paper, we presented ACO-eMOC: Ant Colony Optimisation with an Eclectic Multi-criteria Ordinal Classifier. ACO-eMOC was designed to address continuous and unconstrained many-objective problems through interactive preference incorporation. This algorithm interactivity skews the search toward the Region of Interest (RoI), the region of the Pareto frontier containing the most satisfactory solutions according to the DM's preferences. The selective pressure toward the RoI is reached by embedding an ordinal classifier in the pheromone matrix.

ACO-eMOC is innovative in several ways:

i. The embedded ordinal classifier is far-reaching: it can model a wide range of properties of the functional and relational paradigms, including non-compensatory, partially compensatory, and fully compensatory.
ii. The model parameters are interval numbers; consequently, imprecision and hesitation can be tolerated. A many-objective metaheuristic with such robustness is a valuable contribution to the specialised literature.
iii. ACO-eMOC interacts with the DM to update the ordinal classifier according to their latest preference expression. This interaction is based on classifying solutions as 'satisfactory' or 'dissatisfactory.' The binary classification is the least cognitively demanding approach. Moreover, the DM only needs to identify the characteristic profiles for each class (the most representative solutions for the classes 'satisfactory' and 'dissatisfactory'). Such interaction in a many-objective metaheuristic is also a contribution of this paper.
iv. The output of ACO-eMOC is a short set of solutions recommended under the DM's preferences. In practice, this feature notably eases the decision analysis to choose the only solution to implement. From the perspective of prescriptive analytics, ACO-eMOC is a more integral approach than the Pareto-based MaOEAs.

Our proposed ACO-eMOC was validated on the DTLZ and WFG test suites. These benchmarks have become a standard to validate many-objective optimisation approaches. Results in comparison with RVEA-iGNG and FDEA II—evolutionary algorithms with outstanding results in the range of a posteriori approaches—support the advantages of our algorithm. ACO-eMOC provided higher quality solutions when a suitable number of interactions was considered, typically nine or more. Additionally, the ACO-eMOC's performance became more remarkable as the number of objectives increased.

In addition, two limitations were observed:

i. ACO-eMOC is likely unsuitable if the DM cannot or does not want to interact at least nine times; in our opinion, this fact would be an external threat to our approach.
ii. A construct limitation is that the experimentation is valid under the assumption that the DM is compatible with the eclectic model based on a weighted sum function enriched with veto conditions.

In future research, we will study other properties in the preference model, expressly reflexive preference relations. Also, ACO-eMOC could be extended by considering different ways to describe the classes in ordinal classification (e.g., limiting profiles). Moreover, in practice, ACO-eMOC could incorporate a preference elicitation method to infer the model parameters ($\boldsymbol{w}$ and $\boldsymbol{v}$); even these parameters could be updated in some selected interactions. Lastly, an extension with interacting criteria is also a promising direction for future research.

Funding

This research received no external funding.

CRediT authorship contribution statement

Gilberto Rivera: Conceptualization, Software, Formal analysis, Writing – review & editing. **Laura Cruz-Reyes:** Methodology, Investigation, Writing – review & editing. **Eduardo Fernandez:** Conceptualization, Formal analysis, Writing – review & editing. **Claudia Gomez-Santillan:** Methodology, Investigation, Writing – review & editing, Investigation. **Nelson Rangel-Valdez:** Investigation, Software, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgements

The authors want to thank Laboratorio Nacional de Tecnologías de la Información ("Call 2021 Reaccreditation of Conacyt National Laboratories" Project 321248) and the support of: (a) Cátedras CONACYT Program Number 3058, and TecNM through research networks Red de Investigación de Tecnologías Computacionales Aplicadas (Project 16804.23-P) y Red de Investigación de Electro Movilidad.

