Dimensional Analysis Under Pythagorean Fuzzy Set with Hesitant Linguists Term Entropy Information

A. J. Villa Silva, L. A. Pérez Domínguez, E. Martínez Gómez, R. Romero López, and D. J. Valles Rosales

Abstract Dimensional Analysis (DA) is a method that consider an association of all the criteria involved in a problem, able to capture the interrelationship usually presents in multi-criteria problems. At the same time Pythagorean Fuzzy Set (PFS), is a recent tool used for handling fuzziness and vagueness, due is able to provide greater flexibility for decision makers to give their assessments. In addition, Multi-criteria decision making (MCDM) problems involves criteria predetermined weights and difficulty when information given is unknown or incomplete. This paper proposes the application and combination of three important tools: Dimensional Analysis, Pythagorean fuzzy sets and entropy measure for hesitant fuzzy linguistic term sets (HFLTSs) in order to solve the qualitative criteria, the interrelationship among the multiple criteria, and weights calculation when are unknown. Finally, an example case is given in order to show the functioning of the proposed hybrid method, and comparison with other weight methods.

Keywords Multi-Criteria Decision Making (MDCM) · Pythagorean Fuzzy Set (PFS) · Analysis Dimensional (DA) · Entropy

1 Introduction

Since theory of fuzzy sets was introduced by Zadeh in 1965 has reached an important success in several fields and became an important approach to handle uncertainty and inaccurate information that appears in several real life problems [1–3]. Since then, different versions of fuzzy set have been studied and proposed by some researchers [1]. Under this context, researchers are working with Pythagorean Fuzzy Set (PFS) in order to improve and developing studies concerning decision-makers are truly familiar with the criteria and alternatives evaluated [4–6].

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After introducing PFS, Yager and Abbasov studied the relationship between the Pythagorean fuzzy numbers (PFNs) and the complex number [7]. In addition other proposed works has been introduced: Zhang and Xu [8] extended the TOPSIS approach concerning to hand the Multi-criteria decision-making (MCDM) problems in terms of Pythagorean fuzzy environment. Peng and Yang [9] proposed the division and subtraction equations for PFNs. Zhang [10] developed a closeness index for Pythagorean fuzzy number (PFN) and for interval-valued Pythagorean fuzzy number (IVPFN) based on distance measures of PFNs and IVPFNs. Garg [11] developed a new approach of Pythagorean fuzzy, using information aggregation by Einstein equations and applied it to decision making.

PFS is symbolized by three values: membership, non-membership and indeterminacy [3], but the main difference consist: the addition of the degree of membership and non-membership given by experts it can be more than unit, but its square sum is the same to or less than unit [12]. Particularly, if decision makers gives their valuations or perceptions information where membership grade is 0.9 and degree of non-membership is 0.5, you can know that the Intuitionistic Fussy Sets (IFS) does not address adequately this problem because $0.9 + 0.5 > 1$ their sum exceeds 1, IFS fail to handle such situations [13]. However, $0.9^2 + 0.5^2 < 1$ therefore the PFS has capacity to represent evaluation and characterize better the uncertainty by lack of clarity than IFS [14], this advantage provides a more powerful representation of uncertainty established by the Fuzzy intuitionist and therefore Fuzzy sets are best and proved tools for modeling uncertainty [3, 15].

In other hand Dimensional Analysis (DA) is a method with capacity to consider the mutual influence between several criteria [16], then, makes it properly in multi-criteria decision making (MCDM) problems in different scales of measuring, within of a single dimensionless index [16–18]. The most remarkable advantage of DA is concerning to join the valuations or perceptions of a group of decision makers (DM) based on different information, such as alternatives, criteria and their importance [16]. It should be noted that the DA is widely mentioned and applied in different industries, but there is a vast literature on its application in the agricultural and automotive sector [19]. Nevertheless, DA presents the weakness to operate with qualitative information usually involved in MCDM problems [19].

In other hand, entropy measure for hesitant fuzzy linguistic term sets (HFLTSs), is applied when information is missing, incomplete, or lots of information are lost [20, 21], due expressing decision maker´s opinion in uncertainty (caused by subjective weights) is hard to provide accurate values, therefore real numbers would change to linguistic terms which are closer to the human cognitive processes thru a proper predefined linguistic evaluation scale, is more adequate reasonable and applicable in real life problems [20, 22, 23].

