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ORIGINAL PAPER

Reliability analysis for DC motors under voltage step-stress scenario

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Abstract
In reliability analysis, different stress techniques are used to know the lifetime and performance of electrical devices via accelerated life testing. One of these stress technique is the step stress, which combines the traditional reliability testing and over-stress testing; with this method, it is easy to obtain the failure time in a short time. Nonetheless, the analysis of step-stress data can be difficult, and the specialist has usually have to trust on shortcuts or estimations to obtain reliability information from step-stress data. In this paper, a model based on Weibull distribution, inverse power law, cumulative damage model and step-tress technique is proposed to analyze the behavior of electronic devices under a voltage step-stress scenario. The parameters of the model were analyzed via a maximum likelihood. A case of study is based on DC motors is presented in this paper. The results obtained in this paper helped to design department in order to improve the lifetime and performance of the device under analysis.

Keywords Reliability · Step-stress · Weibull distribution · Inverse power law · DC motor

1 Introduction

Today electrical devices (ED) can be analyzed under reliability techniques in order to obtain the behavior when a stress is applied. Nevertheless, in reliability analysis exists different test plans to know the lifetime and performance of devices via accelerated life testing (ALT). Typically, ALT plans are based on constant stress; since most of the reliability models are formulated with this kind of stress, most of the applications used constant stress when it is full operation. Notwithstanding, an ALT for ED with a Weibull distribution behavior (WED), the test time and the number of pieces under experiment can be limited due to the manufacturing cost and just in time policy. In those situations, an ALT with constant stress for ED with WED cannot be a good choice due to the test consume considerable time and a large number of pieces to obtain data to feed reliability model and make the inference of performance. An alternative for this problem is to use an step-stress accelerated life testing (SSALT), and this type of ALT applies stress to devices in the way that stress level will be changed at a pre-specified time [1].

SSALT with WED has been studied by many researches in reliability. For example, Khamis [2] made a comparison between constant ALT and SSALT and shows the benefit of SSALT in reliability analysis. Nelson [3] proposed the bases for SSALT, method of estimation based on maximum likelihood estimation (MLE) and test plans for ED under WED and inverse power law (IPL). Miller and Nelson [4] present the optimum test plan for SSALT; the objective of this test plan is to minimize the asymptotic variance of ALT and the mean life at design stress induced by WED and MLE. The proposed studied is based on the cumulative exposure model (CEM). Meanwhile, Bai et al. [5] and Bai and Chun [6] extended the results of Miller and Nelson [4] and present a SSALT with a WED model with a closed form for censoring schemes. Further authors have proposed other methodologies based on SSALT and WED; for example, Zhao and Elsayed [7] present a general approach for SSALT based on the acceleration factors. Alhadeed and Yang [8] proposed an SSALT with Khamis–Higgins model, which is an alternative of SSALT Weibull model. This model provides formula a reasonable approximation to the actual optimal times of changing stress

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levels within a specific range of values of the stress levels and model parameters. Benavides [9] defines an SSALT via retaining the leading term from a series expansion of a general cumulative hazard function and WED. Other applications of SSALT and WED distribution in reliability can be found in Kateri and Balakrishnan [10], EL-Sagheer et al. [11], Hirose et al. [12], Yuan et al. [13], Tang et al. [14], Rackauskas et al. [15]. Li et al. [16], Ling [1], Samanta et al. [17] and Han [18].

Based on the background and literature review, in this paper, an SSALT analysis via cumulative damage model (CDM) is proposed. The goal of this paper to estimate the lifetime and performance of DC motors under a voltage step-stress scheme. The data obtained from the experiment were without censoring and assuming a Weibull distribution. The estimation of the parameters for the reliability model will be obtained via maximum likelihood estimation (MLE).

Finally, this paper is organized as follows. Section 2 presents a preliminary notation related to CDM. Section 3 presents the construction of the reliability model. Section 4 presents the likelihood function to calculate the parameters proposed in Sect. 3. Section 5 presents a practical case study based on the DC motors. Section 6 presents the discussion of the results and parameter interpretation. The last section provides concluding remarks and future work.