Appendix A. . Table of symbols

$\boldsymbol{A}, \boldsymbol{B}$	Interval numbers
$Poss(\boldsymbol{B} \geq \boldsymbol{A})$	Possibility function
α	Degree of credibility
O	A set of potential actions (in our context, feasible solutions to a MOP)
$C = \{C_1, C_2, \cdots, C_k\}$	A set of k classes
R_1 and R_2	Sets of characteristic profiles for the 'satisfactory' and 'dissatisfactory' classes (resp. C_2 and C_1).
x	An action (alternative/object) of a decision problem
$\boldsymbol{U}(x)$	Interval value function for x

(continued on next page)

(*continued*)

$\boldsymbol{A}, \boldsymbol{B}$	Interval numbers
m	Number of objective functions
n	Number of decision variables
$f(x) = \langle f_1(x), f_2(x), f_3(x), \cdots, f_m(x) \rangle$	The vector objective function of x
$w_j = \left[\underline{w_j}, \overline{w_j}\right]$	The interval weight of the jth criterion
$\mathscr{P}_\alpha$	The binary asymmetric preference relation
$y \mathscr{V} x$	A binary relation modelling the statement 'y vetoes x'
v_j	An interval number representing the veto threshold of the jth criterion
A	The ant colony
τ	The pheromone matrix
κ	The size of the pheromone matrix, which is equivalent to the number of ants (colony)
ι	In Algorithm 1, a row of the pheromone matrix ($1 \leq \iota \leq \kappa$)
t	In Algorithm 1, the current iteration to construct κ solutions by ants ($1 \leq t \leq iter_{\max}$)
$\tau_\iota = \langle \tau_{\iota,1}, \ \tau_{\iota,2}, \ \tau_{\iota,3}, \ \ldots, \ \tau_{\iota,n} \rangle$	The vector with the values of the decision variables in the ι th solution archived in τ
ω_ι	The weight associated to τ_ι based on its position
ς	A parameter defining the balance between exploration and exploitation
x_ι	A selected solution. $x_\iota = \langle x_{\iota,1}, \ x_{\iota,2}, \ x_{\iota,3}, \ \ldots, \ x_{\iota,n} \rangle$ where ι is a row of the pheromone matrix and $1 \leq \iota \leq n$
p_ι	Probability of choosing τ_ι
$g_\iota^i(x)$	The Gaussian probability function of decision variable i in solution x_ι
s_ι^i	Standard deviation of $g_\iota^i(x)$
ξ	Evaporation rate
$\acute{\iota}$	Counter variable to indicate when an interaction should be performed

References

Abouhawwash, M., & Deb, K. (2021). Reference point based evolutionary multi-objective optimization algorithms with convergence properties using KKTPM and ASF metrics. *Journal of Heuristics, 27*, 575–614. https://doi.org/10.1007/s10732-021-09470-4

S.F. Adra I. Griffin P.J. Fleming A Comparative Study of Progressive Preference Articulation Techniques for Multiobjective Optimisation S. Obayashi K. Deb C. Poloni T. Hiroyasu T. Murata Evolutionary Multi-Criterion Optimization. EMO 2007 Lecture Notes in Computer Science vol 4403 2007 Springer Berlin, Heidelberg 10.1007/978-3-540-70928-2_67.

Afsar, B., Ruiz, A. B., & Miettinen, K. (2021). Comparing interactive evolutionary multiobjective optimization methods with an artificial decision maker. *Complex & Intelligent Systems, 7*(6), 1–17. https://doi.org/10.1007/s40747-021-00586-5

Balderas, F., Fernandez, E., Gomez, C., Rangel, N., & Cruz-Reyes, L. (2019). An interval-based approach for evolutionary multi-objective optimisation of project portfolios. *International Journal of Information Technology & Decision Making, 18*(4), 1317–1358. https://doi.org/10.1142/S021962201950024X

Balderas, F., Fernández, E., Cruz-Reyes, L., Gómez-Santillán, C., & Rangel-Valdez, N. (2022). Solving group multi-objective optimization problems by optimizing consensus through multi-criteria ordinal classification. *European Journal of Operational Research, 297*(3), 1014–1029. https://doi.org/10.1016/j.ejor.2021.05.032