However, in the literature [24–27] has been found a great amount of methods including fuzzy versions and others, but almost the majority of them has limitations, basically our purpose try to overcome the next limitations on MCDM problems:

- Consider that all the input criteria are independent and cannot consider the interrelationship among input criteria [28–30]. Few researches are concerned about consider the interrelationship among criteria in MDCM [28, 30, 31].
- There are limitations about the handling of subjective/ uncertainly information [31–33] regarding to MCDM problems.
- Weight/preference of the decision makers'. In several situations, performance ratings and weights cannot be given precisely [34], in some methods it is difficult to determine if the weights are used as trade-offs or importance coefficients [20, 35].

Based on the considerations mentioned above, this paper proposes the application and combination of three important tools: Dimensional Analysis, Pythagorean fuzzy sets and entropy measure for hesitant fuzzy linguistic term sets, in order to solve the qualitative criteria, the interrelationship among the multiple criteria, and weights calculation when are unknown. The structure of the paper is summarized as follows: in Sect. 2, basic concepts of Pythagorean Fuzzy sets (PFS), Dimensional Analysis (DA) and entropy measure for hesitant fuzzy linguistic term sets (HFLTSs) are described. In Sect. 3, description of integration of (PFS) and (DA) proposes a hybrid method, and an algorithm is given. In Sect. 4 an application is presented numerical example to illustrate our technique Dimensional Analysis-Pythagorean fuzzy (DA-PF), we present a comparison between DA-PFS with calculated weights and Entropy weights. Finally, the conclusion is provided in Sect. 5.

2 Preliminaries

In the following sections some fundamental concepts of PFS [9–15], DA [16–19] and Entropy [20–22] are described.

2.1 *Pythagorean Fuzzy Sets*

Definition 1 [35–46], if S, R ∈ PFSs equations are described as follows:

$$R \oplus S = \{< T, \sqrt{\mu_R^2(T) + \mu_S^2(T) - \mu_R^2(T)\mu_S^2(T)}, \nu_R(\mathrm{T})\nu_S(\mathrm{T})\}; \tag{1}$$

$$R \ominus S = \{< T, \sqrt{\frac{\mu_R^2(T) - \mu_S^2(T)}{1 - \mu_S^2(T)}}, \frac{\nu_R(T)}{\nu_S(T)} > |T \in X\} \tag{2}$$

$$R \oslash S = \{< T, \frac{\mu_R(T)}{\mu_S(T)}, \sqrt{\frac{\nu_R^2(T) - \nu_S^2(T)}{1 - \upsilon_S^2(T)}} > |T \in X\} \tag{3}$$

$$R \otimes S = \{< T, \mu_R(T)\mu_S(T), \sqrt{\nu_R^2(T) + \nu_S^2(T) - \nu_R^2(T)\nu_S^2(T)} > |T \in X\} \tag{4}$$

$$p^{\lambda} = (\mu^{\lambda}, \sqrt{1 - (1 - \nu^2)^{\lambda}}) \quad (5)$$

$$\lambda p = (\sqrt{1 - (1 - \mu^2)^{\lambda}}, \nu^{\lambda}) \quad (6)$$

$$s(p) = (\mu)^2 - (\nu)^2 \quad (7)$$

2.2 Dimensional Analysis

DA is a MCDM method applied in the decision-making process, that operates with an optimal solution better or chosen in a set of alternatives. DA operates with a comparison of each alternative in evaluation against optimal alternative and calculate an index of similarity, where the highest index of similarity is consider as the best alternative [16, 18].

Definition 1
Be $a_l^N(N = 1, \ldots, n)(M = 1, \ldots m)$ and $S_l^* = a_j^*(M = 1, \ldots, m)$ represent a data base of crisp values. DA equation is described as follows:

$$IS_i\left(a_1^i, a_2^i, \ldots, a_m^i\right) = \prod_{j=1}^{m} \left(\frac{a_j^i}{S_l^*}\right)^{w_j} \quad (8)$$

where IS_i represents the index of similarity for alternative *i*.

Where a_l^k represents crisp values of criterion *l* for alternative *i*.

Where S_l^* represents crisp values of the optimal alternative for criterion *l*.

Where $w_j(z = 1, \ldots, m)$ represents crisp weight value for criterion l.

