Article

Fatigue-Life Prediction of Mechanical Element by Using the Weibull Distribution

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Abstract: Applying Goodman, Gerber, Soderberg and Elliptical failure theories does not make it possible to determine the span of failure times (cycles to failure-N_i) of a mechanical element, and so in this paper a fatigue-life/Weibull method to predict the span of the N_i values is formulated. The input's method are: (1) the equivalent stress (σ_{eq}) value given by the used failure theory; (2) the expected N_{eq} value determined by the Basquin equation; and (3) the Weibull shape β and scale η parameters that are fitted directly from the applied principal stress σ_1 and σ_2 values. The efficiency of the proposed method is based on the following facts: (1) the β and η parameters completely reproduce the applied σ_1 and σ_2 values. (2) The method allows us to determine the reliability index $R(t)$, that corresponds to any applied σ_{1i} value or observed N_i value. (3) The method can be applied to any mechanical element's analysis where the corresponding σ_1 and σ_2, σ_{eq} and N_{eq} values are known. In the performed application, the σ_1 and σ_2 values were determined by finite element analysis (FEA) and from the static stress analysis. Results of both approaches are compared. The steps to determine the expected N_i values by using the Weibull distribution are given.

Keywords: static and fatigue reliability; mechanical design; Weibull distribution; finite element analysis; principal stresses

1. Introduction

Fatigue is a random phenomenon [1,2] that causes a mechanical component to fail [3] at a stress level lower than the material strength limit (S_e) [4]. Although fatigue is random and the mechanical component is subject to variable amplitude and cyclic load [σ_1, σ_2], based on single equivalent stress σ_{eq} value, its failure analysis can be performed by applying a failure theory such as the Goodman, Gerber, Soderberg, and ASME (American Society of Mechanical Engineers) elliptical theories [5] (pp. 185,186,188) [6] (pp. 313–316). Depending on the specific research, different statistical models based on (stress, strain, crack growth, etc.) have been proposed to determine the cycles to failure [7–9]. Therefore, because rather than using the random behavior the analysis uses only a stress value, its application does not enable either the reliability of the component or its expected cycles to failure (N_i) values to be found. However, because the expected N_i values can be determined from the related stress-cycles S–N curve of the used material [10], and since the ultimate strength (S_{ut}) and S_e values represent the strength of the material used, then based on the theory given in [11], and on the S_{ut} and S_e values, the Weibull shape β and scale η parameters can be determined. Therefore, the novelty of the proposed method lies in that it uses the Weibull parameters to determine both the component's reliability and the expected N_i values. Because the reliability of the component depends on both the applied stress and the inherent strength [12,13] in the product to overcome it, then the proposed method to determine the random behavior of N can be based on the Weibull distribution used to model the stress, and on the Weibull distribution used to model the N_i values.

Based on the theory given in [11], the Weibull stress β_s and η_s parameters are both fitted directly from the applied maximum and minimum principal stress σ_1 and σ_2 values, established here by static and finite element analyses (FEA). Thus, based on the fact that the reliability of the component is completely determined for either the applied stress or for the corresponding cycles to failure, the Weibull cycle to failure ($\beta_t = \beta_s$, η_t) parameters are found based on both the addressed (β_s, η_s) parameters and on the cycles to failure N value, where N is obtained from the S-N curve by inserting the σ_{eq} that corresponds to the applied failure theory in Basquin's equation. This makes the proposed method highly efficient in determining the random behavior of N and the corresponding reliability indices. This efficiency is based on the fact that the Weibull stress and the Weibull cycle distributions both converge to the same reliability index [14].

To show numerically how the proposed method works, in the practical analysis the used principal stresses σ_1 and σ_2 values were determined by FEA and static analyses [15]. Because the proposed method depends only on the principal stress values, but not on the complex analysis used to determine them, then in the application a simple flat spring component made of steel AISI 4340 OQT 1300 subjected to tensile cycling stress was used. Since from the FEA simulation, the addressed principal stresses values are σ_1 = 491.75 MPa and σ_2 = 184.8 MPa, then by applying the proposed method, the corresponding Weibull stress parameters were (β_s = 2.248519, η_s = 301.455469 MPa) . However, because β_s and η_s are expressed in stress units, then in order to determine the related Weibull cycle distribution, the corresponding expected cycle to failure (N) value of the flat spring was determined by using Basquin's equation [see Equation (32)], where the σ_{eq} value was estimated by using the ASME elliptical failure theory criterion [see Equation (29)]. Therefore, based on the stress (β_s, η_s) parameters and on the addressed (N) value, the resulting cycle Weibull parameters were β_s = 2.248519, η_t = 1,243,427,849 cycles. As can be seen, although the reliability of each one of the expected cycles to failure can be determined by either the stress or the cycle Weibull families, it is important to notice that the random behavior of N can only be determined by the Weibull cycle family. Finally, it is important to mention that based on the Weibull stress (β_s, η_s) parameters, the steps to determine the minimal strength Sy value that a component must have to feature at least the desired *R(t)* index, and vice versa, are also given.

The structure of the paper is as follows. Section 2 offers the generalities of the fatigue analysis; and in Section 3, the generalities of the Weibull analysis is presented. The proposed method to determine the expected cycle to failure is formulated in Section 4. Section 5 contains the numerical application and a comparison between the FEA results and those given by the static analysis method. Finally, the conclusions are given in Section 5.