2 Preliminary notation

In step-stress testing, units are subjected to a stress level held constant for a specified period of time, at the end of which, if some units survive, the stress level is increased and held constant for another specified period. This process is continued until a predetermined number of units fail or until a predetermined test time is reached. This kind of test requires special reliability models to support the analysis. One of these models is the CDM, which according to Nelson [3] needs to follow these assumptions:

1. The life of the product under test depends only on the current accumulated fraction.
2. If the current stress is maintained in the test, the pieces that are under this stress will fail according to the CDF of that stress, but from the previously accumulated fraction.

Based on the assumptions of the model, let F be a function of a nonnegative random variable with a stress variable V and with V_{th} as threshold which denotes the maximum level of the stress in the piece; the distribution function D of a random variable T for the failure time and by denoting $V\imath$ the stress that the product is under test in an interval of time $(t_\imath, t]$, the distribution can be defined for the first step $t_0 \leq t \leq t_1$ as:

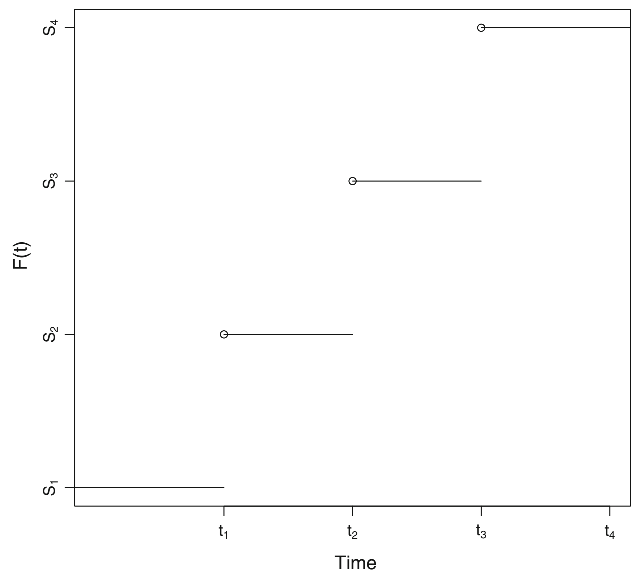

Fig. 1 Representation of reliability step-stress profile (SSP)

$$D(t) = \begin{cases} F(t - t_0; V_1) & (V_1 > V_{\mathrm{th}}) \\ 0 & (V_1 \leq V_{\mathrm{th}}) \end{cases} \tag{1}$$

For the next step, $t_1 \leq t \leq t_2$, from Eq. (1), can be expressed as:

$$D(t) = \begin{cases} F(t - t_1 + s_1; V_2) & (V_2 > V_{\mathrm{th}}) \\ F(s_1; V_2) & (V_2 \leq V_i) \end{cases} \tag{2}$$

Now, based on Komori [19], if S_1 is satisfying $G(t_1) = F(S_1; V_2)$ and the step of the stress in present time is higher than the step in past time, which means $V_\jmath \leq V_{\mathrm{th}}$ for any $\jmath < \imath$ when $V_\imath \leq V_{\mathrm{th}}$, it can be expressed as:

$$D(t) = \begin{cases} F(t - t_{\imath-1} + s_{\imath-1}; \imath) & (V_\imath > V_{\mathrm{th}}) \\ 0 & (V_\imath \leq V_{\mathrm{th}}) \end{cases} \tag{3}$$

Finally, the probability density function (PDF) of the model can be obtained as:

$$f_\imath(t, s_\imath) = \frac{\mathrm{d}}{\mathrm{d}t}[F_\imath, s_\imath] \tag{4}$$

A graphical representation of Eq. (3) is presented in Fig. 1. In the following section, we use the CDM described in Eq. (3), the IPL and WED in order to describe the behavior of DC motors under SSALT.

