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A reliability analysis for electronic devices under an extension of exponentiated perks distribution

Luis Carlos Méndez-González[1] | **Luis Alberto Rodríguez-Picón**[1] | **Ivan Juan Carlos Pérez-Olguin**[1] | **Vicente Garcia**[2] | **Abel Eduardo Quezada-Carreón**[2]

[1]Department of Industrial Engineering and Manufacturing, Institute of Engineering and Technology, Universidad Autónoma de Ciudad Juárez, Ciudad Juárez, Chihuahua, México

[2]Department of Electrical and Computer Engineering, Institute of Engineering and Technology, Universidad Autónoma de Ciudad Juárez, Ciudad Juárez, Chihuahua, México

Correspondence
Luis Carlos Méndez-González, Department of Industrial Engineering and Manufacturing, Institute of Engineering and Technology, Universidad Autónoma de Ciudad Juárez, Ciudad Juárez, Chihuahua 32310, México.
Email: luis.mendez@uacj.mx

Abstract

This paper presents a reliability analysis for electronic devices (ED) with bathtub curve-shaped failure times. An extension of the exponentiated perks distribution (EPD) is proposed for the analysis. The extension of this new distribution is based on the Alpha Power Transformation, so the Alpha Exponentiated Perks Distribution (AEXP) is introduced. The AEXP has three shape parameters and one scale parameter, allowing greater flexibility to represent failure rates in an increasing, decreasing, or bathtub curve form. Some useful properties in the reliability engineering context are presented. AEXP parameters were estimated via the Maximum Likelihood Method. Finally, two case studies focused on ED are used to compare the proposed distribution and other distributions with similar failure rate representation properties. The obtained results show that the AEXP better describes the behavior of ED than the distributions considered in the analysis.

KEYWORDS

alpha exponentiated perks distribution, bathtub shape distribution, non-monotone failure rate, perks distribution, reliability of electronic devices

1 | INTRODUCTION

Nowadays, technological advances have substantially modified the designing and manufacturing of electronic devices (ED) due to the number of semiconductors based on Silicon and Germanium with which they are built. These modifications generate that the ED is a complex system. From the reliability engineering point of view, the information obtained to represent the failure rate with the bathtub shape may be inaccurate if the used statistical distribution does not have enough flexibility in its parameters to model such behavior. The above causes some important reliability or quality metrics, such as maintenance or warranty times being incorrect or not close to real behavior, leading to increased manufacturing costs due to excessive repair or replacement of EDs when operating with customers.

Different mathematical and statistical methodologies applied to reliability analysis have been proposed to represent the non-monotonic failure rates of ED. Aarset[1] proposed that it is possible to know failure times' behavior through probability graphs through the Total Time on Test (TTT). He et al.[2] developed a methodology to identify the inflection point between infant mortality and the normal operating life of EDs within the bathtub curve. Wang et al.[3] proposed general modeling for the representation of failure times considering, in terms of reliability, the different sources of failure that can be generated in a device. Hjorth[4] proposed three parameters mixed distributions with Increasing Failure Rate giving the possibility that this model can be used for small samples. Arshad et al.[5] obtains a three-parameter model based on the modification of the Mustapha Type-I distribution; through the Lehmann-II distribution's general formulation, the extra shape parameter allows better control of the skewness and kurtosis of the density function. Dombi et al.[6] proposes the Omega distribution with three parameters formulated from the Lambert W-function; this new distribution has two essential characteristics, it lacks exponential terms, and its domain does not have infinity. Other authors propose the modification of classified reliability distributions such as the Weibull distribution (WD), an example of these works can be seen in Xie et al.,[7] Jia et al.,[8] Liao et al.,[9] Lai et al.,[10] Bebbington et al.,[11] Nassar et al.,[12] Peng and Yan,[13] Abd El-Monsef et al.,[14] Shakhatreh et al.[15]

Other methodologies propose the sum of the risk functions of two distributions with similar properties to represent the failure rates in the shape of a bathtub curve. Xie and Lai[16] presented the base methodology for additive distributions and introduced the Additive Weibull Distribution (AWD) based on a summation of two WD hazard rate functions. Zaindin and Sarhan[17] proposed the Sarhan and Zaindin modified Weibull (SZMW), which has better flexibility to model failure rates; the SZMW starts from being a special case of the AWD. Abd EL-Baset and Ghazal[18] proposes a generalization for the additive forms that involve the WD, thus introducing the Exponentiated additive Weibull Distribution EaddWD; this distribution of five parameters has several sub-models capable of representing the bathtub curve failure times exhibited by the ED. Other additive distributions with applications for ED can be seen in Thanh Thach and Briš,[19] Lemonte et al.,[20] He et al.[21]

Some authors propose hybrid distributions with reliability applications, with which they combine two distributions to describe non-monotone failure times. For example Lee et al.,[22] Silva et al.,[23] Cordeiro et al.,[24] Pal et al.,[25] Ali et al.,[26] Ali et al.[27] establish the use of the Exponential and Beta distributions in combination with the WD, with which they can describe the non-monotonic failures exhibited by the ED.

The methodologies presented above have a fundamental problem; in some cases, the modeling of bathtub curve failure times tends to have a "J", "U", or "V" shape. The preceding statement establishes that the normal life of the ED is represented as a short portion of the failure rate that the statistical distribution for reliability analysis offers, so perhaps the choice of said distribution is not appropriate.

Recently, reliability analyses have been presented using the Perks Mortality Equation (PME),[28] which is used in the actuarial field to determine people's average life in insurance studies. Zeng et al.[29] proposed a pair of distributions with four (Perks-4) and five (Perks-5) parameters based on PME. They conclude that a four-parameter Perks distribution accurately describes the ED failure times during their analysis. Using the additive methodology, Singh[30] proposed the four-parameter Additive Perks-Weibull (APW) distribution. The results obtained by the APW, when applied to different case studies to represent the ED failures, show that the description of the presented failure rates resembles the reliability bathtub curve. Singh and Choudhary[31] proposes an extension of the PME by exponentially adding a parameter to the hazard rate function, presenting the exponentiated perks distribution (EXP). This distribution establishes only three parameters and can model non-monotonic failure rates but does not represent the bathtub curve, which opens the possibility of further analysis of this distribution and its relationship with modeling ED failure times.

Other methodologies propose using transformations to probability distributions to represent non-monotone behavior. Mahdavi and Kundu[32] proposed to use the Alpha Power Transformation (APT), with which a new shape parameter is added to the distributions, providing greater flexibility to the representation of the data under analysis. Mead et al.[33] establishes the statistical properties of the APT family and how this transformation can be implemented for analysis in some branches of science, such as reliability engineering. Elbatal et al.,[34] Nassar et al.,[35] Méndez-González et al.,[36] Chettri et al.,[37] Alotaibi et al.,[38] proposed reliability analysis using transforming the WD through the APT, thus forming the APTW, where they obtained a three-parameter distribution with which it was possible to obtain non-monotonic behaviors with data from experiments with and without information acceleration. This distribution gives a competitive advantage to the APTW for representing the failure times of different classes of devices. Other related works that use the APT with some other distribution and have very competitive results can be seen in Ali et al.,[39] Basheer,[40] Ibrahim et al.,[41] Bulut et al.,[42] Basheer,[43] Hassan et al.[44]

Based on the above background, this paper presents a model that proposes an EXP extension through APT. Thus, the Alpha Power Transformation Exponentiated Perks Distribution (AEXP) is introduced as an alternative for analyzing failure times in EDs. One of the most remarkable properties of the AEXP is its three shape parameters, which allow it to represent non-monotone behaviors very close to the concepts of the reliability bathtub curve. The AEXP is more flexible with the experimental data that possess a non-monotonic failure than some distributions presented in the literature review. The mathematical flexibility shows that it can be a good option for reliability analysis practitioners. To verify the proposed methodology, three case studies focused on determining the reliability of ED are present. The obtained results by the AEXP and some of the distributions established in the literature review that have non-monotonic behaviors are compared. The Maximum Likelihood Method (MLE) was used to estimate each parameter for all the distributions under analysis. The Akaike information criterion (AIC), the Bayesian information criterion (BIC), Kolmogorov Smirnov test (K-S), and P-value were used to compare the distributions under analysis.

Finally, this paper is organized as follows. Section 2 presents the AEXP model construction. Section 3 presents relevant statistical equations to perform a reliability analysis. Section 4 presents an approach of AEXP under Accelerated Life Testing (ALT) and time-varying modeling centered in ED. Section 5 presents the Maximum likelihood Estimator (MLE) to calculate the parameters proposed in section 2. Section 6 presents the case studies of the paper. The last section provides concluding remarks and future work about the proposed model.