2. Generalities of Fatigue Analysis

Since the data from the accumulated fatigue damage analysis allow us to determine how the mechanical elements are weakened over time, then the main points considered in fatigue analysis are: (i) load, (ii) component geometry, (iii) material properties, (iv) stress-life analysis and (v) fatigue life analysis. However, fatigue analysis is performed by considering that the fatigue life depends on the average value and on the amplitude of the fluctuating loads [16]. However, since several mechanical components are subjected to fluctuating load conditions, then fatigue analysis is required in order to estimate their lifetime [17,18]. On the other hand, fatigue analysis requires an estimation of the cycles to failure, which is achieved when a set of n samples are tested to failure. Thus, when the fracture occurs, the corresponding cycles are recorded as the corresponding cycle to failure. In high-cycle fatigue situations, material performance is commonly characterized by an S–N curve [19], also known as Wöhler's curve. The S–N is a graph of the magnitude of the alternating stress (σ_a) against the logarithmic of cycles to failure (N) [3]. Therefore, the principal stresses σ_1 and σ_2 values are used to determine the stress amplitude σ_a and mean stress σ_m values, which are used in fatigue analysis. Then the σ_a and σ_m values are both used in a failure theory criterion to determine the corresponding equivalent stress σ_{eq} value used in Basquin's equation and, thus, determine the N value for the expected

cycles to failure [20]. Because cycles to failure occur due to weakness of the strength, they can be represented by the weakest link theory represented by the Weibull distribution, which offers the smallest extreme value distribution [14]. The most useful properties of the Weibull distribution to represent fatigue data are its stability property (referred to as stability to changes of the location parameter), its stability by using the minimum operations, and its stability under scale transformation (see [14] chapter 2 for details).

Since the Weibull stress (β_s, η_s) parameters used in the proposed method are both estimated directly from the applied principal stresses σ_1 and σ_2 values [21,22], let us present how they are determined from the static stress analysis.

2.1. Static Stress Analysis

The objective of this section is to show how to determine the principal stresses σ_1 and σ_2 values that we used in the proposed method (1) to determine the corresponding Weibull stress β_s and η_s parameters and (2) to perform the fatigue analysis. Although the σ_1 and σ_2 values are both determined depending on the axial, tension, compression, bending, torsion and shear stresses values, because the case analyzed herein deals only with bending, then σ_1 and σ_2 are both determined by the maximum and minimum generated moment. The procedure to determine them is explained below:

(i) By using the maximum M value determine the maximum bending stress σ_1 value as:

$$\sigma = \frac{Mc}{I} \tag{1}$$

(ii) Determine the maximum and minimum deflection of the element (Y) as:

$$Y = FL^3/3EI \tag{2}$$

where, F is the maximum and minimum applied force, E is the material's elasticity modulus, and L is the component's length.

(iii) By solving Equation (3) for F, by using the maximum and minimum Y values, determine the corresponding maximum and minimum applied forces as:

$$F = 3EIY/L^3 \tag{3}$$

(iv) Determine the inertia area moment I. It is calculated as in Equation (4), where b is the element's width, and t is the element's thickness.

$$I = bt^3/12 \tag{4}$$

(v) By using the maximum and minimum F value, determine the maximum and minimum bending moment (M) as:

$$M = F \times L \tag{5}$$

Finally, by using the minimum M value in Equation (1), determine the corresponding minimum bending stress σ_2 value. The proposed method is now described by using the σ_1 and σ_2 values.

3. Proposed Method

This section explains the proposed method to determine the cycle to failure of a mechanical component. Because the proposed method is based on both the principal applied stresses and the corresponding cycle to failure values, the section is structured to present first the steps first to determine the *Weibull stress* family, and second to determine the corresponding *Weibull cycle to failure* family. However, since the Weibull distribution is used, its generalities are stated below.

3.1. Generalities of the Weibull Analysis

The aim of this section is to present the generalities of the Weibull distribution based on which we can directly determine the β and η parameters of the *Weibull stress* (β_s, η_s) and the *Weibull cycles* ($\beta_t = \beta_s$, η_t) families. The Weibull probability density function (*pdf*) and its reliability function $R(t)$ [23] are given as:

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{6}$$

and

$$R(t) = exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{7}$$

From the addressed σ_1 and σ_2 values, the (β_s, η_s) parameters are directly estimated [24]. β_s is determined as:

$$\beta_s = \frac{-4\mu_y}{0.99176 * ln\left(\frac{\sigma_1}{\sigma_2}\right)} \tag{8}$$

where the constant 0.99176 is specified for the analyzed case, and the method to determine it for any case, is given in Section 4.1 in [24]. Also, μ_y is the mean of the used Y vector, determined based on the median rank approach. Notice that if $n = 21$ elements are tested, then $\mu_y = -0.545624$; and η_s is determined as:

$$\eta_s = exp(\mu_x) \tag{9}$$

where from [24] μx in Equation (9) is the log-mean of the observed failure-time data which based on the addressed σ_1 and σ_2 values is determined as:

$$\mu_x = ln(\sigma_1\sigma_2)^{\frac{1}{2}} \tag{10}$$

To summarize, note that by using the σ_1 and σ_2 values in Equations (8)–(10), the Weibull stress parameters are both completely determined, and that because $\mu_y = -0.545624$ is constant, then the efficiency of β_s and η_s only depends on the efficiency on which the σ_1 and σ_2 values were determined. Based on the addressed stress (β_s, η_s) parameters, let us present the method to determine the random behavior of the addressed σ_1 and σ_2 values, based on which the random behavior of the expected cycles to failure is formulated.

3.2. Weibull Stress Family Estimation and Its Random Behavior Analysis

Before determining the random behavior of the addressed σ_1 and σ_2 values, it is necessary to mention that they should be validated by using them with the material yield strength S_y value, and the desired safety factor (*SF*) in the yielding maximum shear stress theory (*MSS*) and the distortion energy theory (*DE*) criteria [25] as follows:

$$MSS\ theory = (\sigma_1) < \frac{S_y}{SF} \tag{11}$$

and,

$$DE\ theory = \left(\sigma_1^2 - \sigma_1 * \sigma_2 + \sigma_2^2\right)^{0.5} < \frac{S_y}{SF} \tag{12}$$

If Equations (11) and (12) do not hold, then a material with higher S_y for which Equations (11) and (12) holds, must be selected.

Note 1. Note that from Equations (11) and (12), we only know whether the design is safe or not, but at this point it is not possible to determine either the designed reliability or the corresponding expected failure times.