3 Reliability model

In reliability, to describe the behavior and performance of ED under a voltage profile scenario, the IPL model is used. The IPL model is written as follows:

$$\lambda = \frac{1}{kV^n} \tag{5}$$

where $k > 0$ is a characteristic parameter and depends on material properties, product design and other factors in the product under analysis. Parameter $n > 0$ measures the effect of the stress on the device's life. Parameter $V > 0$ represents the voltage stress level applied in the piece. But, for the CDM case Eq. (5) is written as:

$$\lambda = \left[\frac{\alpha}{x(t)}\right]^n \tag{6}$$

where $x(t)$ represents the equation or constant value which describes the SS profile in the ALT.

Now based on the WED, Eqs. (4) and (6), the PDF of the model step-stress Weibull-inverse power law (SSWIPL) is written as:

$$f(t, x(t)) = \left\{\beta\left[\frac{x(t)}{\alpha}\right]^n\left[\int_0^t\left[\frac{x(y)}{\alpha}\right]^n \mathrm{d}y\right]^{\beta-1}\right\}\cdot e^{-\left[\int_0^t\left[\frac{x(y)}{\alpha}\right]^n \mathrm{d}y\right]^\beta} \tag{7}$$

Based on (7), the CDF ($F(x(t))$, the survival ($S(x(t))$, hazard ($H(x(t))$ and quantile ($Q(u)$) functions are given by:

$$F(t, x(t)) = 1 - e^{-\left[\int_0^t\left[\frac{x(y)}{\alpha}\right]^n \mathrm{d}y\right]^\beta} \tag{8}$$

$$S(t, x(t)) = e^{-\left[\int_0^t\left[\frac{x(y)}{a}\right]^n \mathrm{d}y\right]^\beta} \tag{9}$$

$$H(t, x(t)) = \left\{\beta\left[\frac{x(t)}{\alpha}\right]^n\left[\int_0^t\left[\frac{x(y)}{\alpha}\right]^n \mathrm{d}y\right]^{\beta-1}\right\} \tag{10}$$

$$Q(\rho) = F^{-1}(\rho) = \frac{1}{\beta}\cdot\ln(1-\rho)\,\alpha^n \quad 0 \le \rho \le 1 \tag{11}$$

The equations from Eq. (7) to Eq. (11) describe the lifetime and performance of any ED on SSALT with WED and voltage step-stress scenario.

For this case, the meaning of the parameters of the model presented in 7 to 11 is as follows. The parameter α measures the damage of the terminal resistance in the DC motor. On the other hand, the parameter n measures the effects of the voltage into the DC motor. A high value of parameter n means that the saturation voltage in the DC motor increases the internal temperature. Finally, parameter β represents a shape parameter and marked effects on the performance of the lifetime and failure time distribution of DC motor.

4 Parameter estimation of the model

Based on Eq. (7), let t_ι, $\iota = 1, 2, \ldots, \omega$, the failure time of the piece induced by the voltage stress level $x(t)$, and thus, the ln-likelihood function is defined as:

$$\begin{aligned}\Lambda = {} & \omega\,[ln(\beta) - n\cdot ln(\alpha)] + n\sum_{\iota=1}^{\omega}\ln(x(t_i)) \\ & + \left[(\beta-1)\sum_{\iota=1}^{\omega}\ln\left(\int_0^{t_i}\left[\frac{x_i(u)}{\alpha}\right]^n \mathrm{d}u\right)\right] \\ & - \sum_{\iota=1}^{\omega}\left[\int_0^{t_\iota}\left(\frac{x_\iota(u)}{\alpha}\right)^n \mathrm{d}u\right]^\beta\end{aligned} \tag{12}$$