Bao, Q., Wang, M., Dai, G., Chen, X., & Song, Z. (2023). Dynamical decomposition and selection based evolutionary algorithm for many-objective optimization. *Applied Soft Computing, 141*, Article 110295. https://doi.org/10.1016/j.asoc.2023.110295

Bechikh, S., Elarbi, M., & Said, L. B. (2017). Many-objective optimization using evolutionary algorithms: A survey. In S. Bechikh, R. Datta, & A. Gupta (Eds.), *Recent advances in evolutionary multi-objective optimization* (pp. 105–137). Switzerland: Springer. https://doi.org/10.1007/978-3-319-42978-6_4. ISBN: 978-3-319-42977-9.

Bezerra, L. C., López-Ibáñez, M., & Stützle, T. (2018). A large-scale experimental evaluation of high-performing multi-and many-objective evolutionary algorithms. *Evolutionary computation, 26*(4), 621–656. https://doi.org/10.1162/evco_a_00217

Bouyssou, D., Marchant, T., Pirlot, M., Tsoukias, A., & Vincke, P. (2006). *Evaluation and decision models with multiple criteria: Stepping stones for the analyst* (Vol. 86). https://doi.org/10.1007/0-387-31099-1

Branke, J., & Deb, K. (2005). Integrating User Preferences into Evolutionary Multi-Objective Optimization. In Y. Jin (Ed.), *Knowledge Incorporation in Evolutionary Computation. Studies in Fuzziness and Soft Computing* (vol 167). Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-540-44511-1_21.

Branke, J., Corrente, S., Greco, S., Słowiński, R., & Zielniewicz, P. (2016). Using Choquet integral as preference model in interactive evolutionary multiobjective optimization. *European Journal of Operational Research, 250*(3), 884–901. https://doi.org/10.1016/j.ejor.2015.10.027

Brockhoff, D., Wagner, T., & Trautmann, H. (2012). On the properties of the R2 indicator. In 2012 Genetic and evolutionary computation conference (GECCO'2012) (pp. 465–472). Philadelphia: ACM Press. ISBN: 978-1-4503-1177-9. Doi: 10.1145/2330163.2330230.

Brockhoff, D., Bader, J., Thiele, L., & Zitzler, E. (2013). Directed multiobjective optimization based on the weighted hypervolume indicator. *Journal of Multi-Criteria Decision Analysis, 20*(5–6), 291–317. https://doi.org/10.1002/mcda.1502

Castellanos-Alvarez, A., Cruz-Reyes, L., Fernandez, E., Rangel-Valdez, N., Gómez-Santillán, C., Fraire, H., et al. (2021). A Method for Integration of Preferences to a Multi-Objective Evolutionary Algorithm Using Ordinal Multi-Criteria Classification. *Mathematical and Computational Applications, 26*(2), 27. https://doi.org/10.3390/mca26020027

Castellanos, A., Cruz-Reyes, L., Fernández, E., Rivera, G., Gomez-Santillan, C., & Rangel-Valdez, N. (2022). Hybridisation of Swarm Intelligence Algorithms with Multi-Criteria Ordinal Classification: A Strategy to Address Many-Objective Optimisation. *Mathematics, 10*(3), 322. https://doi.org/10.3390/math10030322

Cheng, R., Rodemann, T., Fischer, M., Olhofer, M., & Jin, Y. (2017). Evolutionary many-objective optimization of hybrid electric vehicle control: From general optimization to preference articulation. *IEEE Transactions on Emerging Topics in Computational Intelligence, 1*(2), 97–111. https://doi.org/10.1109/TETCI.2017.2669104

Coello Coello, C. A., González Brambila, S., Figueroa Gamboa, J., Castillo Tapia, M. G., & Hernández Gómez, R. (2020). Evolutionary multiobjective optimization: Open research areas and some challenges lying ahead. *Complex & Intelligent Systems, 6*, 221–236. https://doi.org/10.1007/s40747-019-0113-4