2 | AEXP MODEL CONSTRUCTION

The APT is a family for generating univariate distributions. Based on Mahdavi and Kundu,[32] let x be a random variable that has a Continuous Density Function (CDF) $G(x)$, and defined Probability Density Function (PDF) $g(x)$, then the f_{APT} can be written as:

$$f_{APT}(x) = \frac{\ln(\alpha)}{\alpha - 1} \cdot g(x) \cdot \alpha^{G(x)} \;\; if \; \alpha > 0, \alpha \neq 1. \tag{1}$$

As established by Singh and Choudhary,[31] the EXP CDF, it can be expressed as:

$$F_{EXP}(x) = \left[1 - \left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta}. \tag{2}$$

Substituting Equation (2) in Equation (1) the PDF of the proposed model is obtained and is described as:

$$f(x) = \frac{\ln(\alpha)}{\alpha - 1} \cdot \beta\lambda\theta \cdot \frac{(1+\beta)e^{\lambda x}}{\left(1+\beta e^{\lambda x}\right)^2} \cdot \left[1 - \left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta - 1} \alpha^{\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta}}, \tag{3}$$

where α, β, and θ are the shape parameters, and λ is a scale parameter of the AEXP.

Derived from Equation (3), the CDF $F(x)$, Reliability function $S(x)$ and the Harzard function $h(x)$ and the Cumulative Hazard Function $H(x)$ can be obtained as:

$$F(x) = \frac{\alpha^{\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta}} - 1}{\alpha - 1}, \tag{4}$$

$$S(x) = \frac{\alpha}{\alpha - 1}\left[1 - \alpha^{\left\{\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta} - 1\right\}}\right], \tag{5}$$

$$h(x) = \frac{\ln(\alpha)\beta\lambda\theta(1+\beta)e^{\lambda x}\left(1 - \frac{1+\beta}{1+\beta\, e^{\lambda x}}\right)^{\theta - 1} \alpha^{\left(1-\frac{1+\beta}{1+\beta\, e^{\lambda x}}\right)^{\theta}}}{\left(1+\beta\, e^{\lambda x}\right)^2 \alpha\left(1 - \alpha^{\left(1-\frac{1+\beta}{1+\beta\, e^{\lambda x}}\right)^{\theta} - 1}\right)}, \tag{6}$$

$$H(x) = -ln\left\{\frac{\alpha}{\alpha - 1}\left[1 - \alpha^{\left\{\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta} - 1\right\}}\right]\right\}. \tag{7}$$

Figures 1 and 2 show the behavior of the PDF and h(x) derived from Equation (3) and Equation (6).

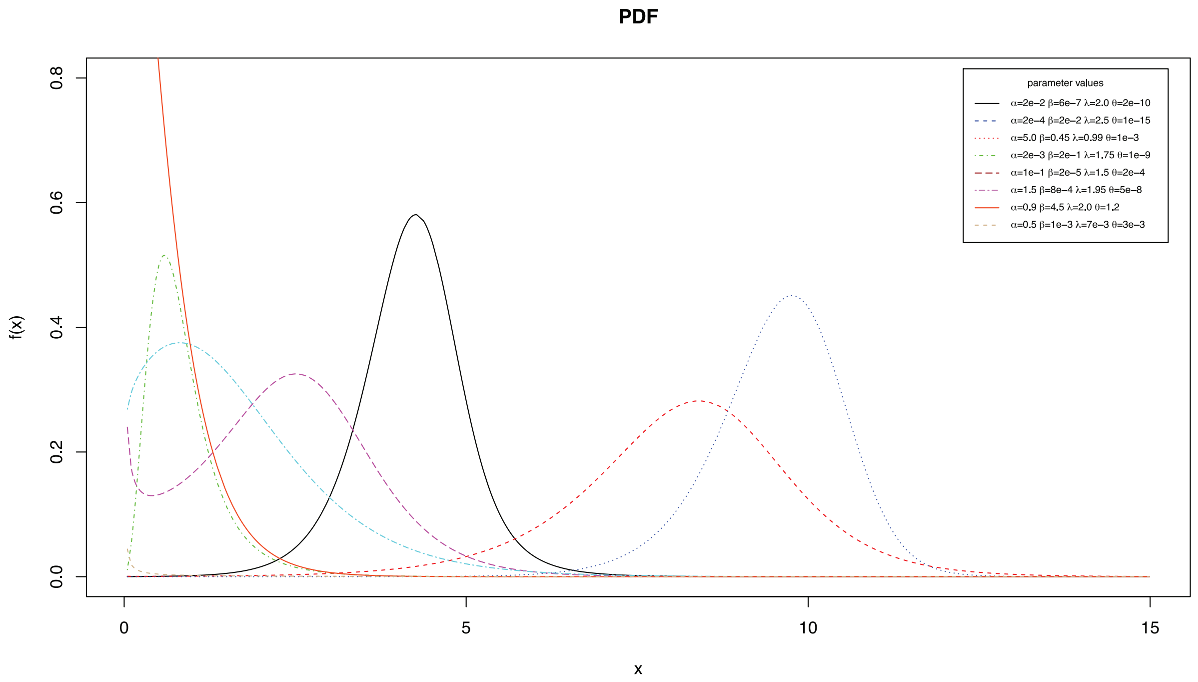


FIGURE 1 PDF for some parameter values derived of AEXP

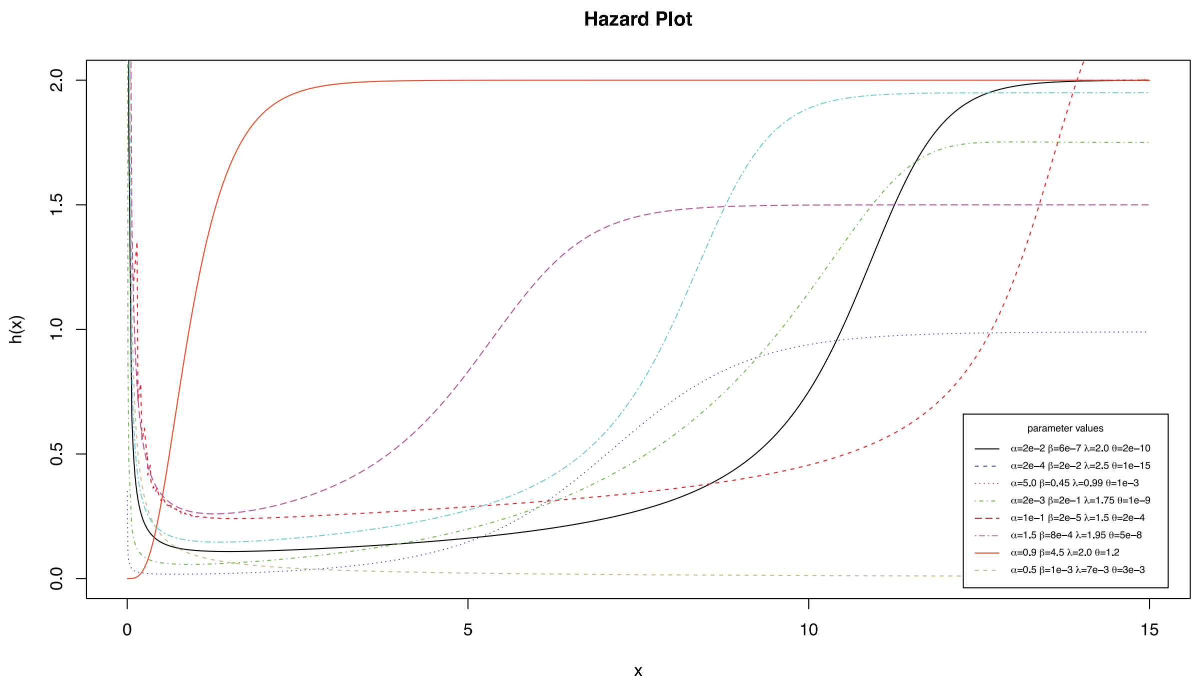


FIGURE 2 Hazard Rate for some parameter values derived of AEXP

3 | USEFUL RELIABILITY EQUATIONS FOR AEXP

This section details some essential statistical equations to perform a reliability study based on the AEXP.

3.1 | Quantile function and random generating data

The $p-th$ quantile function x_q of AEXP can be derived from the F^{-1} of Equation (4) and is given by:

$$p = \frac{1}{\lambda} \cdot \ln \left[\frac{-\beta - \left(\frac{\ln(1+q(\alpha-1))}{\ln(\alpha)} \right)^{\frac{1}{\theta}}}{\beta \left\{ \left(\frac{\ln(1+q(\alpha-1))}{\ln(\alpha)} \right)^{\frac{1}{\theta}} - 1 \right\}} \right], \quad 0 < q < 1. \tag{8}$$

The median can be obtained from Equation (8) by setting $q = 0.5$.

3.2 | Mode

The mode can be obtained as a non-negative solution of the following nonlinear equation:

$$\frac{\theta \ln(\alpha) \alpha^{\zeta^{\theta}} \left(\theta \ln(\alpha) e^{\lambda x} (1+\beta) \zeta^{2\theta} + \zeta^{\theta} \varrho \right) e^{\lambda x} (1+\beta) \lambda^2}{\left(1+\beta e^{\lambda x}\right)^2 (\alpha-1) \left(e^{\lambda x}-1\right)^2} = 0, \tag{9}$$

where

$$\zeta = \frac{\beta \left(\mathrm{e}^{\lambda x} - 1 \right)}{1 + \beta \, \mathrm{e}^{\lambda x}}, \tag{10}$$

$$\varrho = -\beta \, \mathrm{e}^{2\lambda x} - 1 + \theta(1+\beta)\mathrm{e}^{\lambda x}. \tag{11}$$

The Equation (9) does not have an analytical solution in general, so the mode of AEXP has to be estimated by numerical methods.