To estimate parameters β,n and α via MLE , we calculated the first partial derivative,

$$\begin{aligned}\frac{\partial\Lambda}{\partial\beta} = {} & \frac{\omega}{\beta} + \sum_{\iota=1}^{\omega} ln\left[\int_0^{t\iota}\left(\frac{x_\iota(u)}{\alpha}\right)^n \mathrm{d}u\right] \\ & - \sum_{\iota=1}^{\omega}\left[\int_0^{t\iota}\left[\left(\frac{x_\iota(u)}{\alpha}\right)^n \mathrm{d}u\right]^\beta \cdot ln\int_0^{t_\iota}\left(\frac{x_\iota(u)}{\alpha}\right)^n \mathrm{d}u\right]\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial\Lambda}{\partial n} = {} & -\omega\cdot ln(\alpha) + \sum_{\iota=1}^{\omega} ln(x_\iota(t_\iota)) \\ & + \left\{\frac{(\beta-1)\left[(n+1)\cdot\left(ln(\alpha) - \sum_{\iota=1}^{\omega} ln(x_\iota(t_\iota)) + 1\right)\right]}{n+1}\right\} \\ & - \frac{\beta\cdot\alpha^{-\beta n}\cdot\sum_{\iota=1}^{\omega}(x_\iota(t_\iota))^{\beta n+1}\cdot\left[(\beta n+1)\cdot\sum_{\iota=1}^{\omega} ln(x_\iota(t_\iota)) - 1\right]}{(\beta n+1)^2}\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial\Lambda}{\partial\alpha} = {} & -\frac{n\omega}{\alpha} + \frac{(\beta-1)n}{\alpha} \\ & + \frac{\beta\cdot n\cdot\alpha^{-\beta n}\cdot\sum_{\iota=1}^{\omega}\left[(x_\iota t_\iota)^{\beta n+1}\right]}{\alpha\cdot(\beta n+1)}\end{aligned} \tag{15}$$

The fisher information matrix (FIM) is given by:

$$J(\delta) = -\begin{bmatrix} O_{\beta\beta} & O_{\beta n} & O_{\beta\alpha} \\ O_{n\beta} & O_{nn} & O_{n\alpha} \\ O_{\alpha\beta} & O_{\alpha n} & O_{\alpha\alpha}\end{bmatrix} \tag{16}$$

where the elements of the matrix can be obtained from second partial derivatives from Eq. (13) to Eq. (15), and in "Appendix A" can be found all elements of FIM.

On the other hand, to obtain the confidence bounds we based on the asymptotic normality of MLE's. The $100(1-\rho)\,\%$ confidence bounds for β, $n$and α are given by:

$$\left(\hat\beta \pm z_{1-\rho/2}\sqrt{\operatorname{var}(\hat\beta)}\right) \tag{17}$$

$$\left(\hat n \pm z_{1-\rho/2}\sqrt{\operatorname{var}(\hat n)}\right) \tag{18}$$

Table 1 Technical specs of DC motor under reliability analysis

Values at nominal voltage		Characteristics	
Nominal voltage	5 V	Terminal resistance	0.282 Ω
No load speed	5900 rpm	Terminal inductance	0.0390 mH
No load current	81.5 mA	Torque constant	8.5 mNm/A
Nominal speed	5000 rpm	Speed constant	1250 rpm/V
Nominal torque (max. continuous torque)	12.1 mA	Speed/torque gradient	42.6 rpm/mNm
Nominal Current (max continuous current)	2.1 A	Mechanical constant	5.24 ms
Stall torque	160 mNm	Rotor inertia	12.1 gcm^2

$$\left(\hat{\alpha} \pm z_{1-\rho/2}\sqrt{\operatorname{var}\left(\hat{\alpha}\right)}\right) \tag{19}$$

where $z_{1-\rho/2}$ is the upper $(\rho/2)$ percentile of the standard normal distribution. And $\operatorname{var}(\hat{\beta})$, $\operatorname{var}(\hat{n})$ and $\operatorname{var}(\hat{\alpha})$ can be found on the main diagonal of the FIM.