2.3 Entropy with Unknown Weights in Hesitant Fuzzy Linguistic Term Setting

According with Gou et al. [21], usually MCDM concerning two important steps: first: calculate criteria weights, and second: obtain an adequate ranking of alternatives. In accordance with Farhadinia [20]; described entropy measures, are applied to treat with the MCDM problems where information concerning criteria weights is missing or lack of data. The following equations stand for entropy measure based on generalized distance:

$$E_{dg}\left(H_{\xi}\right) = 1 - \frac{2}{N}\sum_{i=1}^{N}\left[\left(\frac{1}{L}\sum_{I=1}^{L}\left(\frac{|\delta_l}{2\tau}\right)^{\lambda}\right)^{\lambda}\right], \lambda > 0 \tag{9}$$

Then, to calculate the entropy weights as follows:

$$W_j = \frac{1 - E_j}{m - \sum_{j=1}^{m} E_j}, j = 1, \ldots, m \tag{10}$$

Then we use linguistic labels that represents the preferences given by decision makers therefore a predefined linguistic evaluation scale is needed. For this, in accordance with Xu [47] a discrete linguistic term set is described as: $\vartheta = \{S_\alpha | \alpha = -T, \ldots, -1, 0, 1, \ldots, T\}$ where S_α represents a linguistic variable.

3 DA-PFS with Hesitant Entropy

In this section, we propose the hybrid method of DA and PFSs is given for dealing with both types of information, and an algorithm is proposed.

3.1 Dimensional Analysis Under Pythagorean Fuzzy Set (DA-PFS)

Based on Eq. (8) of DA described in Sect. 2, the definition of DA-PFS is described as follows: Let $\omega_j^i = \left(\mu_{w_j^i}, \nu_{w_j^i}\right)(i = 1, 2, \ldots n)(j = 1, 2 \ldots m)$ and $S_j^i = \left(\mu_{w_j^i}, \nu_{w_j^i}\right)(j = 1, 2 \ldots m)$ be a collection of P F_s, if:

$$PFIS_i\left(\omega_1^i, \omega_2^i, \ldots, \omega_m^i\right) = (\otimes_{j=1}^{m}(\frac{\omega_j^i}{S_j^*})^{T_j}) = (\otimes_{j=1}^{m}(\psi_j^i)^{T_j} \tag{11}$$

According with Eqs. (3–6) of the PFS given in Sect. 2 and Eq. (11), we deduct the next results.

Theorem 1 Let $\psi_j^i = (\mu_{\psi_j^i}, \nu_{\psi_j^i})(i = 1, 2, \ldots, n)(j = 1, 2, \ldots, m)$ be a set of PFS. Therefore, the aggregated value, by using $PFIS$, is also an IFN, and

$$PFIS_i\left(\omega_1^i, \omega_2^i, \ldots., \omega_m^i\right) = \left(\underset{j=1}{\overset{m}{\otimes}} \left(\psi_j^i\right)^{T_j}\right)$$

$$= \left(\prod_{j=1}^{m} \left(\mu_{\psi_j^i} \right)^{T_j}, \sqrt{1 - \prod_{j=1}^{m} \left(1 - \left(\nu_{\psi_j^i} \right)^2 \right)^{T_j}} \right) \quad (12)$$

3.2 Algorithm for DA-PFS with Hesitant Entropy

According with above analysis, DA-PFS is described as follows:

- Step 1: Build Pythagorean decision matrix, preferences given by decision makers.
- Step 2: Choose optimal solution according to (BN) or (C) criteria values.
- Step 3: Calculate hesitant entropy criteria weights, by Eq. (9 and 10)
- Step 4: Calculate standardized matrix: according to (BN) criteria, or (C) criteria
- Step 5: Standardized matrix elevated according to entropy criteria weights, use Eq. (5)
- Step 6: Calculate Pythagorean fuzzy index, by Eq. (12)
- Step 7: Calculate the highest similarity index or defuzzy, by score Eq. (7)
- Step 8: The index similarity must be organized in descending order and choose the alternative with the highest value.

4 Application

4.1 Numerical Example

A Company, needs to evaluate and select the most properly Forklift machine, to choose the best forklift that offers maximum performance at lowest cost. There are five alternatives or brands to select (A1, A2, A3, A4, and A5), and eight criteria to consider: C1: Load capacity (pounds), C2: Maximum travel speed full load (mph), C3: Maximum lift speed full load (fpm), C4: Maximum grade ability full load (%), C5: Basic right angle stack (in), C6: More Intelligent, C7: Safer and C8: Robuster. Criteria C1-C5 are quantitative data that can be treated with simple AD, in other hand criteria C6-C8 are qualitative data, in this case AD-PFS is applied (Table 1).