Based on the above, the expected behavior of σ_1 and σ_2 is determined as follows:

(i) From [24], determine the sample size n value to be used in the analysis.

$$n = \frac{-1}{ln(R(t))} \tag{13}$$

where $R(t)$ is the desired *reliability of the analysis*, here $R(t)$ could be seen as the equivalent of a confidential interval level used in the quality field. Note that the reason why $R(t) = 0.9535$ is used because for this value, n is an integer ($n = 21$), but any desired $R(t)$ index can be used.

(ii) Using the n value from Equation (13) in the median rank approach function [26] given in Equation (14), determine the corresponding cumulated failure percentile $F(t_i)$ elements as:

$$F(t_i) = \frac{i - 0.3}{n + 0.4} \tag{14}$$

(iii) by using the $F(t_i)$ elements in the linear form of the reliability function defined in Equation (7), determine the corresponding Y_i elements as shown in Equation (15). Then, from the Y_i elements, determine its corresponding mean value, obtained by using Equation (16).

$$Y_i = LN(-LN(1 - ((i - 0.3)/(n + 0.4)))) \tag{15}$$

and,

$$\mu_y = \sum_{i=1}^{n} \frac{Y_i}{n} \tag{16}$$

(iii) by inserting the μ_y value and the principal stress σ_1 and σ_2 values into Equations (8) and (9), determine the corresponding *Weibull stress parameters* (β_s, η_s). These estimated (β_s, η_s) parameters represent the *Weibull stress* family used to model the random behavior of the estimated principal stresses σ_1 and σ_2 values that represent the minimum required strength the used material must present in order for it to have the designed reliability. Thus, σ_1 and σ_2 are the applied stresses from which the σ_{eq} value was determined; σ_{eq} is the equivalent constant applied stress value, that by its cyclical application demands the material presents at least the minimum strength of σ_{2i} value. Based on the β_s and η_s parameters, the minimum strength σ_{2i} values are determined by using the t_{0i} value that corresponds for each one of the Y_i elements as:

$$t_{0i} = exp\{Y_i/\beta_s\} \tag{17}$$

Therefore, the minimum strength of σ_{1i} value that corresponds to each one of the Y_i elements is given as:

$$\sigma_{2i} = \eta_s * t_{0i} \tag{18}$$

Additionally, it is important to mention that because from [24], the Weibull β_s and η_s parameters also let determine the minimum Weibull strength parameter that the used material should present if the applied constant stress is the η_s value, then it is determined as:

$$\sigma_{1i} = \eta_s/t_{0i} \tag{19}$$

Thus, in Equation (18) σ_{2i} is the minimum stress the used material must present if the applied stress is constant at the value given by the σ_{eq} value. In Equation (19), σ_{1i} is the minimal Weibull scale parameter that that used material must present if the applied stress is constant at the value given by the η_s value (for details see Equation (61) in [24]), consequently by using the σ_{1i} and σ_{2i} values in Equation (25), the corresponding mean value is determined.

Also, by using the σ_1 value, the t_{01} element that corresponds to the σ_1 and σ_2 values, is calculated.

$$t_{01} = \eta_s/\sigma_1 \tag{20}$$

Consequently, by using the t_{0max} and the β_s values, the corresponding Y_1 value is established.

$$Y_1 = ln(t_{01}) * \beta_s \tag{21}$$

Finally, the reliability index corresponding to the Y_1 value is obtained.

$$R(t) = exp\{-exp\{Y_1\}\} \tag{22}$$

Note 2. It is important to mention that the *R(t)* index obtained in Equation (22) corresponds to an element which has a strength equivalent to the σ_1 value. Thus, if $S_y \neq \sigma_1$, the reliability of the designed element is found by replacing the S_y value with σ_1 in Equation (20).

However, note that while the reliability of the designed element can be obtained from Equation (22), the corresponding expected cycles to failure remain unknown. In this paper the cycle to failure are determined as follows:

3.3. Weibull Cycle Family Estimation and Its Random Behavior

To determine the Weibull cycle family and the corresponding random cycles to failure behavior, we need first to determine the cycle (*N*) value which corresponds to the equivalent stress σ_{eq}, by using Basquin's formula. The following is the explanation of how the *N* value is found.

3.3.1. Estimation of the *N* Value that Corresponds to the Equivalent Stress Value

The Weibull analysis of the component is performed based on the dynamic loads. That is why finding the corresponding cycles to failure occurs as follows. Depending on the component's material, the yield stress S_y and ultimate stress S_{ut} values are obtained. Then the corresponding endurance limit S_e value is calculated as:

$$S'_e = 0.5(S_{ut}) \tag{23}$$

Then based on the correction factor values, the corrected endurance limit is determined as:

$$S_e = K_a K_b K_c K_d K_e K_f S'_e \tag{24}$$

where, K_a = surface condition modification factor, K_b = size modification factor, K_c = load modification factor, K_d = temperature modification factor, K_e = reliability factor and K_f = miscellaneous-effects modification factor. After that, using Equations (25) and (26) and the values for principal stresses σ_1 and σ_2 from Section 2.1, the corresponding mid-range stress σ_m and the alternating stress σ_a values are set.

$$\sigma_m = \frac{(\sigma_1 + \sigma_2)}{2} \tag{25}$$

and,

$$\sigma_a = \frac{(\sigma_1 - \sigma_2)}{2} \tag{26}$$

Then, by using the S_y, S_e, σ_a and σ_m values in the used failure theory criterion, determine whether the designed element is safe or not. (Although as shown in Figure 1, there are several criteria, the one used here was the ASME Elliptic criterion). Thus, to determine the fatigue safety factor n_f, in the case of the ASME elliptical theory criterion, the n_f function is:

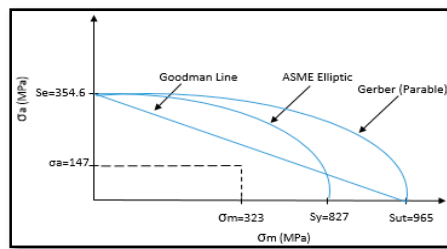$$n_f = \sqrt{\frac{1}{\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2}} \tag{27}$$

Figure 1. Goodman, Gerber and ASME failure criteria.