Corrente, S., Greco, S., Matarazzo, B., & Slowinski, R. (2021). Explainable Interactive Evolutionary Multiobjective Optimization (2021). *Social Science Research Network, 3792994*. https://doi.org/10.2139/ssrn.3792994

Covantes, E., Fernández, E., & Navarro, J. (2016). Handling the multiplicity of solutions in a MOEA based PDA-THESEUS framework for multi-criteria sorting. *Foundations of Computing and Decision Sciences, 41*(4), 213–235. https://doi.org/10.1515/fcds-2016-0013

Cruz-Reyes, L., Fernandez, E., Sanchez, P., Coello, C. A. C., & Gomez, C. (2017). Incorporation of implicit decision-maker preferences in multi-objective evolutionary optimization using a multi-criteria classification method. *Applied Soft Computing, 50*, 48–57. https://doi.org/10.1016/j.asoc.2016.10.037

Cruz-Reyes, L., Fernandez, E., Sanchez-Solis, J. P., Coello Coello, C. A., & Gomez, C. (2020). Hybrid evolutionary multi-objective optimisation using outranking-based ordinal classification methods. *Swarm and Evolutionary Computation, 54*, Article 100652. https://doi.org/10.1016/j.swevo.2020.100652

Cvetkovic, D., & Parmee, I. C. (2002). Preferences and their application in evolutionary multiobjective optimization. *IEEE Transactions on evolutionary computation, 6*(1), 42–57. https://doi.org/10.1109/4235.985691

Deb, K., Thiele, L., Laumanns, M., & Zitzler, E. (2002). Scalable test problems for evolutionary multiobjective optimization. In *Evolutionary Multiobjective Optimization* (pp. 105–145). London: Springer. https://doi.org/10.1109/CEC.2002.1007032.

Douissa, M. R., & Jabeur, K. (2020). A non-compensatory classification approach for multi-criteria ABC analysis. *Soft Computing, 24*(13), 9525–9556. https://doi.org/10.1007/s00500-019-04462-w

Doumpos, M., Marinakis, Y., Marinaki, M., & Zopounidis, C. (2009). An evolutionary approach to construction of outranking models for multicriteria classification: The case of the ELECTRE TRI method. *European Journal of Operational Research, 199*(2), 496–505. https://doi.org/10.1016/j.ejor.2008.11.035

Ezugwu, A. E., Adeleke, O. J., Akinyelu, A. A., & Viriri, S. (2020). A conceptual comparison of several metaheuristic algorithms on continuous optimisation problems. *Neural Computing and Applications, 32*, 6207–6251. https://doi.org/10.1007/s00521-019-04132-w

Falcón-Cardona, J. G., & Coello Coello, C. A. (2020). Indicator-based multi-objective evolutionary algorithms: A comprehensive survey. *ACM Computing Surveys (CSUR), 53*(2), 1–35. https://doi.org/10.1145/3376916

Fernandez, E., Navarro, J., & Bernal, S. (2009). Multicriteria sorting using a valued indifference relation under a preference disaggregation paradigm. *European Journal of Operational Research, 198*(2), 602–609. https://doi.org/10.1016/j.ejor.2008.09.020

Fernandez, E., Lopez, E., Bernal, S., Coello Coello, C. A., & Navarro, J. (2010). Evolutionary multiobjective optimization using an outranking-based dominance generalization. *Computers and Operations Research, 37*(2), 390–395. https://doi.org/10.1016/j.cor.2009.06.004

Fernandez, E., Lopez, E., Lopez, F., & Coello Coello, C. A. (2011). Increasing selective pressure towards the best compromise in evolutionary multiobjective optimization: The extended NOSGA method. *Information Sciences, 181*(1), 44–56. https://doi.org/10.1016/j.ins.2010.09.007