According to algorithm proposed in Sect. 3, it is important to note that they should be treated separately criteria C1 to C5 using simple DA, and criteria C6-C8 using DA-PFS, then steps are the following:

In this part DA-PFS is applied: Step 1: In accordance with DM evaluations, the Pythagorean fuzzy decision matrix is defined as follows:

Table 1 Alternative and criteria Forklift selection

Options	C1	C2	C3	C4	C5	C6	C7	C8
A1	3000.491	10.9	110	43	110	{0.7,0.6}	{0.8,0.44}	{0.5,0.8}
A2	3999.185	10.9	110	35	87.6	{1.0,0}	{0.8,0.44}	{0.7,0.60}
A3	3999.185	10.6	120	36	92.3	{0.5,0.8}	{0.7,0.6}	{0.6,0.71}
A4	5000.084	11	125	31	95	{0.8,0.44}	{1.0,0}	{0.8,0.44}
A5	5511.557	14	130	55	115	{0.8,0.44}	{0.8,0.44}	{1.0,0}

$$\begin{bmatrix} \{0.70, 0.60\} & \{0.80, 0.44\} & \{0.50, 0.44\} \\ \{1.00, 0.00\} & \{0.80, 0.44\} & \{0.70, 0.60\} \\ \{0.50, 0.80\} & \{0.70, 0.60\} & \{0.60, 0.71\} \\ \{0.80, 0.44\} & \{1.00, 0.00\} & \{0.80, 0.44\} \\ \{0.80, 0.44\} & \{0.80, 0.44\} & \{1.00, 0.00\} \end{bmatrix}$$

Step 2: Establish ideal solution in accordance to criteria values:

$$S^{+} : \{1.00, 0.00\}\{1.00, 0.00\}\{1.00, 0.00\}$$

Step 3: Establish the entropy criteria weights use Eq. (9) and (10):

$$W_{\{C_6,C_7,C_8\}} = \begin{bmatrix} \{0.1345\} \\ \{0.1578\} \\ \{0.0935\} \end{bmatrix}$$

Step 4: In order to standardized matrix, use Eqs. (3) in accordance to BN or C:

$$\begin{bmatrix} \{0.70, 0.60\} & \{0.80, 0.44\} & \{0.50, 0.80\} \\ \{1.00, 0.00\} & \{0.80, 0.44\} & \{0.70, 0.60\} \\ \{0.50, 0.80\} & \{0.70, 0.60\} & \{0.60, 0.71\} \\ \{0.80, 0.44\} & \{1.00, 0.00\} & \{0.80, 0.44\} \\ \{0.80, 0.44\} & \{0.80, 0.44\} & \{1.00, 0.00\} \end{bmatrix}$$

Step 5: Then, each criteria column in standardized matrix is elevated with entropy criteria weights, use Eq. (5):

$$\begin{bmatrix} \{0.9532, 0.2414\} & \{0.9650, 0.1828\} & \{0.9370, 0.3019\} \\ \{1.0000, 0.0000\} & \{0.9650, 0.1828\} & \{0.9670, 0.2022\} \\ \{0.9110, 0.3583\} & \{0.9450, 0.2608\} & \{0.9530, 0.2520\} \\ \{0.9704, 0.1689\} & \{1.0000, 0.0000\} & \{0.9790, 0.1412\} \\ \{0.9704, 0.1689\} & \{0.9650, 0.1828\} & \{1.0000, 0.0000\} \end{bmatrix}$$

Step 6: Then, to generate an index of similarity $PFIS_i$ use Eq. (12):

$$\begin{bmatrix} 0.8624 & 0.4980 \\ 0.9337 & 0.3330 \\ 0.8209 & 0.5360 \\ 0.9504 & 0.2590 \\ 0.9368 & 0.2470 \end{bmatrix}$$

Step 7: To get the highest index of similarity use Eq. (7), however for this case we need to get the IS from the simple DA using Eq. (8) in order to solve criteria C1-C5 due they are quantitative data. Then we have the following matrix in accordance with forklift specifications:

$$\begin{bmatrix} 3000.491 & 10.9 & 110 & 43 & 110 \\ 3999.185 & 10.9 & 110 & 35 & 87.6 \\ 3999.185 & 10.6 & 120 & 36 & 92.3 \\ 5000.084 & 11 & 125 & 31 & 95 \\ 5511.557 & 14 & 130 & 55 & 115 \end{bmatrix}$$