If n_f is higher than one ($n_f > 1$), the designed element is considered to have an infinite life ($N = \infty$). Otherwise, it is considered to have a finite life. However, because the aspect being worked with is the probabilistic behavior of the stress, what proceeds is to determine the corresponding reversed equivalent stress σ_{eq} value [27]. This is obtained from the ASME elliptical failure theory criterion represented by:

$$\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2 = 1 \tag{28}$$

The σ_{eq} is placed instead of the S_e element into Equation (28); thus, its value is set as:

$$\sigma_{eq} = \frac{\sigma_a}{\sqrt{1 - \left(\frac{\sigma_m}{S_y}\right)^2}} \tag{29}$$

Since the calculation of the fatigue constant a and b parameters of Basquin's equation is required, such a calculation is undertaken by using the fatigue strength factor f that is taken from [6], the ultimate stress S_{ut}, and the endurance limit S_e values.

$$a = \frac{(fS_{ut})^2}{S_e} \tag{30}$$

and,

$$b = -\frac{1}{3}log\left(\frac{fS_{ut}}{S_e}\right) \tag{31}$$

Then, by inserting the *a*, *b* and σ_{eq} estimated values into Equation (32), the expected average cycles to failure *N* value is obtained:

$$N = \left(\frac{\sigma_{eq}}{a}\right)^{\frac{1}{b}} \tag{32}$$

From Equation (32), the *N* value that corresponds to the equivalent stress is set. That value will be used to determine the corresponding Weibull cycle parameters.

3.3.2. Determination of the Weibull Cycle Parameters

By using the cycle to failure *N* value and the *Weibull element* t_{0eq} value that corresponds to the equivalent stress value in Equation (20), the corresponding *Weibull scale cycle to failure* η_t parameter is set.

$$\eta_t = \frac{N}{t_{oeq}} \tag{33}$$

At this point, by using the Weibull cycle η_t parameter, the corresponding expected cycle to failure values that correspond to each one of the Y_i elements from Equation (15) are determined.

$$N_i = \eta_t * t_{0i} \tag{34}$$

Thus, by using the Weibull stress β_s value, the Weibull cycle to failure family is W (β_s, η_t).

Note 3. The β_s value is used on the assumption that the failure mode remains constant.

Finally, the N_i value that corresponds to the used S_y value is determined based on the t_{0sy} element given as:

$$t_{0sy} = \eta_s / S_y \tag{35}$$

Then by using the t_{0sy} value in Equation (34), the N_i value that corresponds to the S_y value is determined. The numerical application is as follows.

4. Mechanical Application with Weibull/Finite Element Analysis (FEA)

The numerical application is performed by using a flat spring undergoing cyclic variable stress that causes tensile deflection. The load is considered as a cantilever and is the direct cause of failure affecting a product's reliability. By way of initial conditions, the following data are established: the minimal operational deflection of the flat spring is 3.0 mm, and the maximal operational deflection of the spring is 8.0 mm. It is also necessary to perform the spring stress analysis at point A. At this point, the convex side of the base of the spring experiences the varying tensile stresses [5]. The relevant data to determine the main applied stresses are, length $L = 65$ mm, thickness $t = 0.80$ mm, width $b = 6$ mm, and the used material is a steel AISI 4340 OQT 1300 with elasticity modulus $E = 207$ GPa. The spring is joint to the base by using the double-sided longitudinal welded joint that corresponds to the category B of joints in AWS D1.1 [28]. The structure geometry (see Figures 2 and 3) is subjected to variable cyclic load, within the elastic range, of frequency and magnitude sufficient to initiate cracking and progressive failure due to fatigue.
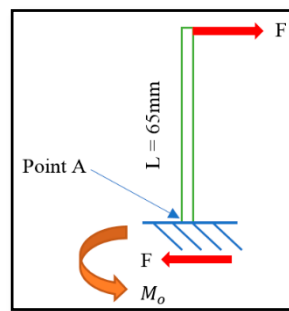

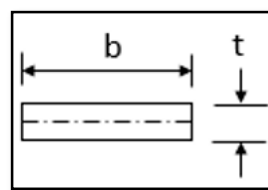
Figure 2. Free-Body diagram of flat spring.

Figure 3. Cross section of spring.

The relation between stress range F_{SR} and the allowable cycle number N under variable amplitude load condition [29] is:

$$F_{SR} = \left(\frac{C_f \, X \, 329}{N} \right)^{0.333} \tag{36}$$

$$F_{SR} = \sigma_{max} - \sigma_{min} \tag{37}$$

in which:

F_{SR} = Allowable stress range, C_f = Fatigue constant load, N = Number of cycles of stress range.

With the relevant data provided, the σ_1 and σ_2 principal stress can be determined to perform the Weibull FEA stress data analysis as follows.

4.1. Weibull FEA Stress Data Analysis

The proposed Weibull analysis is performed by using the σ_1 and σ_2 principal stress values that are acting on the flat spring device; thus, in this study they are determined by performing a FEA simulation. The analysis is performed by presenting the FEA stress data procedure [30] first, in Section 4.1, and then the corresponding cycles to failure (Ni) analysis in Section 4.2. The procedure will begin by following the steps to the static stress outlined in Section 2. Therefore, from Equation (4) the estimated inertia moment is $I = 0.256$ mm^4. Then, by using the minimum and maximum generated deflection $Y_2 = 3$ mm and $Y_1 = 8$ mm values and the elasticity module of E = 207 GPa in Equation (3), the deflection forces are $F_1 = 4.63$ N and $F_2 = 1.74$ N Those forces are introduced in the FEA simulation to determine the corresponding principal stress values used to determine the Weibull parameters. Once this is done, the principal stresses values obtained are, σ_1 = 491.75 MPa and σ_2 = 184.8 MPa.