3.3 | Moments

The rth row moment of AEXP is given by:

$$E(x^r) = \int_0^{\infty} x^r f(x) dx. \tag{12}$$

By substituting the Equation (3) in the Equation (12) we obtain:

$$= \int_0^{\infty} x^r \cdot \left\{ \frac{\ln(\alpha)}{\alpha-1} \cdot \beta\lambda\theta \cdot \frac{(1+\beta)e^{\lambda x}}{\left(1+\beta e^{\lambda x}\right)^2} \cdot \left[1 - \left(\frac{1+\beta}{1+\beta e^{\lambda x}} \right) \right]^{\theta-1} \alpha^{\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta}} \right\} dx. \tag{13}$$

Using Taylor series expansion $a^b = \sum_{\iota=0}^{\infty} \frac{log(a)^{\iota}}{\iota!} \cdot b^{\iota}$ and binomial series expansion $(1-z)^{\theta-1} = \sum_{\jmath=0}^{\infty} \binom{\theta-1}{\jmath} (-1)^{\jmath} z^{\jmath}$ and after perform some math in Equation (13) the following is get:

$$E(x^r) = \frac{\lambda\theta}{\alpha-1} \cdot \sum_{\iota,\jmath,\rho,u=0}^{\infty} \binom{(\iota+1)\theta-1}{\jmath} \cdot \frac{\ln(\alpha)^{\iota+1} (\beta+1)^{\jmath+1} \beta(-\beta(\jmath+2))^{\rho} (\lambda\rho)^{u} \Gamma\left(\lambda^2+r+u+1\right)}{\iota! \Gamma(\rho+1)}. \tag{14}$$

From Equation (14), the Mean Time To Failure (MTTF) can be estimated by calculating the first moment, which is vital to determine the warranty and maintenance times in ED. Similarly, from Equation (12), the Moment Generating Function can be calculated through the series expansion of $e^{tx} = \sum_{\upsilon=0}^{\infty} \frac{(tx)^{\upsilon}}{r!}$.

3.4 | Residual and reversed lifetime functions

The Residual and Reversed Lifetime functions can be estimated to understand the lifetime and failure time of an ED under an operational environment.

The Residual Lifetime (RSL) is defined by the conditional random variable $R_t = X - t|X > t, t \geq 0$, and represents the period from t until the time of failure of the device. The RSL is defined as:

$$S_{R(t)}X = \frac{R(x+t)}{R(t)}. \tag{15}$$

Based on Equation (15) and by taking the Equation (3), the RSL of AEXP is given by:

$$S_{R(t)}X = \frac{\alpha^{\left(1+\beta\, e^{\lambda(x+t)}\right)^{-\theta}\beta^{\theta}\left(e^{\lambda(x+t)}-1\right)^{\theta}} - \alpha}{\alpha^{\left(1+\beta\, e^{\lambda x}\right)^{-\theta}\beta^{\theta}\left(e^{\lambda x}-1\right)^{\theta}} - \alpha}. \tag{16}$$

The reversed lifetime (RVL) represents the time elapsed from the failure of a component in the ED given its life $\leq t$. The RVL for $F_t = t - X|X \leq t$ can be written as:

$$S_{F(t)}X = \frac{F(t-x)}{F(t)}. \tag{17}$$

By taking the Equation (4) and substituting in the Equation (17), the RVL can be calculated as:

$$S_{F(t)}X = \frac{\alpha^{\left(1+\beta\, e^{\lambda(x+t)}\right)^{-\theta}\beta^{\theta}\left(e^{\lambda(x+t)}-1\right)^{\theta}} - 1}{\alpha^{\left(1+\beta\, e^{\lambda x}\right)^{-\theta}\beta^{\theta}\left(e^{\lambda x}-1\right)^{\theta}} - 1}. \tag{18}$$

3.5 | Order statistics

The k-out-n configuration is highly relevant for devices that have redundant systems with which the device can work properly. This configuration is trendy in ED to maintain optimal performance as much as possible. Let $x_1, x_2 \ldots x_n$ a random sample from the AEXP with PDF. Thus, the PDF of the kth order statistic x_k is given by:

$$f_{k:n}(x) = n\binom{n-1}{k-1}F(x)^{k-1}(1-F(x))^{n-k}f(x). \tag{19}$$

By taking Equation (3) and Equation (4) and substituting in Equation (19)

$$f_{k:n}(x) = n\left(\frac{n-1}{k-1}\right)\left\{\frac{\alpha^{\left(1-\frac{1+\beta}{1+\beta\, e^{\lambda x}}\right)^{\theta}} - 1}{\alpha - 1}\right\}^{k-1} \left\{\frac{\alpha}{\alpha-1}\left[1-\alpha^{\left\{\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta}-1\right\}}\right]\right\}^{n-k}$$
$$\cdot \frac{\ln(\alpha)}{\alpha-1}\cdot\beta\lambda\theta\cdot\frac{(1+\beta)\,e^{\lambda x}}{\left(1+\beta e^{\lambda x}\right)^{2}}\cdot\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta-1}\alpha^{\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta}} \tag{20}$$

By expanding Equation (20) in Taylor and Binomial series, the following is obtained:

$$f_{k:n}(x) = n\binom{n-1}{k-1}\frac{\beta\lambda(1+\beta)e^{\lambda x}}{(\alpha-1)(1+\beta e^{\lambda x})^2}\sum_{\iota,\jmath,\rho=0}^{\infty}\frac{\ln(\alpha)^{\iota+1}(n-k)^{\jmath}}{\iota!}\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x}}\right)\right]^{\theta\iota}\left[\frac{\alpha^{\left(1-\frac{1+\beta}{1+\beta e^{\lambda x}}\right)^{\theta}}-1}{\alpha-1}\right]^{\jmath+\rho} \tag{21}$$

4 | AEXP UNDER ALT AND TIME-VARYING SCENARIO

In this section, two models are proposed that can be useful for practitioners of reliability engineering. The proposed models focus on determining the behavior of ED under some conditions that are carried out in practice.

4.1 | AEXP-ALT

Reliability practitioners use ALT to obtain relatively short failure times. When the experimental data come from an ALT, a life-stress relationship must be built, which is formed from a statistical distribution and a relationship that describes the behavior of physical stress. For the case of EDs, it is widespread to use the Inverse Power Law (IPL) as the relationship that describes the device's behavior under analysis under stress such as voltage, current, or load. As is well known, the IPL can be described as:

$$L(\upsilon) = \frac{1}{\sigma\upsilon^{\gamma}}. \tag{22}$$

By taking Equation (22) and substituting it in Equation (3), the life-stress relationship used by AEXP and IPL is described as:

$$f_{AEXP-IPL}(x) = \frac{\ln(\alpha)\lambda(\sigma\,\upsilon^{\gamma}+1)e^{\lambda x}\psi^{\frac{1}{\sigma\upsilon^{\gamma}}-1}\alpha^{(\psi)^{\frac{1}{\sigma\upsilon^{\gamma}}}}}{\sigma\,\upsilon^{\gamma}(\alpha-1)\left(\sigma\,\upsilon^{\gamma}+e^{\lambda x}\right)^{2}}, \tag{23}$$

where $\psi = (\frac{e^{\lambda x}-1}{\sigma\,\upsilon^{\gamma}+e^{\lambda x}})$.

From Equation (23), it is possible to establish a physical interpretation focused on the reliability of ED. Parameter $\sigma > 0$ is a parameter that characterizes the physical properties of the components with which the ED is built. In practice, this parameter measures the device's efficiency and is a fundamental part of determining the MTTF of the ED. Parameter $\gamma > 0$ measures the effect of the stress variable applied to the ED during the lifetime. This parameter is essential since it can establish the threshold level of electrical resistance and temperature with which the device will maintain its operation before reaching the limit of a catastrophic failure. Finally, the parameter υ represents the stress level applied to the ED during the ALT.

4.2 | Time-varying AEXP modeling

Another aspect to consider is whether the ED failure times acquired during ALT are time-varying stress. To achieve this modeling, the IPL must be modified as follows:

$$L(\upsilon) = \left[\frac{\sigma}{x(t)}\right]^{\gamma}, \tag{24}$$

where $x(t)$ represents the parametric form of stress in time-varying. Also, $x(t)$ must have a defined derivate.