5 Case study

In this section, the model established in Eq. (7) is used to estimate the lifetime of DC motors via SSALT. The SSALT was performed with the following parameters considerations:

- 16 DC motors were under SSALT analysis.
- The technical specifications of the DC motor are described in Table 1.
- The setup of the SSALT was defined and is given in Table 2. This test plan was based on the methodology proposed by Nelson [20]. This table represents the $x(t)$ presented in Eq. 7. For this case, $x(t)$ is an integer value (stress level) which is a constant value
- All units were run to failure, where failure is said to have occurred when the internal resistance drifts more than 30%.
- No censored data were obtained in this experiment.
- The ambient temperature of the experiment was set to 22 °C.

Once the stress profile has been defined for the accelerated life test, the lifetime of the DC motors is acquired. For this propose, it was necessary to build a device for acquiring the failure times of the product under analysis, and a block diagram of the acquisition process is shown in Fig. 2.

On another hand, the SSALT was conducted as follows; all 16 DC motors started the test under the first step established in Table 2 if any of the DC motors achieved a loss of 30% of the internal resistance; the device is removed from the analysis; and its time are taken. The survival DC motors in the analysis follow the next step and so on until each motor reaches the specification established in the considerations.

Table 2 Setup of SSALT for DC motors

Stress level	Exposed time (in h)
15	$0 < t \leq 250$
18	$250 < t \leq 550$
21	$550 < t \leq 790$
24	$790 < t \leq 910$
27	$910 < t \leq 1120$
30	$1120 < t \leq +\infty$

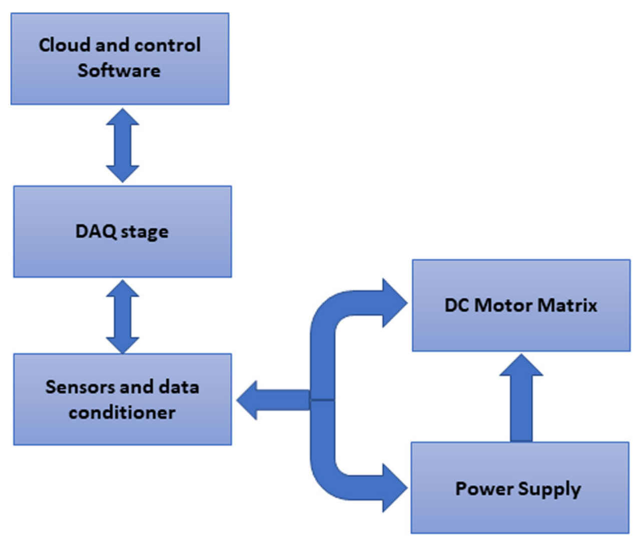

Fig. 2 Representation of reliability SSP for the SSALT

The SSP for the ALT performed by Table 1 is shown in Fig. 3.

By performing the SSALT via the SSP defined in Table 2, the failure time of the DC motors was as follows:

5.1 Estimation of SSWIPL

To estimate the parameters α, β and n established in Eq. (7), the MLE equation was programmed in R by using the MaxLik package. The results of the estimation obtained from the data obtained in Table 3 are presented in Table 4.

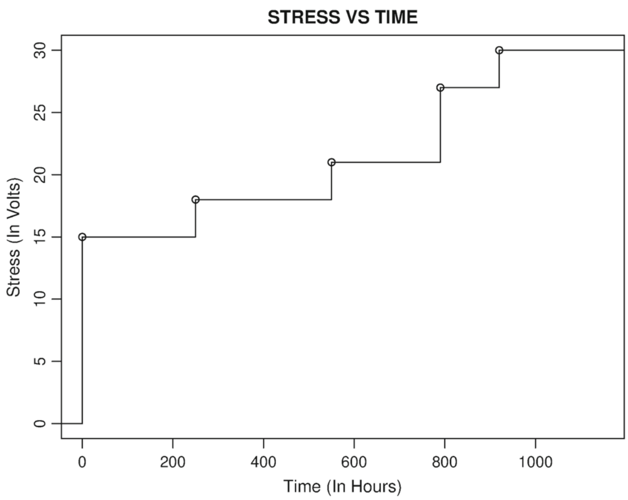

Fig. 3 Representation of SSALT data acquisition process

Table 4 Results of estimations obtained via MLE of data provided in Table 2

Parameter	Estimation
β	1.319
α	442.801
n	2.406

Derived from the results obtained in Table 4, reliability graphs which define the behavior and performance of DC motors under SSALT can be drawn by substituting the estimations in Eqs. (7), (9) and (10); results are shown in Fig. 4.