In this part we use simple DA: Step 7.1: Establish ideal solution in accordance to criteria values:

$$S^+ : [5511.557, 14, 130, 55, 115]$$

Step 7.2: Establish the criteria weights, use Eqs. (9) and (10) from Hesitant Entropy:

$$W_{\{C_1,C_2,C_3,C_4C_5\}} = \begin{bmatrix} 0.0935 \\ 0.1481 \\ 0.1403 \\ 0.1091 \\ 0.1228 \end{bmatrix}$$

Step 7.3: Normalized matrix use Eq. (8):

$$\begin{bmatrix} 0.544 & 0.799 & 0.846 & 0.782 & 0.957 \\ 0.726 & 0.799 & 0.846 & 0.636 & 0.762 \\ 0.726 & 0.757 & 0.923 & 0.655 & 0.803 \\ 0.907 & 0.786 & 0.962 & 0.564 & 0.826 \\ 1.000 & 1.000 & 1.000 & 1.000 & 1.000 \end{bmatrix}$$

Table 2 Rankings with entropy-based weights of criteria

	AD	AD	PF IS	ΛP		SCORE	RANK
IS 1	0.861	0.862	0.498	0.831	0.549	0.389	4
IS 2	0.841	0.934	0.333	0.907	0.397	0.665	3
IS 3	0.856	0.821	0.536	0.785	0.587	0.272	5
IS 4	0.872	0.950	0.259	0.933	0.307	0.775	2
IS 5	1.000	0.937	0.247	0.937	0.247	0.817	1

Step 7.4: Normalized matrix weight elevated:

$$\begin{bmatrix} 0.945\ 0.964\ 0.977\ 0.973\ 0.995 \\ 0.970\ 0.964\ 0.977\ 0.952\ 0.967 \\ 0.970\ 0.960\ 0.989\ 0.955\ 0.973 \\ 0.991\ 0.965\ 0.995\ 0.939\ 0.977 \\ 1.000\ 1.000\ 1.000\ 1.000\ 1.000 \end{bmatrix}$$

Step 7.5: Establish AD index of similarity IS_i:

$$\begin{bmatrix} 0.8609 \\ 0.8409 \\ 0.8558 \\ 0.8725 \\ 1.0000 \end{bmatrix}$$

Step 8: Establish the ranking, use Eq. (6) and (7), and then we got the following calculations (Table 2):

Where: $A_4 > A_3 > A_5 > A_2 > A_1$, therefore A_4 is selected as the best Forklift machine, according to the highest index of similarity.

4.2 Hesitant Entropy Weight

In accordance with Farhadinia [20]; calculations for entropy measure based on generalized distance, are based in the hesitant fuzzy linguistic judgment matrix is given by decision makers in the Tables 3 and 4:

Using Eq. (9 and 10), we have entropy-based weights of criteria as follows:

$w1 = 0.093$, $w2 = 0.148$, $w3 = 0.140$, $w4 = 0.109$, $w5 = 0.122$, $w6 = 0.134$, $w7 = 0.157$, $w8 = 0.093$

Table 3 Hesitant fuzzy linguistic matrix given by the decision makers. Part 1

ALT	C1	C2	C3	C4
A1	{s−1, s−1, s−1}	{s−1, s0, s−1}	{s0, s0, s−1}	{s1, s1, s1}
A2	{s0, s0, s0}	{s1, s0, s0}	{s1, s1, s1}	{s0, s0, s0}
A3	{s0, s0}	{s0, s1, s1}	{s1, s1, s1}	{s1, s0, s0}
A4	{s1, s1, s1}	{s1}	{s1, s1, s2}	{s−1, s−1, s−1}
A5	{s2, s2, s2}	{s1, s2, s2}	{s3, s2, s2}	{s2, s2, s2}

Table 4 Hesitant fuzzy linguistic matrix given by the decision makers, part 2

ALT	C5	C6	C7	C8
A1	{s1, s2}	{s1, s0}	{s1, s1, s1}	{s−1, s−1, s−1}
A2	{s0, s0, s0}	{s2, s2}	{s1, s1, s1}	{s0, s0, s0}
A3	{s0, s0}	{s0}	{s1, s0}	{s0, s0, s0}
A4	{s1, s1, s1}	{s1, s1, s1}	{s2, s2}	{s1, s1, s1}
A5	{s2, s2, s2}	{s1, s1, s1}	{s1, s1, s1}	{s2, s2, s2}

4.3 Sensitivity Analysis

Sensitivity analysis is commonly used to ensure robustness of solutions [48]. In other words the sensitivity analysis can be described as stability or behavior when a solution is subjected to small changes by decision makers, or change the parameters values, and these small changes do not affect the result is consider an efficient multi-criteria decision method referring Pamučar and Ćirović [49].