So, by taking Equation (24) and substituting in Equation (3), the AEXP-IPL model in time-varying can be represented as follow:

$$f(t,x(t)) = \frac{\ln(\alpha)\lambda\, e^{\lambda\left(\int_0^t x(u)du\right)}\kappa^{\left(\frac{\sigma}{x(t)}\right)^{\gamma}}\alpha^{\kappa^{\left(\frac{\sigma}{x(t)}\right)^{\gamma}}}\xi}{(\alpha-1)\left(1+\left(\frac{\sigma}{x(t)}\right)^{\gamma}e^{\lambda\left(\int_0^t x(u)du\right)}\right)^2\left(e^{\lambda\left(\int_0^t x(u)du\right)}-1\right)}, \tag{25}$$

where

$$\kappa = \frac{\left(\frac{\sigma}{x(t)}\right)^{\gamma}\left(e^{\lambda\left(\int_0^t x(u)du\right)}-1\right)}{1+\left(\frac{\sigma}{x(t)}\right)^{\gamma}e^{\lambda\left(\int_0^t x(u)du\right)}}, \tag{26}$$

$$\xi = e^{\lambda\left(\int_0^t x(u)du\right)}\left[\left(\frac{\sigma}{x(t)}\right)^{3\gamma}+\left(\frac{\sigma}{x(t)}\right)^{2\gamma}\right]+\left(\frac{\sigma}{x(t)}\right)^{2\gamma}+\left(\frac{\sigma}{x(t)}\right)^{\gamma}. \tag{27}$$

To apply the model proposed in Equation (25), Méndez-González et al.[45] proposes some of the most common electrical variations for reliability analysis in ED.

5 | PARAMETER ESTIMATION FOR AEXP

For the estimation of the parameters of the model established in Equation (3), the MLE is proposed. Therefore, let $x_1, x_2 \ldots x_m$ be a random sample from the lifetime distribution with PDF $f(x)$ and based on sample of size m. Then the likelihood function of Eq. 3 is written as follows:

$$L = m\cdot\ln\left[\frac{\ln(\alpha)}{\alpha-1}\cdot\beta\lambda\theta(1+\beta)\right]\cdot\prod_{\imath=1}^{m}\left\{\frac{e^{\lambda x_\imath}}{\left(1+\beta e^{\lambda x_\imath}\right)^2}\cdot\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x_\imath}}\right)\right]^{\theta-1}\alpha^{\left[1-\left(\frac{1+\beta}{1+\beta e^{\lambda x_\imath}}\right)\right]^{\theta}}\right\}. \tag{28}$$

By taking the logarithm of the above equation, the log-likelihood function (Λ) becomes:

$$\Lambda = m\left[\ln(\ln(\alpha))-\ln(\alpha-1)+\ln(\beta)+\ln(\lambda)+\ln(\theta)+\ln(1+\beta)\right]+\lambda\sum_{\imath=1}^{m}x_i-2\sum_{\imath=1}^{m}\ln\left(1+\beta\, e^{\lambda x_\imath}\right)$$
$$+\,(\theta-1)\sum_{\imath=1}^{m}\ln\left(1-\frac{1+\beta}{1+\beta\, e^{\lambda x_\imath}}\right)+\ln(\alpha)\sum_{\imath=1}^{m}\left(1-\frac{1+\beta}{1+\beta\, e^{\lambda x_\imath}}\right)^{\theta}. \tag{29}$$

By taking the first partial derivative of Eq. 29, the estimation for each parameter α, β, λ, and θ can be calculated as follow:

$$\frac{\partial\Lambda}{\partial\alpha} = m\left(\frac{1}{\alpha\ln(\alpha)}-\frac{1}{\alpha-1}\right)+\frac{1}{\alpha}\sum_{\imath=1}^{m}\left[\left(1-\frac{1+\beta}{1+\beta\, e^{\lambda x_\imath}}\right)^{\theta}\right]. \tag{30}$$

$$\frac{\partial\Lambda}{\partial\beta} = m\left(\frac{1}{\beta}+\frac{1}{1+\beta}\right)+\sum_{\imath=1}^{m}\left[\frac{\ln(\alpha)\left(1-\frac{1+\beta}{1+\beta\, e^{\lambda x_\imath}}\right)^{\theta}\theta-2\beta\, e^{\lambda x_\imath}+\theta-1}{\left(1+\beta\, e^{\lambda x_\imath}\right)\beta}\right]. \tag{31}$$

$$\frac{\partial\Lambda}{\partial\lambda} = \frac{m}{\lambda}+\sum_{\imath=1}^{m}\left[\frac{x_\imath\left(\ln(\alpha)\theta(1+\beta)e^{\lambda x_\imath}\left(\frac{\beta\left(e^{\lambda x_\imath}-1\right)}{1+\beta\, e^{\lambda x_\imath}}\right)^{\theta}-\beta\, e^{2\lambda x_\imath}-1+\theta(1+\beta)e^{\lambda x_\imath}\right)}{\left(e^{\lambda x_\imath}-1\right)\left(1+\beta\, e^{\lambda x_\imath}\right)}\right]. \tag{32}$$

TABLE 1 Bathtube shape distributions

Model	h(x)
Perks4	$\frac{\theta+e^{(\beta x+\alpha)}}{1-e^{(-\beta x+\lambda)}}$
APW	$\frac{\alpha\lambda e^{\lambda x}}{1+\alpha e^{\lambda x}}+\theta\beta x^{\beta-1}$
EXP	$\frac{\theta\lambda\beta^{\theta}(1+\beta)e^{\lambda x}(e^{\lambda x}-1)^{\theta-1}}{(1+\beta e^{\lambda x})[(1+\beta e^{\lambda x})^{\theta}-\beta^{\theta}(e^{\lambda x}-1)^{\beta}]}$
AWD	$\alpha\lambda x^{\lambda-1}+\beta\theta x^{\theta-1}$
SZMW	$\alpha+\beta\lambda x^{\lambda-1}$
BW	$\frac{\theta\lambda^{\theta}x^{\theta-1}e^{-\beta(\lambda x)^{\theta}}[1-e^{-(\lambda x)^{\theta}}]^{\alpha-1}}{I_{1-e^{(-\lambda x)^{\theta}}}(\alpha,\beta)}$
APTW	$\ln(\alpha)\lambda\beta x^{\beta-1}e^{-\lambda x^{\beta}}(\alpha^{e^{-\lambda x^{\beta}}})^{-1}$

$$\frac{\partial\Lambda}{\partial\theta}=\frac{m}{\theta}+\sum_{\iota=1}^{m}\left[\ln\left(\frac{\beta\left(e^{\lambda x_{\iota}}-1\right)}{1+\beta\,e^{\lambda x_{\iota}}}\right)\left\{\left(\frac{\beta\left(e^{\lambda x_{\iota}}-1\right)}{1+\beta\,e^{\lambda x_{\iota}}}\right)^{\theta}\ln(\alpha)+1\right\}\right]. \quad (33)$$

The Fisher information matrix based on Eq.30 to Eq.33 is given by:

$$J(\delta)=-\begin{bmatrix} I_{\alpha\alpha} & I_{\alpha\beta} & I_{\alpha\lambda} & I_{\alpha\theta} \\ I_{\beta\alpha} & I_{\beta\beta} & I_{\beta\lambda} & I_{\beta\theta} \\ I_{\lambda\alpha} & I_{\lambda\beta} & I_{\lambda\lambda} & I_{\lambda\theta} \\ I_{\theta\alpha} & I_{\theta\beta} & I_{\theta\lambda} & I_{\theta\theta} \end{bmatrix}. \quad (34)$$

The elements of the fisher matrix are shown in Appendix A.

The asymptotic normality of MLE can be used to estimate the confidence intervals for each AEXP parameter. The $100(1-\eta)\%$ confidence intervals for $\alpha,\ \beta,\ \lambda, and\ \theta$ are given by:

$$\left(\hat{\alpha}\pm z_{1-\rho/2}\sqrt{var(\hat{\alpha})}\right), \quad (35)$$

$$\left(\hat{\beta}\pm z_{1-\rho/2}\sqrt{var\left(\hat{\beta}\right)}\right), \quad (36)$$

$$\left(\hat{\lambda}\pm z_{1-\rho/2}\sqrt{var\left(\hat{\lambda}\right)}\right), \quad (37)$$

$$\left(\hat{\theta}\pm z_{1-\rho/2}\sqrt{var\left(\hat{\theta}\right)}\right), \quad (38)$$

where $var(\hat{\alpha}), var(\hat{\beta}), var(\hat{\lambda})$ and $var(\hat{\theta})$ can be obtained from the Fisher's matrix obtained in Equation (34).

6 | CASE OF STUDY

In this section, three case studies are presented focused on determining the reliability of ED where the AEXP is put to the test and compared with other distributions with the property of representing failure times in the form of a bathtub curve or non-monotone. For the comparative analysis, the following distributions were considered: Perks4,[29] APW,[30] EXP,[31] AWD,[16] SZMW,[17] BW[22] and APTW.[34] Table 1 shows the hazard functions of the distributions used for the comparative analysis.

For all case studies, the parameters of the distributions were estimated via MLE programmed in R. Moreover, the AIC, BIC, K-S, and P-Value were calculated to derive conclusions from the comparative study.