The FEA analysis of the flat spring is shown in Figure 4. The used material is a steel AISI 4340 OQT 1300, with a Young's modulus of 207 GPa and a Poisson's ratio of 0.29.
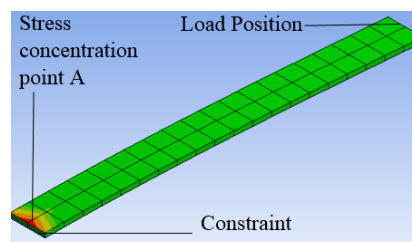

Figure 4. The finite element model of the flat spring.

The application was performed using the ANSYS software. The FEA model of the flat spring device with the dimensional features and the loading condition given in Section 4 was dispersed by using an adaptive sizing mesh of 148 mm, that uses the first-order element, with a total of 1343 nodes and 160 elements. The corresponding stress results are shown in Table 1.
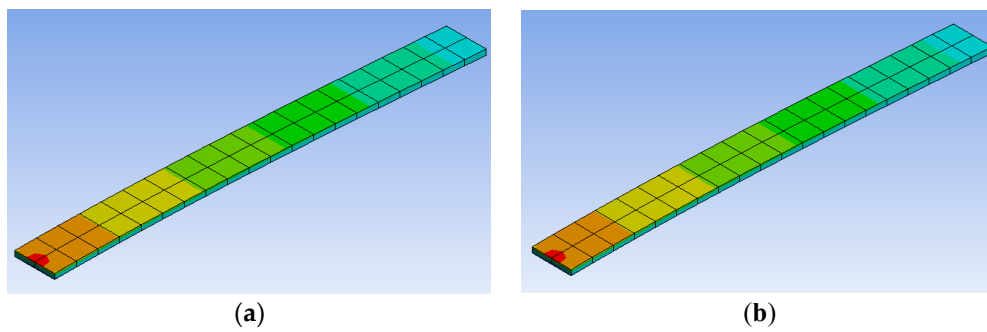
Table 1. Stresses and deformation results from the finite element analysis (FEA).

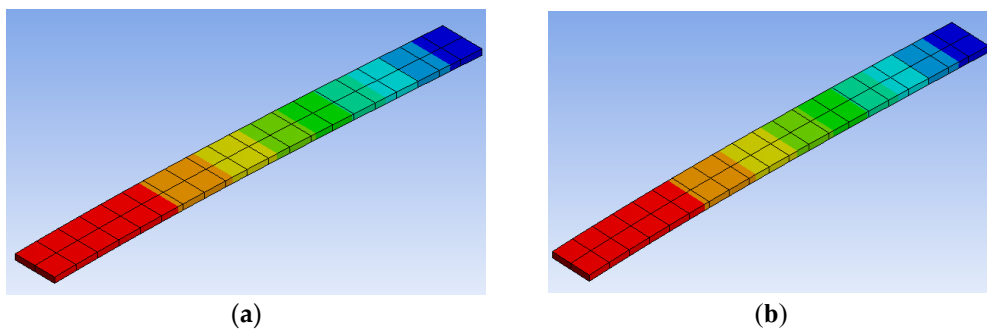| Maximum Principal Stress (MPa) Force $F_1 = 4.63$ N	Minimum Principal Stress (MPa) Force $F_2 = 1.74$ N	Deformation (mm) Force $F_1 = 4.63$ N	Deformation (mm) Force $F_2 = 1.74$ N
491.75	184.8	8.02	2.99

Based on Figure 4, and to reflect the real service loads obtained by Equation (3), the stress calculations are performed based on the maximum and the minimum applied forces of 4.63 N and 1.74 N, respectively. Since to determine the principal stress values, based on which we determine the random behavior of N, we need to know both how the stresses are distributed and how much the element is deflected, then Figure 5 shows the stress distribution; and Figure 6 shows the corresponding deflection.

As can be seen in Figure 5, the stress concentration occurs in the red area (point A); that is why the analysis was performed there.

In Figure 6 it can be observed that the maximum deflection occurs in the blue area where the force is being applied. Then, the red area shown in Figure 5 and the blue area shown in Figure 6 are both analyzed. Finally, the estimated σ_1 and σ_2 stress values of the FEA simulation given in Table 1 is compared to the static stress analysis. Both results are given in Table 2, where it can be seen that no significant difference exists.

Figure 5. Stress distribution picture of the flat spring under the different applied load (**a**) 4.63 N; (**b**) 1.74 N.

Figure 6. Deformation picture of the flat spring under the different applied forces (**a**) 4.63 N; (**b**) 1.74 N.

Table 2. Comparison of the stresses and deformation results of the static stress analysis and FEA simulation.

Application	Maximum Principal Stress (MPa), Force F_1 = 4.63 N	Minimum Principal Stress (MPa), Force F_2 = 1.74 N	Deformation (mm) Force F_1 = 4.63 N	Deformation (mm) Force F_2 = 1.74 N
Static	470	176	8.00	3.00
FEA	491.75	184.8	8.02	2.99

4.1.1. Validation of the Applied Principal Stresses σ_1 and σ_2 Values

With these principal stress values from the FEA application and the material yield strength (AISI 4340 OQT 1300) value S_y = 827 MPa, the yielding criteria for ductile materials is ascertained through the AmesWeb tool, with a design factor of *SF* = 1.65. The maximum shear stress theory (*MSS*) and the distortion of energy theory (*DE*) output criteria are both shown in Table 3.

Table 3. Static Analysis Results using the theories of failure for ductile materials.

Parameter	Condition to Be Met for Safe Design		Status
Maximum Shear Stress	$(\sigma_1) < S_y/SF$	491.8 < 501.8	Ok
Distortion Energy	$\left(\sigma_1^\wedge 2 - \sigma_1*\sigma_2 + \sigma_2^\wedge 2\right)^\wedge 0.5 < S_y/SF$	430.2 < 501.2	Ok

As can be seen from Table 3, because the status of both failure theories is acceptable, the design element can be deemed safe. Unfortunately, it can also be pointed out at this point that although the designed element is considered safe, nothing can be said about its reliability nor about the expected behavior of the corresponding cycles to failure.

The next section will present the Weibull stress family.