Fernandez, E., Figueira, J. R., & Navarro, J. (2019). An indirect elicitation method for the parameters of the ELECTRE TRI-nB model using genetic algorithms. *Applied Soft Computing, 77*, 723–733. https://doi.org/10.1016/j.asoc.2019.01.050

Fernandez, E., Rangel-Valdez, N., Cruz-Reyes, L., Gomez-Santillan, C., Rivera-Zarate, G., & Sanchez-Solis, P. (2019). Inferring parameters of a relational system of preferences from assignment examples using an evolutionary algorithm. *Technological and Economic Development of Economy, 25*(4), 693–715. https://doi.org/10.3846/tede.2019.9475

Fernández, E., Rangel-Valdez, N., Cruz-Reyes, L., Gomez-Santillan, C. G., & Coello-Coello, C. A. (2022). Preference incorporation in MOEA/D using an outranking approach with imprecise model parameters. *Swarm and Evolutionary Computation, 101097*. https://doi.org/10.1016/j.swevo.2022.101097

Fernández, E., Figueira, J. R., Navarro, J., & Solares, E. (2022). A Generalized Approach to Ordinal Classification Based on the Comparison of Actions with Either Limiting or Characteristic Profiles. *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2022.06.055

Fernández, E., Navarro, J., Solares, E., Coello, C. A. C., Díaz, R., & Flores, A. (2023). Inferring preferences for multi-criteria ordinal classification methods using evolutionary algorithms. *IEEE Access, 11*, 3044–3061. https://doi.org/10.1109/ACCESS.2023.3234240

Fliedner, T., & Liesiö, J. (2016). Adjustable robustness for multi-attribute project portfolio selection. *European Journal of Operational Research, 252*(3), 931–946. https://doi.org/10.1016/j.ejor.2016.01.058

Ge, H., Zhao, M., Sun, L., Wang, Z., Tan, G., Zhang, Q., et al. (2019). A many-objective evolutionary algorithm with two interacting processes: Cascade clustering and reference point incremental learning. *IEEE Transactions on Evolutionary Computation, 23*(4), 572–586. https://doi.org/10.1109/TEVC.2018.2874465

Ge, H., Zhao, M., Zhang, K., & Hou, Y. (2019). A two-engine interaction driven many-objective evolutionary algorithm with feasibility-aware adaptation. *Applied Soft Computing, 82*, Article 105588. https://doi.org/10.1016/j.asoc.2019.105588

Gnansounou, E. (2017). Fundamentals of Life Cycle Assessment and Specificity of Biorefineries. In: Edgard Gnansounou and Ashok Pandey (Eds), Life-Cycle Assessment of Biorefineries. Elsevier, 2017; pp. 41–75. https://doi.org/10.1016/B978-0-444-63585-3.00002-4.

Gong, D., Sun, F., Sun, J., & Sun, X. (2017). Set-based many-objective optimization guided by a preferred region. *Neurocomputing, 228*, 241–255. https://doi.org/10.1016/j.neucom.2016.09.081

Greco, S., Matarazzo, B., & Słowiński, R. (2010). Interactive evolutionary multiobjective optimization using dominance-based rough set approach. In *IEEE Congress on Evolutionary Computation* (pp. 1–8). IEEE. https://doi.org/10.1109/CEC.2010.5585982.

Gu, Q., Xu, Q., & Li, X. (2022). An improved NSGA-III algorithm based on distance dominance relation for many-objective optimization. *Expert Systems with Applications, 207*, Article 117738. https://doi.org/10.1016/j.eswa.2022.117738

Gu, Q., Zhou, Q., Wang, Q., & Xiong, N. N. (2023). An indicator preselection based evolutionary algorithm with auxiliary angle selection for many-objective optimization. *Information Sciences, 638*, Article 118996. https://doi.org/10.1016/j.ins.2023.118996

He, Y., He, Z., Kim, K. J., Jeong, I. J., & Lee, D. H. (2021). A robust interactive desirability function approach for multiple response optimization considering model uncertainty. *IEEE Transactions on Reliability, 70*(1), 175–187. https://doi.org/10.1109/TR.2020.2995752

Hernández Gómez, R., & Coello Coello, C. A. (2015). Improved metaheuristic based on the R2 indicator for many-objective optimization. In *Proceedings of the 2015 annual conference on genetic and evolutionary computation* (pp. 679–686). https://doi.org/10.1145/2739480.2754776.