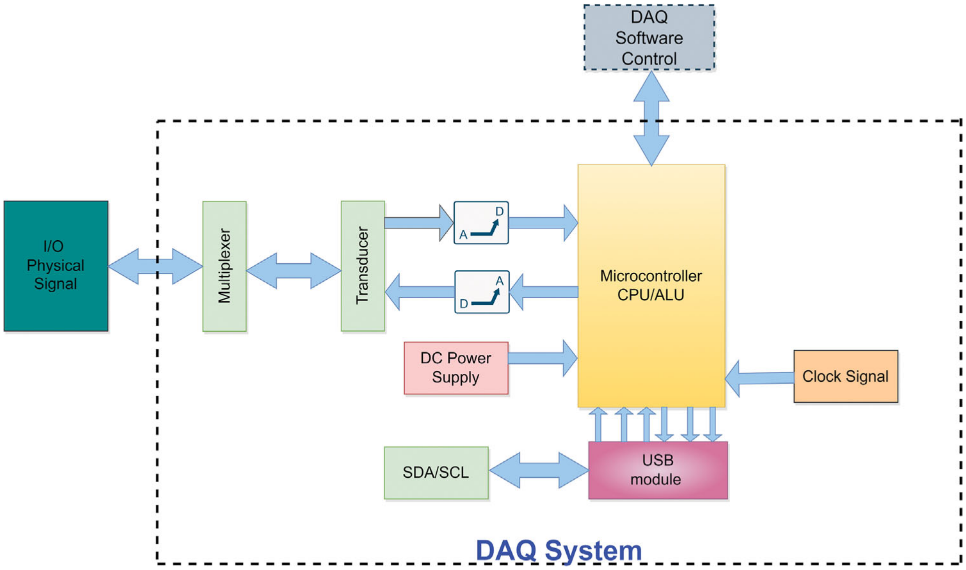


FIGURE 3 Schematic of DAQ system

TABLE 2 Failure time of 30 DAQs

Data									
0.0192	0.0910	0.146	0.346	1.137	1.192	1.339	1.493	2.054	2.185
3.261	3.679	3.891	4.562	4.601	4.819	4.934	5.479	5.822	5.979
5.995	6.006	7.127	7.750	8.089	8.568	11.906	17.041	17.812	17.845

6.1 | Case of study 1: Reliability of an electronic data acquisition system (DAQ)

This case study focuses on determining the behavior of a DAQ card. This ED can transform physical signals such as voltage, current, and temperature coming from sensors into digital signals so that a CPU can process them in real time. The application of a DAQ is mainly in automation systems or test equipment, where the measurement of physical signals is essential to control an industrial process. In figure 3 it can see the internal components and the operation of the DAQ cards used for the analysis.

For this case study, a sample of 30 DAQs was considered; the failure times (voiced in days) of this experiment are presented in Table 3.

The failures exhibited by the DAQ within this case study were established and classified as follows:

- **I/O port failure**: Since this DAQ system has inputs and outputs individually, if any of these fails, the device can continue to operate normally without affecting the DAQ's operation.
- **DAQ without communication**: The USB module establishes the communication between all the parts that make up the DAQ and the PC. This fault is considered critical.
- **DAQ is intermittent**: This fault occurs when the communication between the microcontroller and the I/O ports is not completed, or the readings show the legend $Err in the software.

Figure 4 shows the TTT plot, with which we can empirically verify the behavior of the failure times of the DAQs presented in Table 2. Based on,[1] the results shown in Figure 4 suggest that DAQs failure times follow a bathtub curve shape.

Table 3 shows the result of the parameter estimates, the standard errors, and the statistics of each of the models established in Table 1.

The results obtained in Table 3 establish that the AEXP offers competitive results concerning other distributions with similar properties to describe the failure times of the DAQs and, in turn, to describe the behavior during the useful life of the device under analysis. We can derive the conclusions by obtaining lower values in the AIC and BIC and a higher P-Value indicator.

TABLE 3 Estimated values, standard errors in brackets and statistics metrics for the Case of Study 1

	Parameters				Statistics				
Model	α	β	λ	θ	**Loglik**	**AIC**	**BIC**	**K-S**	**P-value**
AEXP	$1.254e-2(1.778e-4)$	$2.741e-7(3.557e-11)$	1.985(0.145)	$1.471e-10(3.771e-14)$	−79.910	167.820	173.425	$9.741e-3$	0.991
Perks4	−10.414(1.665)	0.587(0.144)	$-0.114(2.265e-2)$	$9.712e-2(1.974e-3)$	−80.449	168.898	174.503	6.5123 − 2	0.894
APW	$0.718(1.445e-2)$	$0.679(3.225e-4)$	$0.238(1.422e-2)$	$6.822e-2(5.774e-5)$	−80.011	168.022	173.626	$1.459e-3$	0.923
AWD	$9.788e-2(3.547e-5)$	$0.259(1.997e-2)$	0.797(0.191)	0.791(0.244)	−83.09	174.197	179.802	0.260	0.801
EXP	–	$0.154(6.774e-3)$	0.741(0.314)	1.1285(0.114)	−80.757	167.514	171.717	0.355	0.654
SZMW	$0.119(3.569e-2)$	$9.088e-2(2.147e-4)$	0.662(0.211)	–	−81.175	168.350	172.553	0.113	0.789
BW	0.593(0.241)	$0.142(1.99e-2)$	1.107(0.335)	0.974(0.112)	−80.368	168.737	174.342	0.442	0.561
APTW	$7.307e-2(6.114e-4)$	0.694(0.332)	0.719(0.114)	–	−81.566	169.132	173.335	0.301	0.692