Graphs presented in Fig. 4 characterize the behavior of the DC motor under step stress via a voltage acceleration. Figure 4a shows the Weibull performance of DC motors under the time. Figure 4b shows the probability of DC motors operating for a certain amount of time without failure, dots in Fig. 4b represent the DC motors submitted in SSALT and their probabilities to survive. Figure 4c represents the performance of the failure rate of each DC motor under SSALT, and the failure rate displays a monotonic rate, which is one of the principal assumptions for models under SSALT.
The results of fisher matrix presented in Eq. 16 are:

$$J(\delta) = \begin{vmatrix} 0.248 & 649.756 & -1.914 \\ 796.969 & 197.346 & -384.142 \\ -1.214 & -372.623 & 1.031 \end{vmatrix} \quad (20)$$

The confidence bounds of the parameters obtained in Table 4 can be calculated from the diagonal matrix presented in Eq. 16 and the equations presented from Eq. (17) to Eq. (19). The results are presented in Table 5 and consider a 95% confidence interval.

6 Discussion

Based on the results obtained in Sect. 5, some conclusions can be obtained. The information provided by Table 4 and drawing in Fig. 4 showed that the probability of failure (see Fig. 4c) in the DC motor increases after 2, 300 h approximately and remains growing until 17, 100 h; after 17, 100 h, the probability of failure remains constant; this is because the wear of the components has reached its limit and the part has reached its maximum useful life. This statement can be explained through the parameters α,n and β.

In practical case, parameter α depends straight on the quality and the fabrication process of the materials that the DC motor was built. A high value of α represents that the internal components suffer a more accelerated wear stage when they enter the final parts of their useful life. Moreover, a high value of parameter n means that the saturation voltage in the DC motor increases the internal temperature when the device is under operation. And finally, parameter β marks the behavior of the DC motor throughout its lifetime, a way to measure the effect of parameter β is calculating the mean

Table 3 Failure time of DC motors under SSALT

Piece number	Stress level failure (In Volts)	Time To failure (In Hours)	Piece number	Stress level failure (In Volts)	Time To failure (In Hours)
1	15	140	9	18	501
2	15	193	10	18	510
3	15	231	11	21	562
4	18	371	12	21	600
5	18	381	13	21	751
6	18	392	14	27	810
7	18	422	15	27	925
8	18	463	16	30	1155