Huband, S., Hingston, P., Barone, L., & While, L. (2006). A review of multiobjective test problems and a scalable test problem toolkit. *IEEE Transactions on Evolutionary Computation, 10*(5), 477–506. https://doi.org/10.1109/TEVC.2005.861417

Huang, Z., & Wang, F. (2021). A Review on Indicator-based Multi-objective Evolutionary Algorithms. In *In 2021 IEEE 7th International Conference on Cloud Computing and Intelligent Systems (CCIS)* (pp. 144–148). IEEE. https://doi.org/10.1109/CCIS53392.2021.9754639.

C.L. Hwang A.S.M. Masud Multiple Objective Decision Making—Methods, and Applications: A State-of-the-Art Survey Lecture Notes in Economics and Mathematical Systems vol. 164 1979 Springer-Verlag 10.1007/978-3-642-45511-7.

Kadziński, M., & Szczepański, A. (2022). Learning the parameters of an outranking-based sorting model with characteristic class profiles from large sets of assignment examples. *Applied Soft Computing, 116*, Article 108312. https://doi.org/10.1016/j.asoc.2021.108312

Kuo, R. J., Luthfiansyah, M. F., Masruroh, N. A., & Zulvia, F. E. (2023). Application of improved multi-objective particle swarm optimization algorithm to solve disruption for the two-stage vehicle routing problem with time windows. *Expert Systems with Applications, 225*, Article 120009. https://doi.org/10.1016/j.eswa.2023.120009

Lahdelma, R., Miettinen, K., & Salminen, P. (2005). Reference point approach for multiple decision makers. *European Journal of Operational Research, 164*(3), 785–791. https://doi.org/10.1016/j.ejor.2004.01.030

Li, B., Li, J., Tang, K., & Yao, X. (2015). Many-objective evolutionary algorithms. *ACM Computing Surveys., 48*(1), 1–35. https://doi.org/10.1145/2792984

K. Li Progressive Preference Learning: Proof-of-Principle Results in MOEA/D. Deb, et al. Evolutionary Multi-Criterion Optimization. EMO 2019 Lecture Notes in Computer Science vol 11411 2019 Springer Cham 10.1007/978-3-030-12598-1_50.

Li, K., Deb, K., & Yao, X. (2018). R-metric: Evaluating the performance of preference-based evolutionary multiobjective optimization using reference points. *IEEE Transactions on Evolutionary Computation, 22*(6), 821–835. https://doi.org/10.1109/TEVC.2017.2737781

Li, K., Liao, M., Deb, K., Min, G., & Yao, X. (2020). Does preference always help? A holistic study on preference-based evolutionary multiobjective optimization using reference points. *IEEE Transactions on Evolutionary Computation, 24*(6), 1078–1096. https://doi.org/10.1109/TEVC.2020.2987559

Li, M., & Yao, X. (2020). Quality evaluation of solution sets in multiobjective optimisation: A survey. *ACM Computing Surveys (CSUR), 52*(2), 1–38. https://doi.org/10.1145/3300148

Li, X., Li, X., Wang, K., & Yang, S. (2023). A Strength Pareto Evolutionary Algorithm Based on Adaptive Reference Points for Solving Irregular fronts. *Information Sciences, 626*, 658–693. https://doi.org/10.1016/j.ins.2023.01.073

Liu, Q., Jin, Y., Heiderich, M., Rodemann, T., & Yu, G. (2022). An adaptive reference vector-guided evolutionary algorithm using growing neural gas for many-objective optimization of irregular problems. *IEEE Transactions on Cybernetics, 52*(5), 2698–2711. https://doi.org/10.1109/TCYB.2020.3020630