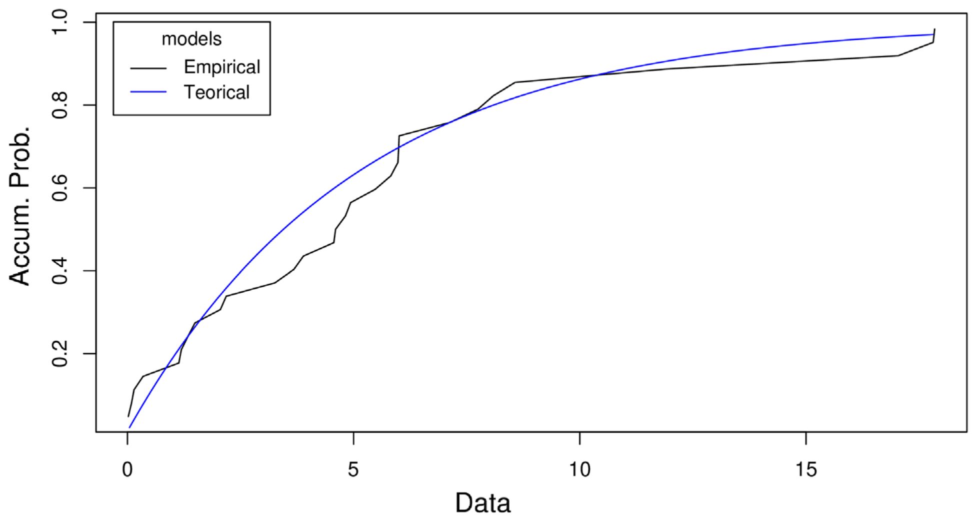


FIGURE 4 TTT plot for data presented in Table 2

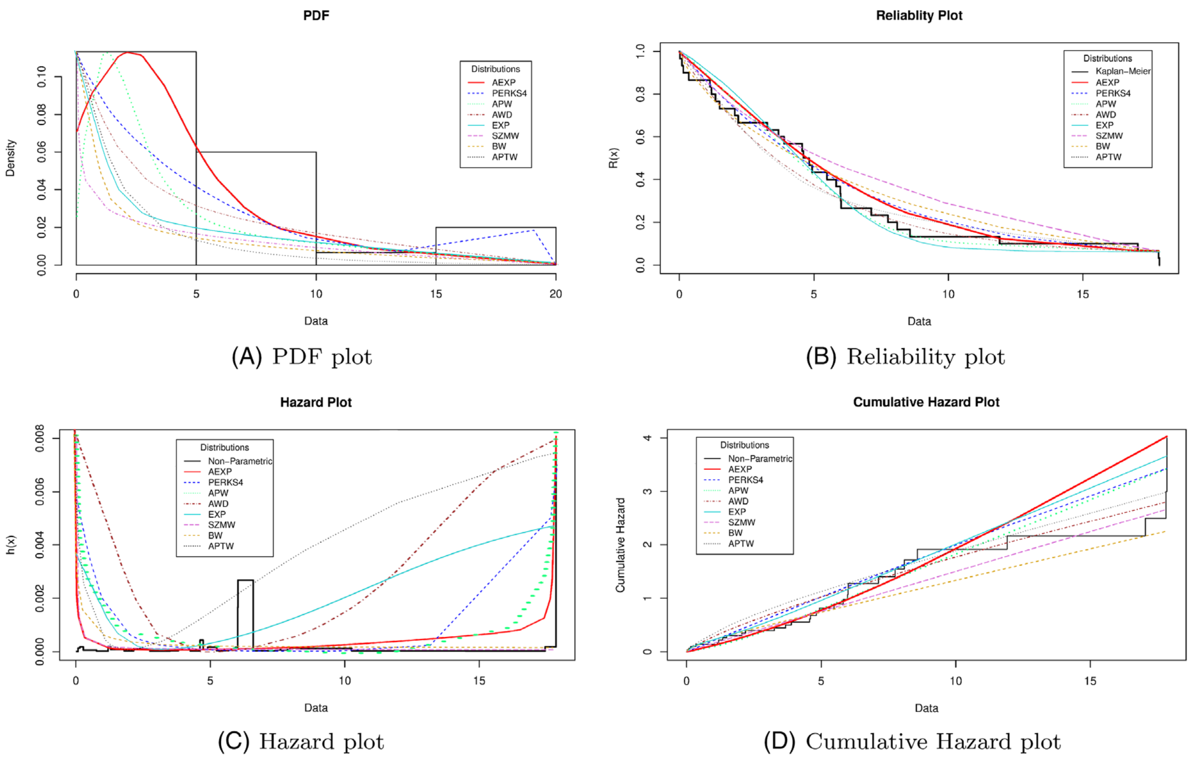


FIGURE 5 Reliability plots for the DAQs

Figure 5 shows the reliability graphs from the estimates obtained in Table 3. Figure 5A shows the histogram and the shape of the PDFs for each of the distributions used in the comparative study. In the first instance, it is observed how the AEXP reflects a better approximation to the histogram; this is due to the inclusion of the Alpha transformation, which adds greater flexibility in the forms of the AEXP by including one more extra form parameter compared to the EXP. Figure 5B shows the reliability graph, where the obtained results show that the AEXP is close to the line of empirical reliability drawn by the Kaplan-Meier method. The preceding means that the MTTF estimated through the AEXP will be closer to when the DAQ will operate during its useful operation life. Figure 5C shows the behavior of DAQ failure times; It stands out that the AEXP, PERKS4, and APW distributions exhibit a bathtub curve-shaped behavior, which makes them candidates to represent the failure times of DAQs.

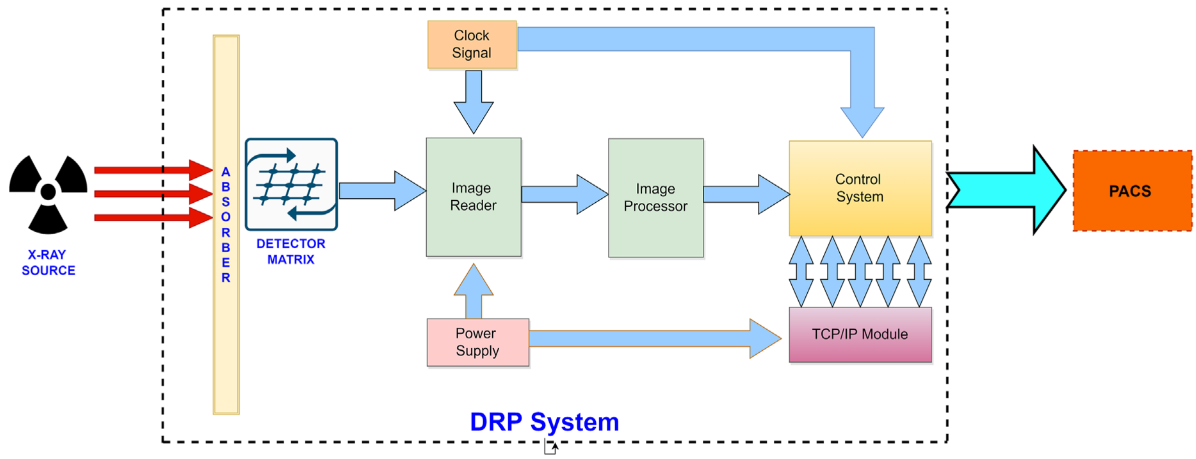


FIGURE 6 Schematic of DRP system

TABLE 4 Failure time of 24 DRP

Data							
1.645	1.981	2.205	2.791	5.627	11.129	18.732	19.012
24.441	38.236	44.599	46.462	52.049	54.867	55.025	66.114
75.314	83.520	95.377	102.378	105.708	110.311	117.141	119.365

Nevertheless, when considering the non-parametric curve, we can establish that the best fit is given by the AEXP, which touches the non-parametric representation in a more significant number of points. The preceding represents that preventive maintenance times can be identified in a more reliable way to reality and extend the useful life of the DAQ. Finally, the cumulative hazard plot is shown in Figure 5D. For this case, it can be seen how the adjustment of the AEXP concerning the non-parametric curve is better than the other distributions used in the analysis; with this, it is shown that the AEXP can establish quite competitive results.

From the manufacturing point of view, the results obtained in Table 3 and Figure 5 help engineers get to know the MTTF more closely. By obtaining an MTTF close to the Kaplan-Meier line like the one with the AEXP, the engineers can more accurately determine the types and times of maintenance carried out in the DAQ. On the other hand, some considerations can be established when analyzing the results obtained in this case study from the product design point of view. In this case, parameters α and θ represent how the internal components (see Figure 3) behave during the infant mortality and wear stages, which experiment with the DAQ throughout its useful life. These factors can be associated with the electrical resistance of the DAQ or the internal resistance of each component. β and λ are associated with the behavior in the average use rate of the DAQ. Those parameters show the behavior of the internal components when the product has stabilized. Generally, this aspect is measured by the number of operating cycles in the ED. With these two aspects, the design engineers can revise the components to improve the learning and stability curves of the ED.

6.2 | Case of study 2: Reliability of digital radiology panel (DRP)

DRP systems have become state-of-the-art medical diagnostic tool involving X-rays to diagnose some diseases such as COVID-19. DRP systems replace traditional silver halide films where radiological images are printed through a chemical process by a system of high-definition images of the part exposed to X-rays; this means that doctors can Instantly obtain an image to diagnose a patient's condition. Figure 6 shows a schematic diagram of the DRP components and the image acquisition process.

For this case study, a sample of 24 DRPs was considered; the failure times (voiced in hours) of this experiment are presented in Table 4.

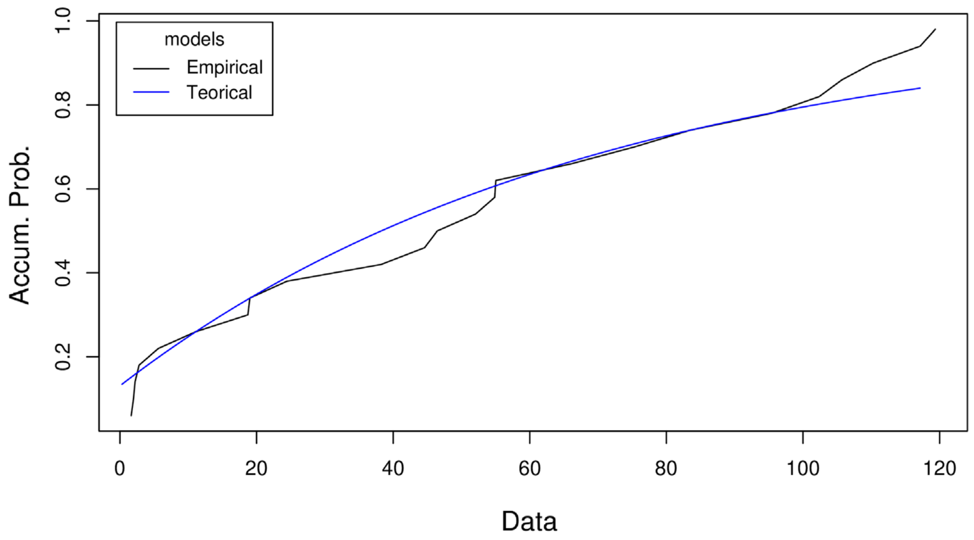


FIGURE 7 TTT plot for data presented in Table 4

The failures exhibited by the DRP within this case study were established and classified as follows:

- **DR system is not ready to receive a new image**: This fault occurs when the image acquisition module does not receive the amount of voltage necessary to pass the threshold value.
- **DRP acquires images but does not process them in the PACS**: This fault occurs when the TCP/IP module does not acquire enough current to establish communication between the image processing stage and the controller.
- **DR loses resolution in the image**: This event occurs because the conductive meshes that receive the x-rays are open. Generally, this fault is associated with the detection matrix, which is sensitive to slight current variations.

Once the failures of the DRPs are known, the behavior of the failure times shown in table 4 is known. Therefore, Figure 7 shows the TTT plot of the failure times belonging to the DRPs. The results obtained in Figure 7 show that the DRP failure times exhibit a non-monotonic behavior.

Table 5 shows the estimates and standard errors obtained for each distribution used in the comparative study.

The results in Table 5 show that the AEXP may be the best option to analyze the behavior of the DRP based on the AIC, BIC, and P-Value values. Therefore, it is concluded that the AEXP offers very competitive results concerning the distributions listed in Table 1, with which engineers can determine how the DRP will act much more closely to the real behavior throughout its working life.