4.1.2. Weibull Stress Family Determination

The corresponding Weibull stress parameters (β_s, η_s) are established as follows.

First, by selecting a reliability of $R(t) = 0.9535$ to perform the analysis, from Equation (13), $n = \frac{-1}{ln(0.9535)} = 21$. Then by using $n = 21$ in Equation (15), the Y_i elements are generated, with a mean of $\mu_y = -0.545624$ and a standard deviation of $\sigma_y = 1.175117$. Finally, from Equation (8), and by using the μ_y, σ_1 and σ_2 FEA values, the Weibull β_s value is:

$$\beta_s = \frac{(-4)(-0.545624)}{0.99176 * ln\left(\frac{491.75}{184.8}\right)} = 2.248519$$

Likewise, since from Equation (10) the logarithm average value is $\mu_x = ln(491.75 * 184.8)^{\frac{1}{2}} = 5.708622$, then from Equation (9) the Weibull η_s parameter is $\eta_s = exp\{5.708622\} = 301.455469$ MPa. Thus, the Weibull stress family used to obtain the corresponding Weibull cycle distribution (Section 4.2) is W (2.248519, 301.455469 MPa). Using the Weibull stress family results, the strength random behavior can be determined. Because these parameters only depend on the applied σ_1 and σ_2 values, then by performing the Weibull analysis their random behavior can be determined as follows.

4.1.3. Stress Random Behavior

Because the determination of the cycles to failure is based on the random behavior of the σ_2 stress value, in this section the random behavior of the stresses σ_1 and σ_2 values are determined by Equations (18) and (19) (see fifth and sixth columns in Table 4). First, by using the β_s and the Y_i values in Equation (17), the basic Weibull elements t_{0i} defined for each one of the Y_i elements are determined, as shown in Table 4. Additionally, by using the η_s and the σ_1 values in Equation (20), the expected Weibull t_{01} value, from which the σ_1 and σ_2 stress values are both reproduced, is determined as $t_{01} = 301.455469/491.75 = 0.617339$, and it is included in Table 4 as well.

Then, by using the β_s value in Equation (21), the Y_1 value that corresponds to the t_{01} value is determined as $Y_1 = ln(0.617339) * (2.248519) = -1.084545$. By using the Y_1 value into the Equation (22), the reliability index that corresponds to the t_{01} element is $R(t) = exp\{-exp\{-1.084545\}\} = 0.713156$.

Note 4. Observe that $R(t) = 0.713156$ is not the reliability of the design component; it only represents the reliability of the t_{0max} element in the Weibull analysis. The $R(t)$ of the element is that determined by using the Y_1 value that corresponds to the S_y value in Equation (22). It is located between rows 2 and 3 in Table 4.

The procedure to find the reliability of the design element by using the S_y value as σ_1 is as follows: From Equation (20), the t_{0Sy} element that corresponds to the S_y value is $t_{0Sy} = 301.455469/827 = 0.364517$ and from Equation (21) the corresponding Y_{Sy} value is $Y_{Sy} = ln(0.364517) * (2.248519) = -2.269166$. Thus, from Equation (22), the reliability index for the Y_{Sy} value is $R(t) = exp\{-exp\{-2.269166\}\} = 0.901768$. From the above it can be concluded that the reliability of the design element is $R(t) = 0.901768$.

Bold data highlight: (1) the $R(t)$ index that corresponds to the used $S_y = \sigma_1 = 827$ MPa value; (2) the reproduced principal stresses values and (3) the stress Weibull eta parameter.

Based on the fact that in the Weibull analysis the reliability of the element is given by either the applied stress or the corresponding life [14], then the t_{oi} elements are used to determine the corresponding random behavior of the expected cycles to failure. The analysis is as follows.

Table 4. Statistics of Weibull FEA stress analysis for data in Section 4.

n Equation (13)	Y_i Equation (15)	t_{oi} Equation (17)	$R(t_{oi})$ Equation (22)	σ_{2i} Equation (18)	σ_{1i} Equation (19)	σ_{med} Equation (25)	σ_{alt} Equation (26)	σ_{eq} (ASME) Equation (29)
1	−3.4034833	0.220104	0.96729	66.35155	1308.124	687.2377	620.8861	1116.157138
	−2.9701952	0.26688	0.95	80.45246	1078.849	579.6507	499.1982	699.8903536
2	−2.491662	0.330174	0.920561	99.53286	872.034	485.7834	386.2506	477.2687796
−2.2691664	**0.364517**	**0.901768**	**109.8856**	**827**	**468.4428**	**358.5572**	**435.0865005**	
3	−2.0034632	0.410239	0.873832	123.6689	701.842	412.7555	289.0865	333.6083957
4	−1.6616459	0.477593	0.827103	143.9731	602.8627	373.4179	229.4448	257.1517697
5	−1.3943983	0.537869	0.780374	162.1435	535.3039	348.7237	186.5802	205.7685332
−1.0845459	**0.617339**	**0.713156**	**184.8**	**491.75**	**338.275**	**153.475**	**168.1886402**	
6	−1.1720537	0.593774	0.733645	178.9966	484.9034	331.95	152.9534	166.9966579
7	−0.9793812	0.646898	0.686916	195.0109	445.0831	320.047	125.0361	135.6021558
8	−0.8074473	0.698303	0.640187	210.5073	412.3183	311.4128	100.9055	108.9229489
9	−0.6504921	0.748789	0.593458	225.7266	384.5185	305.1225	79.39597	85.4226406
10	−0.5045088	0.799016	0.546729	240.8679	360.3471	300.6075	59.73961	64.12598946
11	−0.3665129	0.84959	0.5	256.1134	338.8969	297.5051	41.39172	44.36161829
12	−0.2341223	0.901115	0.453271	271.6459	319.519	295.5825	23.93655	25.62949661
13	−0.1052851	0.954255	0.406542	287.6654	301.7256	294.6955	7.030101	7.524013293
0	**1**	**0.367879**	**301.4555**	**301.4555**	**301.4555**	**0**	**0**	
14	0.0219284	1.0098	0.359813	304.4098	285.129	294.7694	9.640397	10.31807533
15	0.14952577	1.068761	0.313084	322.1837	269.3992	295.7915	26.39226	28.26181481
16	0.279845	1.132534	0.266355	341.4085	254.2293	297.8189	43.58962	46.7245514
17	0.4159621	1.203211	0.219626	362.7145	239.2957	301.0051	61.70939	66.25374946
18	0.56250196	1.284238	0.172897	387.1405	224.1978	305.6691	81.47139	87.68037713
19	0.72761583	1.382091	0.126168	416.639	208.3243	312.4817	104.1573	112.4970518
20	0.92931067	1.511797	0.079439	455.7394	190.451	323.0952	132.6442	144.096238
21	1.22965981	1.727846	0.03271	520.8685	166.6371	343.7528	177.1157	194.7355493