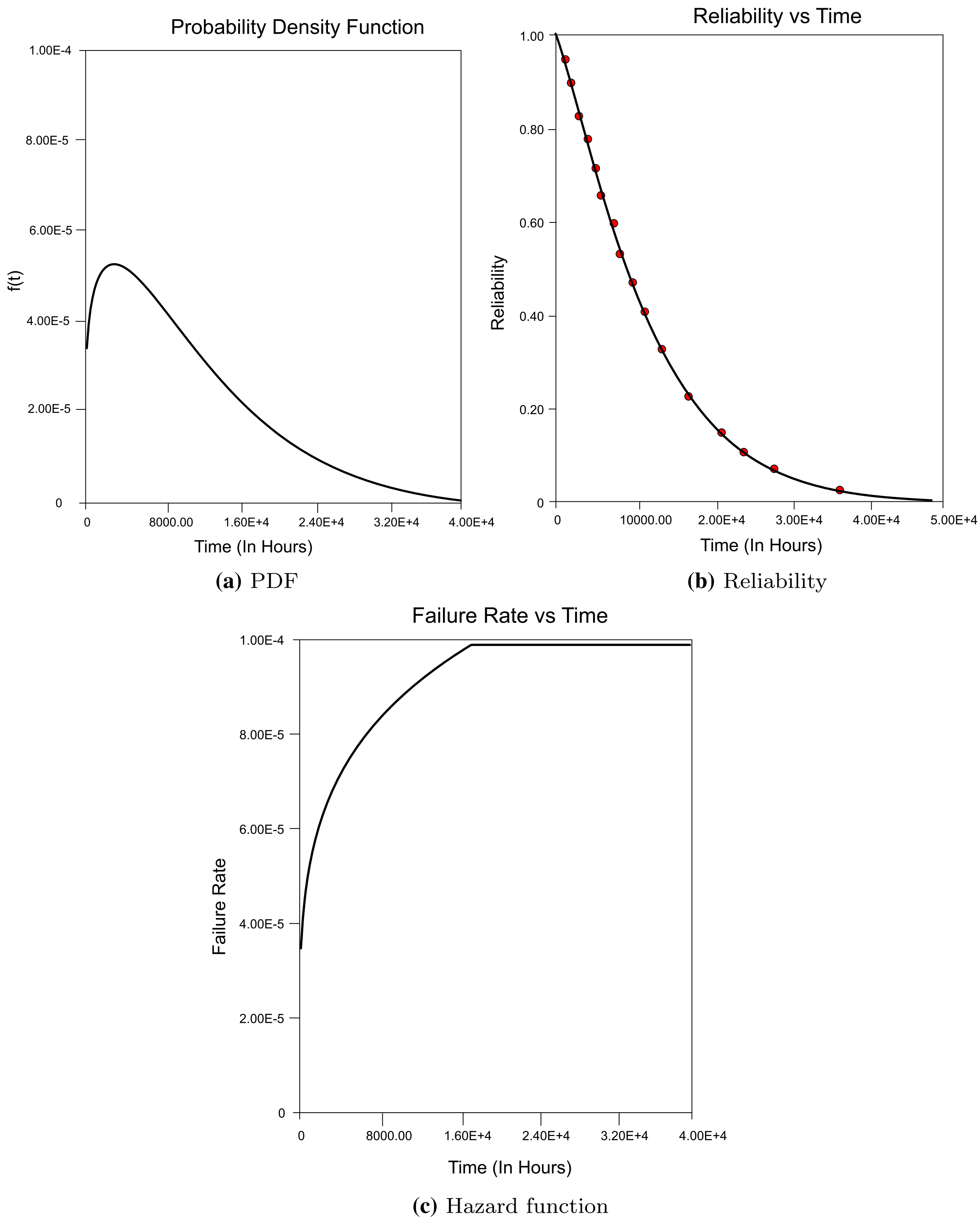

Fig. 4 Reliability graphs obtained from the data presented in Table 4

time to failure (MTTF), which describes the expected time a device will work before failing. The MTTF of DC motor under SSWIPL can be calculated as follows:

$$\text{MTTF} = \left[\frac{\alpha}{x(t)}\right]^n \cdot \Gamma\left(\frac{1}{\beta} + 1\right) \tag{21}$$

Table 5 Confidence bounds for model parameters presented in Eq. 7

Parameter	LCB	Estimation	HCB
β	0.238	1.214	2.191
α	418.571	446.106	473.641
n	0.117	2.108	4.098

Thus, by substituting the values obtained in Table 4 in Eq. (21), and by setting the operating voltage of DC motor at 5 Volts, the MTTF = 12,561 h. This value is vital for the manufacturer because the MTTF can be determinate the warranty time offer to the final user of the DC motor.

7 Conclusion and future work

The presented paper shows a reliability model based on SSALT which analyze the performance of ED under a WED and CDM. Reliability models based on SSALT and CDM can be useful to get more quality information such as the behavior of internal components and how these wear out. On the other hand, reliability models under SSALT and CDM can reduce the experimentation time and simplify the statistical analysis.

The practical case presented in Sect. 5 shows the behavior of a DC motor under an SSALT. For this case of study, an MLE was used to know the values of parameters established in Eq. (7). With these parameters, reliability graphs shown in Fig. 4 can be used for the quality department in order to increase the reliability of the product when a specific conditions in the product reaches.