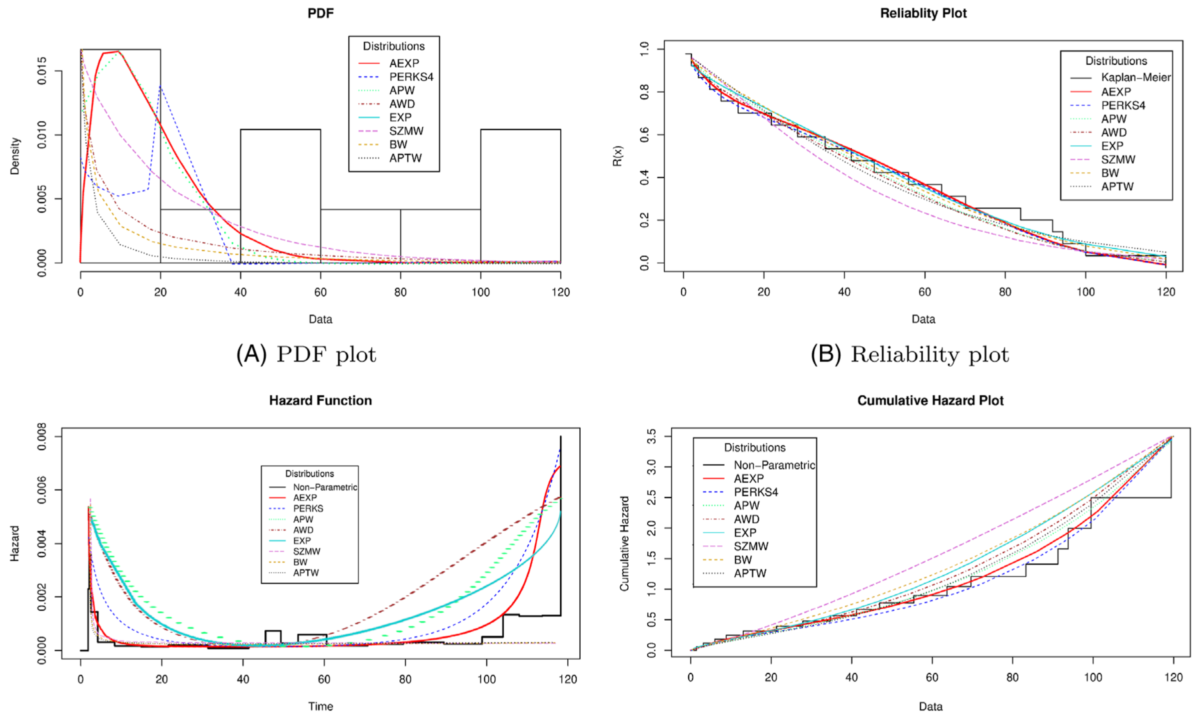
Figure 8 shows the reliability graphs for the DRPs, based on the estimates of the parameters obtained in Table 5. Figure 8A shows the behavior of the PDFs for each of the distributions established in Table 1 and the lifetimes of the DRPs obtained in Table 3. In the Figure mentioned above 8A, it can be seen how the shape of the AEXP concerning the other distributions stands out when obtaining a non-exponential representation, which indicates that during the tests, the early failures of the device are represented by the AEXP. Figure 8B represents the behavior of the reliability of the DRPs under the different distributions. In this case, we can observe how the AEXP closely represents the behavior concerning the Kaplan-Meier curve, followed by a competitive representation of the PERKS4. Figure 8C shows the behavior of the DRP's failure times for each of the distributions analyzed in this case study. In the first instance, it is observed that the AEXP has similar behavior to the bathtub curve and is closer to the non-parametric curve in more points, followed by the PERKS4.

In contrast, some distributions cannot represent the bathtub curve behavior despite having the property. The preceding shows that the AEXP can obtain information closer to the behavior of the DRP under the designed operating conditions. Finally, Figure 8D shows the behavior of the cumulative hazard curve represented non-parametrically. In this case, the AEXP shows that the accumulated hazard in the piece shows a better fit than the other distributions. This fit indicates that the estimated calculations will be close to those obtained in the active life of the DRP when exposed to X-rays.

The information described in the previous paragraph is supported by the evidence obtained in the values described in Table 3.

TABLE 5 Estimated values, standard errors in brackets and statistics metrics for the Case of Study 2

Model	Parameters				Statistics				
	α	β	λ	θ	Loglik	AIC	BIC	K-S	P-value
AEXP	$1.315e-5(2.114e-6)$	$1.725e-2(6.998e-14)$	2.359(0.332)	$2.257e-15(9.411e-17)$	−117.843	243.686	248.398	0.105	0.921
Perks4	−13.113(0.547)	0.425(0.224)	$-0.456(2.265e-2)$	$1.914e-2(2.114e-4)$	−119.331	246.662	251.774	0.195	0.864
APW	0.460(0.101)	0.905(0.211)	$8.892e-3(3.997e-4)$	$9.644e-3(1.114e-5)$	−122.635	253.271	257.983	0.541	0.701
AWD	$1.024e-2(3.141e-3)$	$0.144(3.5543e-2)$	0.319(0.191)	0.489(0.114)	−128.279	264.559	269.271	0.551	0.671
EXP	–	$1.887e-2(1.998e-6)$	$2.351e-2(4.981e-6)$	0.964(0.297)	−122.527	251.054	254.588	0.496	0.734
SZMW	$4.017e-2(1.114e-3)$	0.917(0.101)	$0.255(6.447e-2)$	–	−170.157	346.314	349.848	0.721	0.541
BW	$0.414(1.021e-2)$	$0.224(3.141e-2)$	$0.321(1.114e-2)$	0.914(0.214)	−122.148	252.296	257.008	0.765	0.514
APTW	$5.123e-2(3.014e-3)$	0.547(0.143)	$0.241(1.031e-4)$	–	−133.147	272.274	275.828	0.792	0.494



(A) PDF plot (B) Reliability plot (C) Hazard plot (D) Cumulative Hazard plot

FIGURE 8 Reliability plots for the DRPs

7 | CONCLUSION AND FUTURE WORK

In this paper, a distribution with properties was presented to represent the reliability of ED with bathtub curve behaviors. The proposed analysis methodology was based on the Alpha transformation and the EXP (AEXP); a shape parameter was added to the base EXP distribution with this transformation. This new parameter allowed for a representation of the failure times closer to the bathtub curve that represents the behavior of the ED in reliability. For the proposed distribution, useful statistical properties for the reliability analysis of ED were determined, including time-varying and ALT modeling. The conditions or assumptions in which the AEXP can be implemented within the reliability analysis are for data that have a behavior with especially non-monotonic behavior, but this does not limit the flexibility of the AEXP, which can very competitively represent data with increasing behavior or decreasing. In addition, the time-varying and ALT representations offer reliability practitioners new scenarios to test the properties of the AEXP in different scenarios for device testing that is not just limited to ED. It should be noted that the efficiency of the AEXP depends, as in all statistical models, on the form and treatment given to failure times before being submitted for analysis.

Two case studies were designed to test the AEXP effectiveness to determine the reliability of EDs with failure times in the shape of a bathtub curve. Both case studies conducted a comparative analysis between the AEXP and seven distributions proposed for reliability analysis. The MLE programmed in R was used to calculate the parameters of each of the distributions. The conclusions in both case studies show that the AEXP offers better results in representing failure times, based on the information obtained in Tables 3 and 5.

Future work to extend the results obtained in this paper proposes that a Bayesian analysis be carried out with which they can be optimized. Moreover, given the model's flexibility, it can be used for reliability analysis with data from an ALT, taking into consideration other stress variables that directly affect the life of the ED.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

ORCID

Luis Carlos Méndez-González https://orcid.org/0000-0002-2533-0036
Luis Alberto Rodríguez-Picón https://orcid.org/0000-0003-2951-2344

Ivan Juan Carlos Pérez-Olguin https://orcid.org/0000-0003-2445-0500
Vicente Garcia https://orcid.org/0000-0003-2820-2918
Abel Eduardo Quezada-Carreón https://orcid.org/0000-0001-6971-2848

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AUTHOR BIOGRAPHIES

Luis Carlos Méndez-González Ph.D. is currently a full-time professor at the Department of Industrial Engineering and Manufacturing at the Autonomous University of Ciudad Juárez, México. He received his Ph.D. in Science in Engineering in 2015. He received his M.S in Industrial Engineering from the Technological Institute of Ciudad Juarez in 2011 and his B.S in Electronics Engineering in 2007. He has more than fifteen years in software, hardware design, Applied Statistics, Measurement System Analysis, Reliability Engineering, and Quality Engineering. He is currently a member of the National Researchers System from México as Level 1. His research interests include reliability and degradation modeling, stochastic modeling, hardware, and software design, and Machine Learning.