4.2. Weibull Cycle to Failure (N) Analysis

From the selected material (AISI 4340 OQT 1300) $S_{ut} = 965$ MPa. By using it in Equation (23), $S'_e = 0.5(965 \text{ MPa}) = 482.5$ MPa. From Equation (24) the endurance stress limit is $S_e = 354.6$ MPa. From Equations (25) and (26), the mean and alternating stresses are $\sigma_m = \frac{(491.75+184.8)}{2} = 338.28$ MPa and $\sigma_a = \frac{(491.75−184.8)}{2} = 153.48$ MPa.

Now, by using the values of $S_y = 827$ MPa, $S_e = 354.6$ MPa, $\sigma_a = 153.5$ MPa and $\sigma_m = 338$ MPa in Equation (27), the ASME Elliptical failure theory criterion fatigue factor is $n_f = \sqrt{\frac{1}{\left(\frac{153.5}{354.6}\right)^2 + \left(\frac{338}{827}\right)^2}} = 1.68$. Therefore, since $n_f > 1$, the design element is considered as safe. Next, the endurance limit $S_e = 354.6$ MPa and the f (fatigue strength factor) $f = 0.8$ values are used in Equations (30) and (31), and the fatigue constant a and b values are obtained as $a = \frac{(0.8x965)^2}{354.6} = 1680.72194$ and $b = -\frac{1}{3}log\left(\frac{0.8x965}{354.6}\right) = -0.1125136$. Then, the corresponding reversed equivalent stress is determined by Equation (29), $\sigma_{eq} = \frac{153.48}{\sqrt{1-\left(\frac{338.28}{827}\right)^2}} = 168.2$ MPa. Finally, by using the values $a = 1680.72194$, $b = -0.1125136$ and $\sigma_{eq} = 168.2$ MPa in Basquin's equation, Equation (32), the number of cycles to failure is $N = \left(\frac{168.2}{1680.72194}\right)^{\frac{1}{-0.1125136}} = 767,615,910$ Cycles. Once the value of N is obtained, based on that the corresponding η_t parameter is determined as follows.

4.2.1. Weibull Cycle Parameters

By using the cycles value $N = 767,615,910$ and $t_{oeq} = 0.617339$ in Equation (33), the Weibull cycle η_t parameter is $\eta_t = \frac{767,615,910}{0.617339} = 1,243,427,849$ cycles. Therefore, the Weibull cycle to failure family used to determine the random behavior of N is W $(2.248519, 1,243,427,849 \, Cycles)$. The analysis is as follows.

4.2.2. Cycle Random Behavior

In this section the random behavior of the cycles (N) values is determined. By using the Weibull cycle η_t parameter and the t_{oi} elements in Equation (34), the corresponding expected cycle to failure values for each one of the Y_i elements are determined (see the seventh column in Table 5). Additionally, by using $t_{0sy} = 0.364517$ in Equation (34) the N_{sy} value that corresponds to the S_y value is $N_{sy} = 453,250,454$.

The practical interpretation of data in Table 5 is that data given in the σ_{2i} column represent the minimum expected material strength that will be required when the constant applied stress is $\sigma_{eq} = 168.188640$ (remember that this σ_{eq} is constant for all the rows in Table 5). For instance, in the first row of Table 5 by applying the σ_{eq} stress, the required material strength is 66.35155. Thus, from Equation (7) the expected reliability will be 0.96729 which corresponds to $N_i = 273,683,422.9$. Similarly, data of the σ_{1i} column represent the Weibull scale parameter that the element must present to withstand a constant stress value of $\eta_s = 301.4555$. For instance, in Equation (7) if we use $t = 301.4555$ with $\eta = 1308.124$ the reliability is 0.96729.

Therefore, from the FEA simulation of the flat spring with a yield strength value of $S_y = 827$ MPa we found the principal stresses that are acting on the element are $\sigma_1 = 491.75$ MPa and $\sigma_2 = 184.8$ MPa. And by applying the propose method as a complement of the FEA results we have that the expected cycle to failure of the flat spring is $N = 453,250,454$ cycles with a reliability index of $R(t) = 0.901768$.

On the other hand, although the analysis was performed base on the σ_1 and σ_2 FEA results, because the input of the proposed method is only the σ_1 and σ_2 values, and due to they can also be determined by the conventional static analytical method, then let us now present how the σ_1 and σ_2 values determined by the analytical method are used in the proposed method.

4.2.3. Weibull Stress Analytical Static Family

In order to use the analytical static stress method, the mechanical element presented in Section 4 is applied, with resulting values of $\sigma_1 = 470$ MPa and $\sigma_2 = 176$ MPa. Based on those principal stresses values, the Weibull stress analytical family is W $(2.240388, 287.610848$ MPa). Table 6 shows a summary of the comparison between the static method and the FEA simulation results.

From σ_1 and σ_2 values in Table 6, we conclude that both methods, the FEA and static methods, are equivalent. Additionally, note that for this particular case the fatigue life of the welded joint can be used to estimate the fatigue life of the flat spring mechanism [29]. This can be relevant due to the AWS D1.1 provides joint *S–N* curves whose allowable cycle number N is less than 10^7. The joint *S–N* curve for the welded joint category B is shown in Figure 7.