The proposed model can be used for any application in reliability. A future work proposed for this model is to analyze the effects of the time-varying voltage or combine the SSALT model with some other distribution for another class of electrical or electronic devices. Also, it can be possible to use this model to know the effects of other variables of stress such as temperature, vibration and humidity or combine more of two variables of stress in the same SSALT.

A Observed fisher matrix elements

$$
\begin{aligned}
O_{\beta\beta} &= -\frac{\omega}{\beta^2} + \sum_{\imath=1}^{\omega}\left[ln\int_0^{t_\imath}\left(\frac{x_\imath(u)}{\alpha}\right)^n \mathrm{d}u \cdot \left\{\frac{n\cdot\sum_{\imath=1}^{\omega}\left(\frac{x_\imath(t_\imath)}{\alpha}\right)^{\beta n}\cdot\left[(\beta n+1)\cdot\sum_{\imath=1}^{\omega}\left[\ln\left(\frac{x_\imath(t_\imath)}{\alpha}\right)-1\right]\right]}{(\beta n+1)^2}\right\}\right] \\
O_{\beta n} &= \frac{n-1}{n+1}\cdot\sum_{\imath=1}^{\omega}\ln\left(\frac{x_\imath(t_\imath)}{\alpha}\right) - \frac{\alpha^{-\beta n}\left[ln(\alpha)-\sum_{\imath=1}^{\omega}\left[\ln\left(x_\imath(t_\imath)\right)\right]^{\beta n+2}\cdot\left(\beta n\left(\beta n+1\right)\cdot\left\{ln(\alpha)-\sum_{\imath=1}^{\omega}\left(x_\imath(t_\imath)-1\right)\right)\right\}\right]}{(\beta n+1)^2} \\
O_{\beta\alpha} &= -\frac{n}{\alpha} + \frac{n\cdot\sum_{\imath=1}^{\omega}\left[x_\imath(t_\imath)\cdot\left(\frac{x_\imath(t_\imath)}{\alpha}\right)^{\beta n}\cdot\beta\sum_{\imath=1}^{\omega}\left[ln\left\{\frac{\left(\frac{x_\imath(t_\imath)}{\alpha}\right)^{\beta}}{n+1}\right\}+1\right]\right]}{\alpha(\beta n+1)} \\
O_{n\beta} &= O_{\beta n} \\
O_{nn} &= \frac{(\beta-1)\cdot\sum_{\imath=1}^{\omega}\left[x_\imath(t_\imath)+1\right]}{(n+1)^2} - \frac{\beta^2\cdot\sum_{\imath=1}^{\omega}\left[x_\imath(t_\imath)\cdot\ln\left(x_\imath(t_\imath)-1\right)\cdot\left(\frac{x_\imath(t_\imath)}{\alpha}\right)^{-\beta n}\cdot\left\{(\beta n+1)\cdot\sum_{\imath=1}^{\omega}\left[\left(\frac{x_\imath(t_\imath)}{\alpha}\right)-1\right]\right\}\right]}{(\beta n+1)^2} \\
O_{n\alpha} &= \frac{\beta-\omega-1}{\alpha} + \frac{\beta^2 n\cdot\sum_{\imath=1}^{\omega}\left[x_\imath(t_\imath)\cdot\ln\left(x_\imath(t_\imath)-1\right)\cdot\left(\frac{\alpha}{x_\imath(t_\imath)}\right)^{-\beta n}\right]}{\alpha(\beta n+1)} \\
O_{\alpha\beta} &= O_{\beta\alpha} \\
O_{\alpha n} &= O_{n\alpha} \\
O_{\alpha\alpha} &= \frac{\beta(n-1)+n\omega}{\alpha^2} - \beta n(\alpha^{-\beta n-2})\cdot\sum_{\imath=1}^{\omega}\left[(x_\imath(t_\imath))^{\beta n+1}\right]
\end{aligned}
$$

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