Liu, S., Wang, H., Yao, W., & Peng, W. (2023). Surrogate-Assisted Environmental Selection for Fast Hypervolume-based Many-Objective Optimization. *IEEE Transactions on Evolutionary Computation (Early Access)*. https://doi.org/10.1109/TEVC.2023.3243632

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review, 63*(2), 81–97. https://doi.org/10.1037/h0043158

Molina, J., Santana, L. V., Hernández-Díaz, A. G., Coello Coello, C. A., & Caballero, R. (2009). g-dominance: Reference point-based dominance for multiobjective metaheuristics. *European Journal of Operational Research, 197*(2), 685–692. https://doi.org/10.1016/j.ejor.2008.07.015

R.E. Moore Methods and applications of interval analysis 1979 Society for Industrial and Applied Mathematics, SIAM Philadelphia, USA 10.1137/1.9781611970906.

Okola, I., Omulo, E. O., Ochieng, D. O., & Ouma, G. (2023). A comparison of evolutionary algorithms on a Large Scale Many-Objective Problem in Food–Energy–Water Nexus. *Results in Control and Optimization, 10*, Article 100195. https://doi.org/10.1016/j.rico.2022.100195

Oliveira, E., Antunes, C. H., & Gomes, Á. (2013). A comparative study of different approaches using an outranking relation in a multi-objective evolutionary algorithm. *Computers & Operations Research, 40*(6), 1602–1615. https://doi.org/10.1016/j.cor.2011.09.023

Qi, Y., Li, X., Yu, J., & Miao, Q. (2018). User-preference based decomposition in MOEA/D without using an ideal point. *Swarm and Evolutionary Computation, 44*, 597–611. https://doi.org/10.1016/j.swevo.2018.08.002

Qiu, W., Zhu, J., Wu, G., Fan, M., & Suganthan, P. N. (2021). Evolutionary many-objective algorithm based on fractional dominance relation and improved objective space decomposition strategy. *Swarm and Evolutionary Computation, 60*, Article 100776. https://doi.org/10.1016/j.swevo.2020.100776

Rivera, G., Florencia, R., Guerrero, M., Porras, R., & Sánchez-Solís, J. P. (2021). Online multi-criteria portfolio analysis through compromise programming models built on the underlying principles of fuzzy outranking. *Information Sciences, 580*, 734–755. https://doi.org/10.1016/j.ins.2021.08.087

Rivera, G., Coello Coello, C. A., Cruz-Reyes, L., Fernandez, E., Gomez-Santillan, C., & Rangel-Valdez, N. (2022). Preference incorporation into many-objective optimization: An Ant colony algorithm based on interval outranking. *Swarm and Evolutionary Computation, 69*, Article 101024. https://doi.org/10.1016/j.swevo.2021.101024

Rodríguez-Fdez, I., Canosa, A., Mucientes, M., & Bugarín, A. (2015). In *STAC: a web platform for the comparison of algorithms using statistical tests* (pp. 1–8). IEEE. https://doi.org/10.1109/FUZZ-IEEE.2015.7337889.

Roy, B. (1991). The outranking approach and the foundations of ELECTRE methods. *Theory and decision, 31*, 49–73.

Said, L. B., Bechikh, S., & Ghédira, K. (2010). The *r*-dominance: A new dominance relation for interactive evolutionary multicriteria decision making. *IEEE transactions on Evolutionary Computation, 14*(5), 801–818. https://doi.org/10.1109/TEVC.2010.2041060

Saldanha, W. H., Arrieta, F. R. P., Machado-Coelho, T. M., dos Santos, E. D., Maia, C. B., Ekel, P. I., et al. (2019). Evolutionary algorithms and the Preference Ranking Organization Method for Enrichment Evaluations as applied to a multiobjective design of shell-and-tube heat exchangers. *Case Studies in Thermal Engineering, 17*, Article 100564. https://doi.org/10.1016/j.csite.2019.100564

Selvi, V., & Umarani, R. (2010). Comparative analysis of ant colony and particle swarm optimization techniques. *International Journal of Computer Applications, 5*(4), 1–6.