Table 5. Statistics of Weibull cycle to failure analysis.

n Equation (13)	Y_i Equation (15)	t_{oi} Equation (17)	$R(t_{oi})$ Equation (22)	σ_{2i} Equation (18)	σ_{1i} Equation (19)	*N* (Cycles) Equation (33)
1	−3.4034833	0.220104	0.96729	66.35155	1308.124	273,683,422.9
	−2.9701952	0.26688	0.95	80.45246	1078.849	331,846,104.7
2	−2.491662	0.330174	0.920561	99.53286	872.034	410,547,979.5
	−2.2691664	**0.364517**	**0.901768**	**109.8856**	**827**	**453,250,454.5**
3	−2.0034632	0.410239	0.873832	123.6689	701.842	510,103,112.7
4	−1.6616459	0.477593	0.827103	143.9731	602.8627	593,852,935.8
5	−1.3943983	0.537869	0.780374	162.1435	535.3039	668,801,050.2
	−1.0845459	**0.617339**	**0.713156**	**184.8**	**491.75**	**767,615,910**
6	−1.1720537	0.593774	0.733645	178.9966	484.9034	738,315,719.9
7	−0.9793812	0.646898	0.686916	195.0109	445.0831	804,370,622.7
8	−0.8074473	0.698303	0.640187	210.5073	412.3183	868,289,756.6
9	−0.6504921	0.748789	0.593458	225.7266	384.5185	931,065,175.7
10	−0.5045088	0.799016	0.546729	240.8679	360.3471	993,519,280.8
11	−0.3665129	0.84959	0.5	256.1134	338.8969	1,056,403,357
12	−0.2341223	0.901115	0.453271	271.6459	319.519	1,120,470,957
13	−0.1052851	0.954255	0.406542	287.6654	301.7256	1,186,547,445
	0	**1**	**0.367879**	**301.4555**	**301.4555**	**1,243,427,849**
14	0.0219284	1.0098	0.359813	304.4098	285.129	1,255,613,543
15	0.14952577	1.068761	0.313084	322.1837	269.3992	1,328,926,684
16	0.279845	1.132534	0.266355	341.4085	254.2293	1,408,224,098
17	0.4159621	1.203211	0.219626	362.7145	239.2957	1,496,105,985
18	0.56250196	1.284238	0.172897	387.1405	224.1978	1,596,857,176
19	0.72761583	1.382091	0.126168	416.639	208.3243	1,718,530,710
20	0.92931067	1.511797	0.079439	455.7394	190.451	1,879,810,288
21	1.22965981	1.727846	0.03271	520.8685	166.6371	2,148,451,296

Table 6. Summary of the comparison between the analytical static method and FEA simulation results.

Application	Maximum Principal Stress (MPa), Force F_1 = 4.63 N	Minimum Principal Stress (MPa), Force F_2 = 1.74 N	Weibull β Parameter	Cycles to Failure (*N*)	Reliability R(*t*)
Static	470	176	2.240388	691,910,584	0.91
FEA	491.75	184.8	2.248519	453,250,454	0.90

The relation between the stress range F_{SR} and the allowable cycle number *N* is given by Equation (36). Therefore, by applying Equation (36) in Equation (37) with $C_f = 120 \times 10^8$, the allowable cycles *N* are calculated. Table 7 shows a summary of the comparison between the static method and the FEA simulation fatigue life welded joint results.

As can be observed, the general conclusion is that the proposed method can be applied by using either the FEA or the static analytical stress approach. The efficiency of the proposed method depends on the accuracy on which the principal stresses are estimated. In this way the propose method can be applied in any mechanical analysis where σ_1 and σ_2 are known.
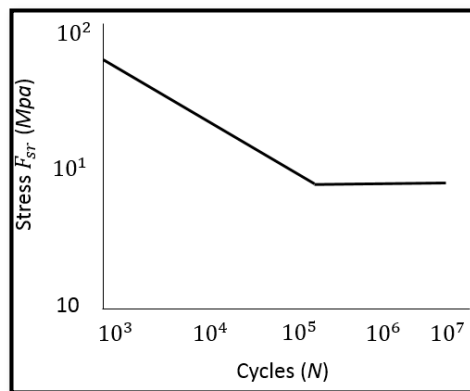
Figure 7. Allowable stress range for cyclically applied load (fatigue) for welded joint category B.

Table 7. Summary fatigue life welded joint *S-N* results.

Application	Allowable Stress Range $F_{SR\ (MPa)}$ Equation (37)	Cycles to Failure (N) Equation (36)	Reliability $R(t)$
Static	294	155,358	0.91
FEA	307	136,446	0.90

5. Conclusions

1. Because the input's method are the σ_1 and σ_2 values, then the proposed method can be applied in any mechanical analysis where σ_1 and σ_2 are known.

2. The efficiency of the proposed method is that the Weibull β (see Equations (8)) and η (see Equation (9)) only depends on the σ_1 and σ_2 values. Consequently, by performing the Weibull analysis we can always reproduce them (see row between rows 5 and 6 Table 4).

3. In the proposed method the mechanical element reliability can be determined by either the stress σ_{21} values (see column σ_{2i} in Table 5) or by the corresponding cycle to failure values.

4. The random behavior of both σ_{2i} and N_i values is determined based on the σ_{eq} and on the its corresponding N_{eq} value, here determined by Basquin's equation, but if they are determined in any other form the proposed method will be also efficient at determining the random behavior around them.

5. In the given application, the addressed σ_1 and σ_2 stress values, determined by the FEA and static stress analysis, were not significant, implying that the Weibull analysis given in Tables 4 and 5 can be used in both cases.

6. As demonstrated in [14], the standard fatigue methodologies based on the stress-strain analysis and on the mechanical fracture both converge to the same solution; it might then be possible to extend the proposed method to be used in the strain and mechanical fracture analysis by related the equivalent stress σ_{eq} to the total deformation given by the elastic (Basquin's equation) and plastic (Coffin–Mason equation) areas, although further research should be undertaken.

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