Luis Alberto Rodríguez-Picón Ph.D. is currently a full-time professor at the Department of Industrial Engineering and Manufacturing at the Autonomous University of Ciudad Juárez, México. He received his Ph.D. in Science in Engineering in 2015. He received his B.S. and M.S. degrees in Industrial Engineering from the Technological Institute of Ciudad Juárez, México, in 2010 and 2012, respectively. He has worked as a professor in industrial engineering, statistics, and mathematics and has several years of professional experience in the automotive industry. He is currently a member of the National Researchers System from México as Level 1. His research interests include reliability and degradation modeling, stochastic modeling, multivariate statistical modeling, and design of experiments.

Iván Juan Carlos Pérez-Olguin received a Doctor of Science degree in industrial engineering from the Technological Institute of Ciudad Juarez, Mexico. He is currently a full-time Professor and a Researcher with the Autonomous University of Ciudad Juarez, Mexico. He has published in journals, conference proceedings, and books more than 50 articles; he also contributed to the automotive industry with two patents and four utility models. His research interests include robust optimization, reliability tests, product optimization, process optimization, and lean manufacturing.

Vicente Garcia received the B.Eng. degree in computer systems from the Technological Institute of Villahermosa, México, in 1995, the M.Sc. degree in computer science from the Technological Institute of Toluca, México, in 2000 and the Ph.D. degree in computer science from the Universitat Jaume I, Castellon, Spain, in 2010. From 2010 to 2013, he was a research fellow with the Institute of New Imaging Technologies, Castellón, Spain. From September 2013 to July 2014, he was a post-doc at the Department of Ingeniería Eléctrica y Computación of the Universidad Autónoma de Ciudad Juárez, Chihuahua, México. Since 2014 serves as a Full Professor of Computer Science, Master and Ph.D. area advisor at the Universidad Autónoma de Ciudad Juárez. His research interest includes classification, performance evaluation metrics, data mining, and medical imaging.

Abel Eduardo Quezada-Carreón received a degree in Electrical Engineering from the Instituto Tecnológico de Ciudad Juárez in 2002 and a Master's degree in Electrical Engineering from Instituto Tecnológico de la Laguna in 2006. He is currently a full-time professor and coordinator of the Electrical Engineering Program of the Universidad Autónoma de Ciudad Juárez. It also forms part of the National technical council for the General Examination of Exit of the Degree in Electrical Engineering. His areas of interest are the analysis of electrical systems of power and the quality of energy.

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APPENDIX A: OBSERVED FISHER MATRIX ELEMENTS FOR AEXP

In this part of the paper, the elements of the fisher matrix established in Equation (33) are shown.

$$I_{\alpha\alpha} = m\left(-\frac{1}{\alpha^2 \ln(\alpha)} - \frac{1}{\alpha^2 \ln(\alpha)^2} + \frac{1}{(\alpha-1)^2}\right) - \frac{1}{\alpha^2} \cdot \sum_{\iota=1}^{m} \left(1 - \frac{1+\beta}{1+\beta\,e^{\lambda x_\iota}}\right)^{\theta} \tag{A1}$$

$$I_{\alpha\beta} = \sum_{\iota=1}^{m} \left[\frac{\theta\left(\frac{\beta(e^{\lambda x_\iota}-1)}{1+\beta\,e^{\lambda x_\iota}}\right)^{\theta}}{\beta\left(1+\beta\,e^{\lambda x_\iota}\right)\alpha}\right] \tag{A2}$$

$$I_{\alpha\lambda} = \sum_{\iota=1}^{m} \left[\frac{\theta\left(1 - \frac{1+\beta}{1+\beta\,e^{\lambda x_\iota}}\right)^{\theta-1} x_\iota \beta(1+\beta)e^{\lambda x_\iota}}{\alpha\left(1+\beta\,e^{\lambda x_\iota}\right)^2}\right] \tag{A3}$$

$$I_{\alpha\theta} = \frac{1}{\alpha}\sum_{\iota=1}^{m} \left[\left(1 - \frac{1+\beta}{1+\beta\,e^{\lambda x_\iota}}\right)^{\theta} \ln\left(1 - \frac{1+\beta}{1+\beta\,e^{\lambda x_\iota}}\right)\right] \tag{A4}$$

$$I_{\beta\alpha} = I_{\alpha\beta} \tag{A5}$$

$$I_{\beta\beta} = m\left(-\frac{1}{\beta^2} - \frac{1}{(1+\beta)^2}\right) + \sum_{\iota=1}^{m}\left[\frac{-2\ln(\alpha)\left(\beta\,e^{\lambda x_\iota} - \frac{\theta}{2} + \frac{1}{2}\right)\theta\left(1 - \frac{1+\beta}{1+\beta\,e^{\lambda x_\iota}}\right)^{\theta} + 2\,e^{2\lambda x_\iota}\beta^2 - 2\left(\beta\,e^{\lambda x_\iota} + \frac{1}{2}\right)(\theta-1)}{\left(1+\beta\,e^{\lambda x_\iota}\right)^2\beta^2}\right] \tag{A6}$$

$$I_{\beta\lambda} = \sum_{\iota=1}^{m}\left[\frac{e^{\lambda x_\iota} x_\iota\left(\ln(\alpha)(-\beta\,e^{\lambda x_\iota} + (\theta+1)\beta + \theta)\theta\left(\frac{\beta(e^{\lambda x_\iota}-1)}{1+\beta\,e^{\lambda x_\iota}}\right)^{\theta} - \beta\left(e^{\lambda x_\iota} - 1\right)(\theta+1)\right)}{\left(1+\beta\,e^{\lambda x_\iota}\right)^2\left(e^{\lambda x_\iota}-1\right)\beta}\right] \tag{A7}$$

$$I_{\beta\theta} = \sum_{\iota=1}^{m}\left[\frac{1 + \ln(\alpha)\left(\frac{\beta(e^{\lambda x_\iota}-1)}{1+\beta\,e^{\lambda x_\iota}}\right)^{\theta}\left(\theta\ln\left(\frac{\beta(e^{\lambda x_\iota}-1)}{1+\beta\,e^{\lambda x_\iota}}\right) + 1\right)}{\beta\left(1+\beta\,e^{\lambda x_\iota}\right)}\right] \tag{A8}$$

$$I_{\lambda\alpha} = I_{\alpha\lambda} \tag{A9}$$

$$I_{\lambda\beta} = I_{\beta\lambda} \tag{A10}$$

$$I_{\lambda\lambda}=\sum_{\imath=1}^{m}\left[\frac{\ln(\alpha)\lambda^{2}(1+\beta)\theta\left(-\beta\,e^{2\lambda x_{\imath}}-1+\theta(1+\beta)e^{\lambda x_{\imath}}\right)\left(\frac{\beta\left(e^{\lambda x_{\imath}}-1\right)}{1+\beta\,e^{\lambda x_{\imath}}}\right)^{\theta}x_{\imath}^{2}e^{\lambda x_{\imath}}}{\left(1+\beta\,e^{\lambda x_{\imath}}\right)^{2}\left(e^{\lambda x_{\imath}}-1\right)^{2}}-\frac{\lambda^{2}(\theta-1)\left(\beta\,e^{2\lambda x_{\imath}}+1\right)e^{\lambda x_{\imath}}x_{\imath}^{2}(1+\beta)}{\left(e^{\lambda x_{\imath}}-1\right)^{2}\left(1+\beta\,e^{\lambda x_{\imath}}\right)^{2}}-\frac{2\lambda^{2}\beta x_{\imath}^{2}e^{\lambda x_{\imath}}}{\left(1+\beta\,e^{\lambda x_{\imath}}\right)^{2}}\right]-\frac{m}{\lambda^{2}} \tag{A11}$$

$$I_{\lambda\theta}=(1+\beta)\sum_{\imath=1}^{m}\left[\frac{\left\{1+\ln(\alpha)\left(\frac{\beta\left(e^{\lambda x_{\imath}}-1\right)}{1+\beta\,e^{\lambda x_{\imath}}}\right)^{\theta}\left(\theta\ln\left(\frac{\beta\left(e^{\lambda x_{\imath}}-1\right)}{1+\beta\,e^{\lambda x_{\imath}}}\right)+1\right)\right\}e^{\lambda x_{\imath}}x_{\imath}}{\left(e^{\lambda x_{\imath}}-1\right)\left(1+\beta\,e^{\lambda x_{\imath}}\right)}\right] \tag{A12}$$

$$I_{\theta\alpha}=I_{\alpha\theta} \tag{A13}$$

$$I_{\theta\beta}=I_{\beta\theta} \tag{A14}$$

$$I_{\theta\lambda}=I_{\lambda\theta} \tag{A15}$$

$$I_{\theta\theta}=\sum_{\imath=1}^{m}\left[\ln(\alpha)\left(1-\frac{1+\beta}{1+\beta\,e^{\lambda x_{\imath}}}\right)^{\theta}\ln\left(\frac{\beta\left(e^{\lambda x_{\imath}}-1\right)}{1+\beta\,e^{\lambda x_{\imath}}}\right)^{2}\right]-\frac{m}{\theta^{2}} \tag{A16}$$