Tomczyk, M. K., & Kadziński, M. (2020). Decomposition-based interactive evolutionary algorithm for multiple objective optimization. *IEEE Transactions on Evolutionary Computation, 24*(2), 320–334. https://doi.org/10.1109/TEVC.2019.2915767

Von Lücken, C., Brizuela, C., & Barán, B. (2019). An overview on evolutionary algorithms for many-objective optimization problems. *Wiley Interdisciplinary reviews: data mining and knowledge discovery, 9*(1), e1267.

Wagner, T., & Trautmann, H. (2010). Integration of preferences in hypervolume-based multiobjective evolutionary algorithms by means of desirability functions. *IEEE Transactions on Evolutionary Computation, 14*(5), 688–701. https://doi.org/10.1109/TEVC.2010.2058119

Wu, Y., Ma, W., Miao, Q., & Wang, S. (2019). Multimodal continuous ant colony optimization for multisensor remote sensing image registration with local search. *Swarm and Evolutionary Computation, 47*, 89–95. https://doi.org/10.1016/j.swevo.2017.07.004

Wu, Y., Liu, J. W., Zhu, C. Z., Bai, Z. F., Miao, Q. G., Ma, W. P., et al. (2021). Computational intelligence in remote sensing image registration: A survey. *International Journal of Automation and Computing, 18*, 1–17. https://doi.org/10.1007/s11633-020-1248-x

Wu, Y., Li, J., Yuan, Y., Qin, A. K., Miao, Q.-G., & Gong, M.-G. (2021). Commonality Autoencoder: Learning Common Features for Change Detection From Heterogeneous Images. *IEEE Transactions on Neural Networks and Learning Systems, 1–14*. https://doi.org/10.1109/tnnls.2021.3056238

Xiang, Y., Zhou, Y., Li, M., & Chen, Z. (2017). A vector angle-based evolutionary algorithm for unconstrained many-objective optimization. *IEEE Transactions Evolutionary Computation, 21*(1), 131–152. https://doi.org/10.1109/TEVC.2016.2587808

Yao, S., Jiang, Z., Li, N., Zhang, H., & Geng, N. (2011). A multi-objective dynamic scheduling approach using multiple attribute decision making in semiconductor manufacturing. *International Journal of Production Economics, 130*(1), 125–133. https://doi.org/10.1016/j.ijpe.2010.12.014

Yuan, M. H., Chiueh, P. T., & Lo, S. L. (2021). Measuring urban food-energy-water nexus sustainability: Finding solutions for cities. *Science of The Total Environment, 752*, Article 141954. https://doi.org/10.1016/j.scitotenv.2020.141954

Zapotecas-Martínez, S., Coello, C. A. C., Aguirre, H. E., & Tanaka, K. (2019). A review of features and limitations of existing scalable multiobjective test suites. *IEEE Transactions on Evolutionary Computation, 23*(1), 130–142. https://doi.org/10.1109/TEVC.2018.2836912

Zhang, W., Liu, J., Tan, S., & Wang, H. (2023). A decomposition-rotation dominance based evolutionary algorithm with reference point adaption for many-objective optimization. *Expert Systems with Applications, 215*, Article 119424. https://doi.org/10.1016/j.eswa.2022.119424

Zhao, H., Zhang, C., Ning, J., Zhang, B., Sun, P., & Feng, Y. (2019). A comparative study of the evolutionary many-objective algorithms. *Progress in Artificial Intelligence, 8*, 15–43. https://doi.org/10.1007/s13748-019-00174-2

Zitzler, E., & Thiele, L. (1998). An evolutionary algorithm for multiobjective optimization: The strength pareto approach. *Eidgenössische Technische Hochschule Zürich, TIK-Report, 43*. https://doi.org/10.3929/ETHZ-A-004288833