4.3.1 Entropy

In accordance with Farhadinia [20]; calculations for entropy measure based on generalized distance, are based in the hesitant fuzzy linguistic judgment matrix provided by decision makers (Table 5).

Step 1: Using Eq. (9), we have the following (Table 6):

Then, using Eq. (9) we get that:

$$E_{1dg} 1 - \frac{2}{5}\sum_{i=1}^{5} E_{dg}\left(h_{\xi}^{il}\right) = 0.733$$

Table 5 The hesitant fuzzy linguistic judgment matrix provided by the decision organization

Alt	C1	C2	C3	C4	C5	C6	C7	C8
A1	{s−1, s−1, s−1}	{s−1, s0, s−1}	{s0, s0, s−1}	{s1, s1, s1}	{s1, s2}	{s1, s0}	{s1, s1, s1}	{s−1, s−1, s−1}
A2	{s0, s0, s0}	{s1, s0, s0}	{s1, s1, s1}	{s0, s0, s0}	{s0, s0, s0}	{s2, s2}	{s1, s1, s1}	{s0, s0, s0}
A3	{s0, s0}	{s0, s1, s1}	{s1, s1, s1}	{s1, s0, so}	{s0, s0}	{s0}	{s1, s0}	{s0, s0, s0}
A4	{s1, s1, s1}	{s1}	{s1, s1, s2}	{s−1, s−1, s−1}	{s1, s1, s1}	{s1, s1, s1}	{s2, s2}	{s1, s1, s1}
A5	{s2, s2, s2}	{s1, s2, s2}	{s3, s2, s2}	{s2, s2, s2}	{s2, s2, s2}	{s1, s1, s1}	{s1, s1, s1}	{s2, s2, s2}

Table 6 Determining the entropy-based weights of criteria by generalized distance

Alt	C1	C2	C3	C4	C5	C6	C7	C8
A1	0.167	0.111	0.056	0.167	0.375	0.125	0.167	0.167
A2	0.000	0.056	0.167	0.000	0.000	0.500	0.167	0.000
A3	0.000	0.111	0.167	0.056	0.000	0.000	0.125	0.000
A4	0.167	0.500	0.222	0.167	0.167	0.167	0.500	0.167
A5	0.333	0.278	0.389	0.389	0.333	0.167	0.167	0.333

In addition, we have:

$E_{2dg} = 0.578$, $E_{3dg} = 0.600$, $E_{4dg} = 0.689$, $E_{5dg} = 0.650$, $E_{6dg} = 0.617$, $E_{7dg} = 0.550$ & $E_{8dg} = 0.733$

Step 2: Consequently, the entropy-based weights of criteria using Eq. (10): c_j(j = 1, 2, 3, 4, 5, 6, 7 and 8) are achieved as:

$$W_1 = \frac{1 - 0.733}{8 - (0.733 - 0.578 - 0.600 - 0.689 - 0.650 - 0.617 - 0.550 - 0.733)}$$

Therefore, we have entropy-based weights of criteria as follows:

$W_1 = 0.093$, $W_2 = 0.148$, $W_3 = 0.140$, $W_4 = 0.109$, $W_5 = 0.122$, $W_6 = 0.134$, $W_7 = 0.157$, $W_8 = 0.093$

4.3.2 Fuzzy Weighted

Step 1: Establish a team of DM and capture preferences. If the $DM_k = \{\mu_{k,} \nu_{k,} \pi_k\}$ is a Pythagorean fuzzy number for DM, then we have the following:

$$\boldsymbol{\delta_k} = \frac{\left(\mu_k + \pi_k\left(\frac{\pi_k}{\mu_k+\nu_k}\right)\right)}{\sum_{k=1}^{l}\left(\mu_k + \pi_k\left(\frac{\pi_k}{\mu_k+\nu_k}\right)\right)} \tag{13}$$

Apply Table 5 for DM preferences (Table 7):

Then we have three DM (Table 8):

Using Eq. (46), we get the following:

$DM_1 = 0.35$, $DM_2 = 0.35$, $DM_3 = 0.30$

Step 2: Establish preferences of criteria. Apply Table 5 now for criteria preferences (Table 9):

Step 3: Using Eq. (14), preferences must be gathered and mixed in just one, we have the following (Table 10):

$$\boldsymbol{PFWA_W} = \left[\left(1 - \prod_{j=i}^{n}(1 - \boldsymbol{\mu}_{\boldsymbol{\alpha}_j}^2)^{\boldsymbol{w}_j}\right)^{\frac{1}{2}}, \prod_{j=1}^{n} \boldsymbol{\nu}_{\boldsymbol{\alpha}_j}^{\boldsymbol{w}_j}\right], \tag{14}$$

Step 4: in addition, we use again Eq. (13):

Table 7 Linguistic scale for DM preferences

Meaning	PFNs (μ, ν)
Apprentice (Ap)/Very Insignificant (VI)	(0.10, 0.90)
Leaner (Lr)/Insignificant (I)	(0.35, 0.60)
Capable (Ct)/Average (A)	(0.50, 0.45)
Skillful (S)/Imperative (Im)	(0.75, 0.40)
Dominant (D)/Very Significative (VS)	(0.90, 0.10)

Table 8 DM preferences

	DM1			DM2			DM3	
μ	ν	π	μ	ν	π	μ	ν	π
0.9	0.1	0.42	0.9	0.1	0.42	0.75	0.4	0.53

Table 9 Criteria preferences

C1	C2	C3	C4	C5	C6	C7	C8
0.50, 0.45	0.90, 0.10	0.75, 0.40	0.50, 0.45	0.90, 0.10	0.75, 0.40	0.90, 0.10	0.75, 0.40
0.90, 0.10	0.35, 0.60	0.90, 0.10	0.90, 0.10	0.35, 0.60	0.90, 0.10	0.35, 0.60	0.90, 0.10
0.75, 0.40	0.90, 0.10	0.90, 0.10	0.75, 0.40	0.90, 0.10	0.90, 0.10	0.90, 0.10	0.90, 0.10

Table 10 Criteria preferences gathered in one

C1	C2	C3	C4	C5	C6	C7	C8
0.78, 0.26	0.82, 0.19	0.86, 0.16	0.78, 0.26	0.82, 0.19	0.86, 0.16	0.82, 0.19	0.86, 0.16

Table 11 Hesitant entropy weight versus fuzzy weighted

C1	C2	C3	C4	C5	C6	C7	C8
0.093	0.148	0.14	0.109	0.122	0.134	0.157	0.093
0.12	0.13	0.13	0.12	0.13	0.13	0.13	0.13

$W_1 = 0.12, W_2 = 0.13, W_3 = 0.13, W_4 = 0.12, W_5 = 0.13, W_6 = 0.13, W_7 = 0.13, W_8 = 0.13$

Then have calculated the weights of criteria in both methods we got the same result:

$$A_4 > A_3 > A_5 > A_2 > A_1$$

In other hand we compare Hesitant Entropy weight and Fuzzy weighted calculated in the previous sections (Table 11).

Results reveal that: DA-PFS with entropy or fuzzy weighted, the alternative A_4 is selected as the best Forklift machine, according to the highest index of similarity and the rankings are consistent.

5 Conclusion

In this paper we have introduced a method DA-PFS with entropy measure for hesitant fuzzy linguistic term sets, in order to solve the qualitative criteria where exist uncertainty and lack of clarity [14], the interrelationship among the multiple criteria [16], and weights calculation when are unknown and prevent the loss of lots or sets information when the process is being carried out [50]. This method combines the best features of DA which consists the capacity to consider the mutual influence between several criteria [16], PFS, has capacity to represent evaluation and characterize better the uncertainty by lack of clarity [15], and HFLTSs by using linguistic labels instead numbers due are closer to the human cognitive processes [20, 22, 23].

As a result of this combination of tools, we obtain a more robust tool capable of considering information not involved in classical methods or their fuzzy extensions, mainly given in TOPSIS and AHP.

In addition we have compared different results concerning to weight concepts: a comparison, between Hesitant Entropy weight against Fuzzy weighted, then we got the same result, however, Hesitant Entropy weight requires less steps, therefore it's more efficient.

For the near future, we advise apply the conjugation of these tools in different fields where exist uncertainty, unknown criteria weights and interrelationship between criteria be an important factor.

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